# Geometric Scaling for the Total $\gamma^* p$ Cross Section in the Low $x$ Region

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(Received 18 July 2000)

We observe that the saturation model of deep inelastic scattering predicts a geometric scaling of the total $\gamma^* p$ cross section in the region of small Bjorken variable $x$. The geometric scaling in this case means that the cross section is a function of only one dimensionless variable $\tau = Q^2 R_0^2(x)$, where the function $R_0(x)$ decreases with decreasing $x$. We show that the experimental data from HERA in the region $x < 0.01$ confirm the expectations of this scaling over a very broad region of $Q^2$. We suggest that the geometric scaling is more general than the saturation model.

DOI: 10.1103/PhysRevLett.86.596 PACS numbers: 13.60.Hb

It has recently been observed that the $ep$ deep inelastic scattering (DIS) data at low $x$ [1,2] can be very economically described with the help of the saturation model [3]. In this model the QCD dipole picture for the total $\gamma^* p$ cross sections was adopted [4–6],

$$\sigma_{T,L}(x,Q^2) = \int d^2\mathbf{r} \int_0^1 dz\, |\Psi_{T,L}(r,z,Q^2)|^2 \hat{\sigma}(r,x)\,, \tag{1}$$

where $\Psi_{T,L}$ is the wave function for splitting of the transverse ($T$) or longitudinal ($L$) polarized virtual photon into a $q\bar{q}$ pair (dipole) and $\hat{\sigma}$ is the dipole cross section, which describes the interaction of the dipole with the proton. In addition, $\mathbf{r}$ is the transverse separation of the quarks in the $q\bar{q}$ pair, and $z$ is the light-cone momentum fraction of the photon carried by the quark (or antiquark). As usual, $-Q^2$ is the photon virtuality and $x$ is the Bjorken variable.

Let us recall that the standard DIS proton structure functions are related to $\sigma_{T,L}$ by

$$F_{T,L}(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\, \sigma_{T,L}(x,Q^2)\,, \tag{2}$$

and $F_2 = F_T + F_L$. The wave function of the virtual photon is given by the following equations:

$$|\Psi_T|^2 = \frac{3\alpha_{em}}{2\pi^2} \sum_f e_f^2 \{[z^2 + (1-z)^2]\overline{Q}_f^2 K_1^2(\overline{Q}_f r) + m_f^2 K_0^2(\overline{Q}_f r)\}\,, \tag{3}$$

$$|\Psi_L|^2 = \frac{3\alpha_{em}}{2\pi^2} \sum_f e_f^2 \{4Q^2 z^2 (1-z)^2 K_0^2(\overline{Q}_f r)\}\,,$$

where the sum is performed over quarks with flavor $f$, charge $e_f$, and mass $m_f$, and $\overline{Q}_f^2 = z(1-z)Q^2 + m_f^2$. The functions $K_{0,1}$ are the Bessel-McDonald functions.

The main assumption of the saturation model concerns the saturation property of the dipole cross section which is incorporated in the approach of Ref. [3] as below:

$$\hat{\sigma}(x,r) = \sigma_0 g\left(\frac{r}{R_0(x)}\right). \tag{4}$$

The function $R_0(x)$ with the dimension of length, called saturation radius, decreases with decreasing $x$, while the normalization $\sigma_0$ is independent of $x$. When $\hat{r} \equiv r/R_0(x) \to \infty$ the function $g(\hat{r})$ saturates to 1, so that $\hat{\sigma}(x,r) \to \sigma_0$. In the realization [3] of the saturation model $g(\hat{r}) = 1 - \exp(-\hat{r}^2/4)$. The fact that the dipole cross section (4) is limited by the energy independent cross section $\sigma_0$ may be regarded as a unitarity bound. This reflects the fact that the strong rise of DIS structure functions as $x \to 0$ has to be tamed by unitarization effects [7–19].

The characteristic feature of Eq. (4) is its "geometric scaling"; i.e., $\hat{\sigma}(x,r)$ depends only on the dimensionless ratio $r/R_0(x)$, and its energy dependence is entirely driven by the saturation radius $R_0(x)$. The scaling property of (4) with $\sigma_0$ independent of $x$ resembles geometric scaling of hadron-hadron scattering [20]. In this case the relevant quantity is the scattering amplitude $G(b^2,s)$, where $b$ is the impact parameter and $s$ is the center-of-mass energy squared. The geometric scaling in this case corresponds to the assumption that

$$G(b^2,s) \to G(\beta)\,, \tag{5}$$

where $\beta = b^2/R^2(s)$ with $R(s)$ corresponding to the interaction radius which increases with increasing energy. The analogy between scaling exhibited in hadron-hadron collisions and in deep inelastic scattering should not be taken too literally. For example, the two radii have different physical interpretations, and, moreover, they show completely different energy dependence since the saturation radius $R_0(x)$ decreases with increasing energy (for $x \simeq Q^2/s \to 0$).

The assumption about the scaling property of the dipole cross section (4) has profound consequences for the measured $\gamma^* p$ cross section $\sigma_{\gamma^* p} = \sigma_T + \sigma_L$. If we neglect the quark masses $m_f$ in the photon wave functions (3) we can rescale the dipole size $r \to r/R_0(x)$ in Eq. (1) such that the integration variables are dimensionless. Thus, after

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the integration $\sigma_{\gamma^* p}$ becomes a function of only one dimensionless variable $\tau = Q^2 R_0^2(x)$,

$$\sigma_{\gamma^* p}(x, Q^2) = \sigma_{\gamma^* p}(\tau). \tag{6}$$

The nonzero light quark mass does not lead to a significant breaking of the scaling (6). Following the discussion in [3] it is easy to show that we smoothly change the behavior of (6),

$$\sigma_{\gamma^* p} \sim \sigma_0 \rightarrow \sigma_{\gamma^* p} \sim \sigma_0/\tau \tag{7}$$

(modulo logarithmic modifications in $\tau$), when $\tau$ changes from small to large values, respectively. The aim of this paper is to demonstrate that the DIS data do indeed approximately exhibit the geometric scaling (6) with the property (7).

In Ref. [3] the saturation radius form was postulated in the form $R_0(x) = 1/Q_0\,(x/x_0)^{\lambda/2}$, where $Q_0 = 1$ GeV, and the parameters $x_0$, $\lambda$, and $\sigma_0$ were determined from a fit to DIS data at small $x$. For a recent related analyses see [21] and also [22]. $R_0(x)$ can also be determined in a less model-dependent way. Let us observe that after suitable extension of the saturation model to the low $Q^2$ region including the photoproduction limit $Q^2 = 0$, the $x$ dependence of the saturation radius $R_0(x)$ can be correlated with the energy dependence of the total photoproduction cross section $\sigma_{\gamma p}$. To do this we replace, following Ref. [3], the argument in $R_0(x)$ by

$$\bar{x} = x\left(1 + \frac{4m_f^2}{Q^2}\right) = \frac{Q^2 + 4m_f^2}{W^2} \tag{8}$$

and keep $m_f \neq 0$. $W$ is the total energy of the $\gamma^* p$ system. We note that the saturation model based on Eqs. (1)–(4) can now be extended down to the region $Q^2 = 0$. The photoproduction cross section is given by Eqs. (1) and (3) with $Q^2 = 0$, $\bar{Q}_f^2 = m_f^2$ and with $x$ replaced by $\bar{x} = 4m_f^2/W^2$. The dominant contribution to the photoproduction cross section comes from the integration region $1/m_f^2 \gg r^2 \gg R_0^2(x)$ in the corresponding integral on the right-hand side in Eq. (1). In this region we can set $m_f^2 K_1^2(m_f r) \simeq 1/r^2$ and $\hat{\sigma}(x, r) \simeq \sigma_0$. This gives the following relation between photoproduction cross section and the saturation radius:

$$\sigma_{\gamma p}(W) = \bar{\sigma}_0 \ln\left(\frac{1}{m_f^2 R_0^2(\bar{x})}\right). \tag{9}$$

The parameter $\bar{\sigma}_0$ is related to the overall normalization of the dipole cross section $\sigma_0$ by $\bar{\sigma}_0 = (2\alpha_{em}/3\pi)\sigma_0$. From Eq. (9) we finally obtain the following prescription for the saturation radius:

$$R_0^2(\bar{x}) = \frac{1}{m_f^2} \exp\left(-\frac{\sigma_{\gamma p}}{\bar{\sigma}_0}\right). \tag{10}$$

For $\sigma_{\gamma p}$ we take the Donnachie-Landshoff parametrization [23]

$$\sigma_{\gamma p} = a\bar{x}^{-0.08}, \tag{11}$$

where we set $m_f = 140$ MeV (following [3]) in Eqs. (8) and (10). Using results of the fit presented in [23] we find $a = 68\ \mu b(4m_f^2/1\ \mathrm{GeV}^2)^{0.08}$. For $\bar{\sigma}_0$ we set 23 $\mu b$ to obtain a good description of data.

Let us now confront the implications of geometric scaling (6) with experimental data on deep inelastic scattering at low $x$. In Fig. 1 we show experimental data [1] on the total cross section $\sigma_{\gamma^* p}$ plotted versus scaling variable $\tau = Q^2 R_0^2(x)$, with $R_0(x)$ obtained from Eq. (10). We include all available data for $x < 0.01$ in the range of $Q^2$ values between 0.045 and 450 GeV$^2$. We see that the data exhibit geometric scaling over a very broad region of $Q^2$. We can also clearly see the change of shape of the dependence of $\sigma_{\gamma^* p}$ on $\tau$ from the approximate $1/\tau$ dependence at large $\tau$ to the less steep dependence at small $\tau$. The asymptotic $1/\tau$ dependence reflects the fact that the cross section $\sigma_{\gamma^* p}$ scales as $1/Q^2$ (modulo logarithmic corrections) and its energy dependence is governed by $1/R_0^2(x)$. Less steep dependence corresponds to the fact that at small values of $\tau$ the total cross section grows weaker with energy than $1/R_0^2(x)$ due to saturation of the dipole cross section; see Eq. (4). We also found a symmetry between the regions of large and small $\tau$ for the function $\sqrt{\tau}\,\sigma_{\gamma^* p}$, which is illustrated in Fig. 2. For the asymptotic values of $\tau$ this is a manifestation of the relations (7). It is remarkable that Fig. 2 seems to indicate the presence of symmetry of $\sqrt{\tau}\,\sigma_{\gamma^* p}$ with respect to the transformation $\tau \leftrightarrow 1/\tau$ in the whole region of $\tau$.

We have also tried the power law parametrization for the radius, $R_0^2(x) \sim x^{\lambda}$, where $0.3 < \lambda < 0.4$, in particular the original form proposed in [3], and found that the data also exhibit the geometric scaling with this choice of parametrization. The approximate $1/\tau$ dependence at large $\tau$ corresponds to the $x^{-\lambda}$ behavior of the proton structure

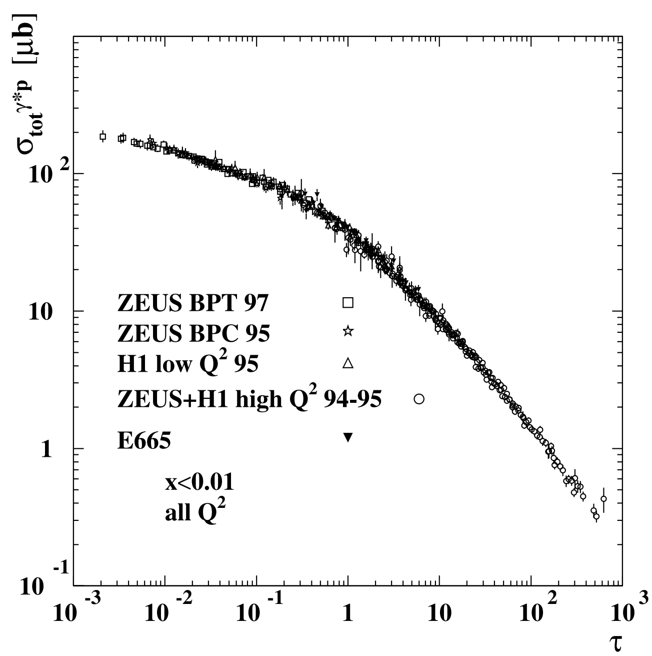


FIG. 1. Experimental data on $\sigma_{\gamma^* p}$ from the region $x < 0.01$ plotted versus the scaling variable $\tau = Q^2 R_0^2(x)$.

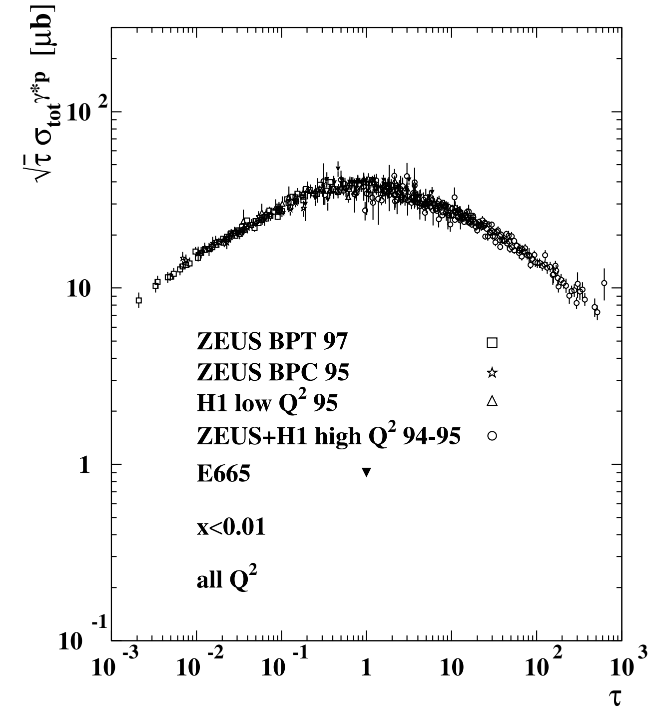


FIG. 2. $\sqrt{\tau}\,\sigma_{\gamma^* p}$ plotted versus the scaling variable $\tau$.

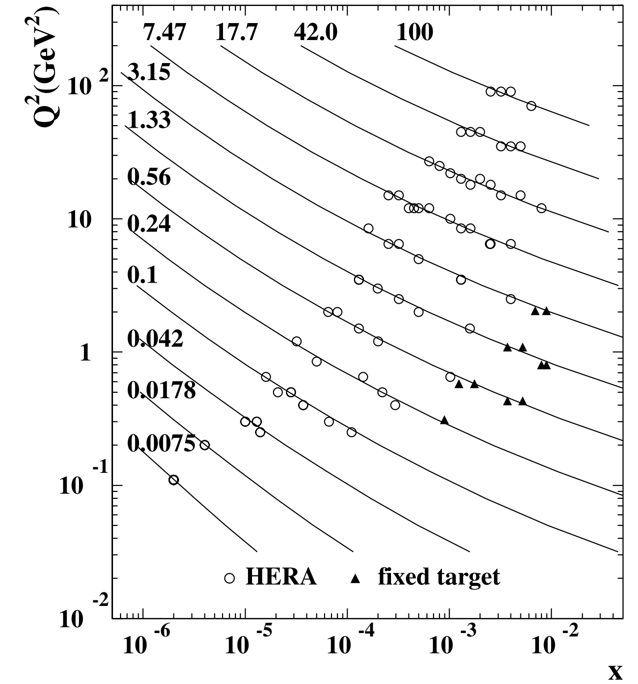


FIG. 3. The lines corresponding to different values of scaling variable $\tau$ (continuous curves) in the $(x, Q^2)$ plane. The points correspond to available experimental data located within the bins $\ln(\tau) \pm \delta$ ($\delta = 0.1$) for each value of $\tau$. The numbers correspond to the value of $\tau$ for each curve.

function $F_2$ at large $Q^2$. In the photoproduction case, the power law parametrization of $R_0(x)$ combined with relation (9) would correspond to the logarithmic dependence on energy of the photoproduction cross section, i.e.,

$$\sigma_{\gamma p} \sim \ln(\bar{x})\,. \tag{12}$$

We do therefore find that both prescriptions for $\sigma_{\gamma p}$, Eqs. (11) and (12), give numerically similar results.

In Fig. 3 we show contours corresponding to different values of variable $\tau$ in $(x, Q^2)$ plane together with experimental points for these values of $\tau$. Geometric scaling means that $\sigma_{\gamma^* p}$ is constant along each line. To be precise for each value of $\tau$ we plot experimental points within the bin $[\ln(\tau) \pm \delta]$ with $\delta = 0.1$. We see from this figure that for each $\tau$ there are several experimental points for which $x$ varies as much as 2 orders of magnitude and $Q^2$ changes by a factor of 4. Despite that, all points along each line in Fig. 3 are transformed to a narrow spread of points for a particular value of $\tau$ in Figs. 1 and 2, thus exhibiting geometric scaling.

In order to show that the geometric scaling is confined to the small $x$ region we plot $\sigma_{\gamma^* p}$ in Fig. 4 as a function of the scaling variable $\tau$ for the experimental data with $x > 0.01$ [1,2]. It is evident that the scaling is significantly violated for large $x$ values. We emphasize that the geometric scaling should predominantly be regarded as a remarkable regularity of DIS experimental data at low $x$. In its essence the new scaling is a manifestation of the presence of an internal scale characterizing dense partonic systems, $Q_s(x) \sim 1/R_0(x)$. This scale emerges from a pioneering work of [7], which was generalized in [8–19]. In the analyses [11,18], the scaling properties similar to those postulated in (4) were found. An independent formulation [12] of the small $x$ processes leads to the same overall picture with the saturation scale. At a deeper level, the

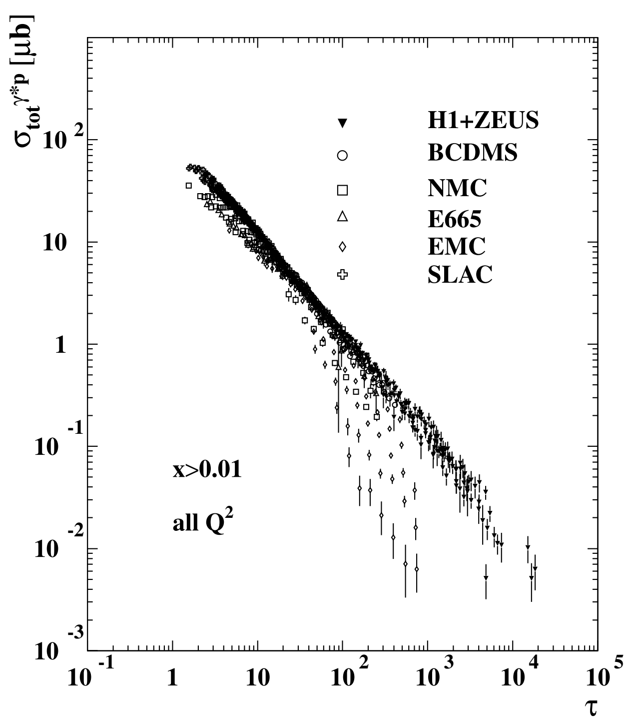


FIG. 4. Experimental data on $\sigma_{\gamma^* p}$ from the region $x > 0.01$ plotted versus the scaling variable $\tau = Q^2 R_0^2(x)$.

geometric scaling for small-$x$ processes may reflect self-similarity or conformal symmetry of the underlying dynamics. More detailed studies are under way; see [16–19].

To sum up we have shown that the experimental data on deep inelastic $ep$ scattering at low $x$ exhibit geometric scaling, i.e., the total cross section $\sigma_{\gamma^* p}(x, Q^2)$ is the function of only one dimensionless variable $\tau = Q^2 R_0^2(x)$. This regularity was found to hold over the very broad range of $Q^2$. It would be interesting to understand in detail a possible dynamical origin of this simple regularity.

We thank J. Bartels, G. Levin, and L. McLerran for useful comments. This research has been partially supported by the EU Framework TMR programme, Contract No. FMRX-CT98-0194, Deutsche Forschungsgemeinschaft, and the Polish Commitee for Scientific Research Grants No. KBN 2P03B 120 19 and No. 2P03B 051 19.

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