Dependence of the Fractional Quantum Hall Edge Critical Exponent on the Range of Interaction

Recent experiments on external electron tunneling into an edge of a fractional quantum Hall (FQH) system find a striking current-voltage power law behavior $I \propto V^{\alpha}$, with exponent $\alpha \approx 2.7$ on the $f = \frac{1}{3}$ FQH plateau [1,2]. Such power law I-V characteristic with $\alpha = 3$ was predicted for electron tunneling into an edge channel at the boundary of the $f = \frac{1}{3}$ FQH system: the low-energy dynamics is effectively 1D, and field-theoretic descriptions of edge channels as chiral Luttinger liquids have been developed [3]. The equality $\alpha = 2i + 1$ for a FQH state at $f = \frac{1}{2i+1}$ ($i = 1, 2, 3, \ldots$) has been demonstrated theoretically in the disk geometry for the Laughlin wave function Ψ_L, which is known to be the exact ground state for certain short-range interactions. The experiments consistently obtain values of $\alpha < 2i + 1$, however [2].

Here we report results of a large numerical study of the microscopic structure of the FQH edge at $f = \frac{1}{3}$. To this end, we diagonalize the interaction Hamiltonian in the disk geometry for up to $N = 12$ spin-polarized electrons restricted to lowest Landau level. We construct numerically [4] the Laughlin Ψ_L and "Coulomb" Ψ_C wave functions as the ground states of the short-range and Coulomb Hamiltonians, respectively, for the total angular momentum $M = \frac{3}{2}N(N - 1)$, which gives filling $f = \frac{1}{3}$ in the thermodynamic limit $N \to \infty$. In particular, we have obtained occupation numbers $\rho(m)$ of the angular momentum basis orbitals

$$\psi_m(r, \theta) = (2\pi 2^m m!)^{-1/2} r^m \exp(im\theta - r^2/4), \quad (1)$$

where radius r is in units of magnetic length $\ell = \sqrt{\hbar/eB}$. The Hilbert space is restricted by consideration of orbitals with angular momentum $m \leq m_{\max}$ only. For the Haldane V_1 short-range interaction, the Laughlin $\rho_L(m)$ with $m > m^L_{\max} = 3(N - 1)$ vanish identically; for Coulomb interaction good convergence of $\rho_C(m)$ is obtained by $m_{\max} = m^L_{\max} + 5$ [5]. For example, for $N = 12$, the largest $f = \frac{1}{3}$ FQH system studied, $M = 198$ and the size of the Hilbert space is 15 293 119 for $m_{\max} = 35$. Details of this study will be published elsewhere [6].

As has been demonstrated by Wen [3], the critical exponent α is equal to the ratio of the occupation numbers

$$\alpha = \rho(m^L_{\max} - 1)/\rho(m^L_{\max}) \quad (2)$$

for the Laughlin state Ψ_L on the disk. Wen has also argued that this relationship must hold for any interaction, so long as the FQH state at the same filling f exists, and is not unique for the Laughlin wave function. In Fig. 1 we present the ratio of the occupation numbers for both Ψ_L (short-range interaction) and Ψ_C (true Coulomb interaction) for $N = 3$ to 12. As expected, we obtain $\alpha_L = 3$ to machine accuracy for Ψ_L. For Coulomb-interacting electrons, the ratio is always less than 3, and an extrapolation to the thermodynamic limit gives $\alpha_C = 2.62$. While we do not know whether the extrapolation shown in Fig. 1 holds for $N > 12$, certain other systematic behavior present in the numerical data [6] allows us to project that in the $N \to \infty$ limit $2.58 \leq \alpha_C \leq 2.75$, and is definitely less than 3.

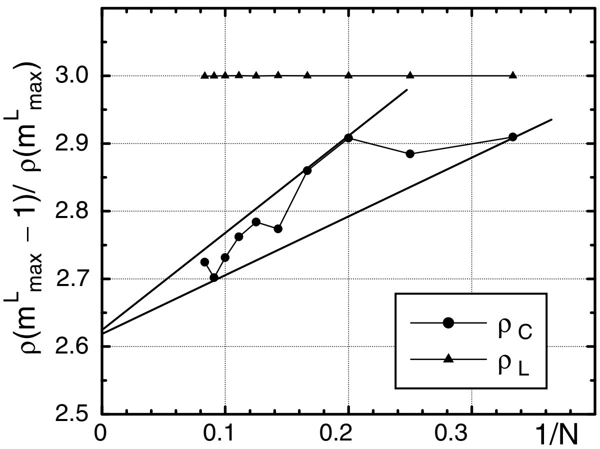

FIG. 1. The ratio of the angular momentum occupation numbers $\rho(m)$ for N interacting electrons on the disk. Shown are the Laughlin state ρ_L for a short-range interaction, and the exact ground state for the Coulomb interaction ρ_C.

Thus we propose that the deviation of the experimental α from the predicted α_L values is not an artifact and is not due to corrections such as finite bulk diagonal conductivity, disorder, or a variation of the electron density in the sample. Rather, we propose, the effect has a fundamental origin: the value of the critical exponent is not universal but depends on the range of particular interaction.

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[5] The sum of $\rho_C(m)$ with $m > m^L_{\max}$ is less than $0.01/N$. This means that fixing total angular momentum $M = \frac{3}{2}N(N - 1)$ selects the $f = \frac{1}{3}$ state for Coulomb interaction, too: it fixes *average* density on the disk $\langle\rho_C\rangle \approx \langle\rho_L\rangle$ (for $r < r_{\rm edge}$) to better than 10^{-3} for any N.
[6] E. V. Tsiper and V. J. Goldman (to be published).

