Neutron Star Structure and the Neutron Radius of ^{208}Pb

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We study relationships between the neutron-rich skin of a heavy nucleus and the properties of neutron-star crusts. Relativistic effective field theories with a thicker neutron skin in ^{208}Pb have a larger electron fraction and a lower liquid-to-solid transition density for neutron-rich matter. These properties are determined by the density dependence of the symmetry energy which we vary by adding nonlinear couplings between isoscalar and isovector mesons. An accurate measurement of the neutron radius in ^{208}Pb—via parity violating electron scattering—may have important implications for the structure of the crust of neutron stars.

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It is an extrapolation of 18 orders of magnitude from the neutron radius of a heavy nucleus—such as ^{208}Pb with $R_n \approx 5.5$ fm—to the approximately 10 km radius of a neutron star. Yet both radii depend on our incomplete knowledge of the equation of state of neutron-rich matter. Therefore, an accurate measurement of the neutron radius in ^{208}Pb may have important implications for some neutron-star properties.

Heavy nuclei are expected to have a neutron-rich skin. This important feature of nuclear structure arises because of the large neutron excess and because the Coulomb barrier reduces the proton density at the surface. The thickness of the neutron skin depends on the pressure of neutron-rich matter: the greater the pressure, the thicker the skin as neutrons are pushed out against surface tension. The same pressure supports a neutron star against gravity [1]. Thus models with thicker neutron skins often produce neutron stars with larger radii.

Neutron stars are expected to have a solid crust of nonuniform neutron-rich matter above a liquid mantle. The phase transition from solid to liquid depends on the properties of neutron-rich matter. Indeed, a high pressure implies a rapid rise of the energy with density making it energetically unfavorable to separate uniform matter into regions of high and low densities. Thus a high pressure typically implies a low transition density from a solid crust to a liquid mantle. This suggests an inverse relationship: the thicker the neutron-rich skin of a heavy nucleus, the thinner the solid crust of a neutron star.

In this Letter we study possible "data-to-data" relations between the neutron-rich skin of a heavy nucleus and the crust of a neutron star. These relations may impact neutron-star observables. Indeed, properties of the crust are important for models of glitches in the rotational period of pulsars [2,3], for the shape and gravitational radiation of nonspherical rotating stars [4] and for neutron-star cooling [5]. Note that the skin of a heavy nucleus and the crust of a neutron star are composed of the same material: neutron-rich matter at similar densities.

The parity radius experiment (PREX) at the Jefferson Laboratory aims to measure the neutron radius in ^{208}Pb via parity violating electron scattering [6,7]. Parity violation is sensitive to the neutron density because the Z^0 boson couples primarily to neutrons. The result of this purely electroweak experiment could be both accurate and model independent. In contrast, all previous measurements of bulk neutron densities used hadronic probes that suffer from controversial uncertainties in the reaction mechanism (see, for example, Ref. [8]). PREX should provide a unique observational constraint on the thickness of the neutron skin in a heavy nucleus. In this Letter we explore some of the implications of this measurement on the structure of neutron stars.

Microscopic calculations of the energy of neutron matter constrain both the neutron skin in ^{208}Pb and the crust of a neutron star; see, for example, Ref. [9]. However, these calculations of infinite neutron matter are not directly tested by observable properties of finite nuclei such as their charge densities or binding energies. Moreover, nonrelativistic calculations of symmetric nuclear matter have not succeeded in predicting the saturation density. It thus becomes necessary to fit some properties of a three-body force in order to reproduce nuclear saturation. Indeed, the properties of $A = 8$ pure neutron drops calculated in Ref. [10] may depend on the three-nucleon force used. Thus we feel that it is important to distinguish direct finite-nucleus measurements—such as PREX—from theoretical neutron-matter "observables" based solely on calculations. Indeed, PREX may provide an important test of these calculations [11].

We start with a relativistic effective field theory [12] that provides a simple description of finite nuclei and a Lorentz covariant extrapolation for the equation of state of dense neutron-rich matter. The theory has an isoscalar-scalar ϕ

(sigma) meson field and three vector fields: an isoscalar V (omega), an isovector b (rho), and the photon A. We now supplement the Lagrangian with new nonlinear sigma-rho and omega-rho couplings. These couplings allow us to change the density dependence of the symmetry energy which changes both the thickness of the neutron skin in ^{208}Pb and the neutron-star crust.

The interacting Lagrangian density is given by [12]

$$\mathcal{L}_{\text{int}} = \bar{\psi}\left[g_s\phi - \left(g_v V_\mu + \frac{g_\rho}{2}\boldsymbol{\tau}\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi - \frac{\kappa}{3!}(g_s\phi)^3 - \frac{\lambda}{4!}(g_s\phi)^4 + \frac{\zeta}{4!}g_v^4(V_\mu V^\mu)^2 + \frac{\xi}{4!}g_\rho^4(\mathbf{b}_\mu\cdot\mathbf{b}^\mu)^2 + g_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu[\Lambda_4 g_s^2\phi^2 + \Lambda_v g_v^2 V_\mu V^\mu]. \quad (1)$$

We consider a number of different parameter sets. First, we note that the nonlinear rho coupling ξ will modify the density dependence of the rho mean field, and this could change the neutron-skin thickness. However, unless ξ is made very large, this term was found to have a small effect [12]. Thus for simplicity, we set $\xi \equiv 0$ in all our parameter sets. Second, note that we could have added a cubic sigma-rho interaction of the form $\mathcal{L}_3 = M\Lambda_3(g_s\phi)(g_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu)$. A nonzero Λ_3 does change the thickness of the neutron skin in ^{208}Pb—but at the expense of a change in the proton density. Therefore, we set $\Lambda_3 \equiv 0$ and focus exclusively on Λ_4 and Λ_v.

We start with the original NL3 parameter set of Lalazissis, König, and Ring [13]. (Note that a small adjustment of the $NN\rho$ coupling constant was needed to fit the symmetry energy of nuclear matter at a Fermi momentum of $k_F = 1.15$ fm^{-1}; see text below.) The NL3 set has $\zeta = \Lambda_4 = \Lambda_v = 0$ and provides a good fit to the ground-state properties of many nuclei. In this model symmetric nuclear matter saturates at $k_F = 1.30$ fm^{-1} with a binding energy per nucleon of $E/A = -16.25$ MeV and an incompressibility of $K = 271$ MeV. All other parameter sets considered here have been fixed to the same saturation properties.

We now add the new nonlinear couplings Λ_4 and/or Λ_v between the isoscalar and the isovector mesons. At the same time we adjust the strength of the $NN\rho$ coupling constant (g_ρ) from its NL3 value to maintain the symmetry energy of nuclear matter unchanged (see text below). Note that neither Λ_4 nor Λ_v affect the properties of symmetric nuclear matter since $\mathbf{b}_\mu \equiv 0$. Hence, the saturation properties remain unchanged. Our goal is to change the neutron density and the neutron-skin thickness in ^{208}Pb while making very small changes to the proton density which is well constrained by the measured charge density [14]. The two new couplings (Λ_4 and Λ_v) change the skin thickness in ^{208}Pb by similar amounts. Yet they have different high-density limits [12]. For $\Lambda_v = 0$ the symmetry energy is proportional to the baryon density ρ in the limit of very high density, while it only grows like $\rho^{1/3}$ for nonzero Λ_v. This can produce noticeable changes in neutron star radii.

Let us start with $\Lambda_4 = 0$. For a given omega-rho coupling Λ_v we readjust only the $NN\rho$ coupling constant g_ρ in order to keep an average symmetry energy fixed. The symmetry energy at saturation density is not well constrained by the binding energy of nuclei. However, some average of the symmetry energy at full density and the surface energy is constrained by binding energies. As a simple approximation we evaluate the symmetry energy not at full density $k_F \approx 1.30$ fm^{-1} but rather at an average density corresponding to $k_F = 1.15$ fm^{-1}. Thus, all our parameter sets have a symmetry energy of 25.68 MeV at $k_F = 1.15$ fm^{-1}. Note that the original NL3 parameter set predicts a symmetry energy of 37.4 MeV at full saturation density and close to 25.68 MeV at $k_F = 1.15$ fm^{-1} [13]. This simple procedure produces a nearly constant binding energy per nucleon for ^{208}Pb as Λ_v is changed, as can be seen in Table I. Moreover, Table I shows that increasing Λ_v reduces the neutron-skin thickness significantly—while maintaining the proton radius nearly constant. In the following we plot our results for a range of Λ_v values for which the proton radius is within 0.01 fm of its $\Lambda_v = 0$ value.

To study the solid crust of a neutron star we make a simple random-phase-approximation calculation of the transition density below which uniform neutron-rich matter becomes unstable against small amplitude density fluctuations. This provides a lower bound to the true transition

TABLE I. Results for the NL3 parameter set. B.E. is the binding energy per particle in ^{208}Pb, R_p is the proton, and $R_n - R_p$ is the difference between neutron and proton radii in Pb. (Note that we do not include center-of-mass corrections.) Finally, ρ_c is our estimate for the transition density of neutron-rich matter from a nonuniform to a uniform phase.

Λ_v	g_ρ^2	B.E. (MeV)	R_p (fm)	$R_n - R_p$ (fm)	ρ_c (fm^{-3})
0.0	79.6	7.85	5.460	0.280	0.052
0.005	84.9	7.86	5.461	0.265	0.056
0.01	90.9	7.87	5.462	0.251	0.061
0.015	97.9	7.88	5.463	0.237	0.067
0.02	106.0	7.88	5.466	0.223	0.075
0.025	115.6	7.89	5.469	0.209	0.081

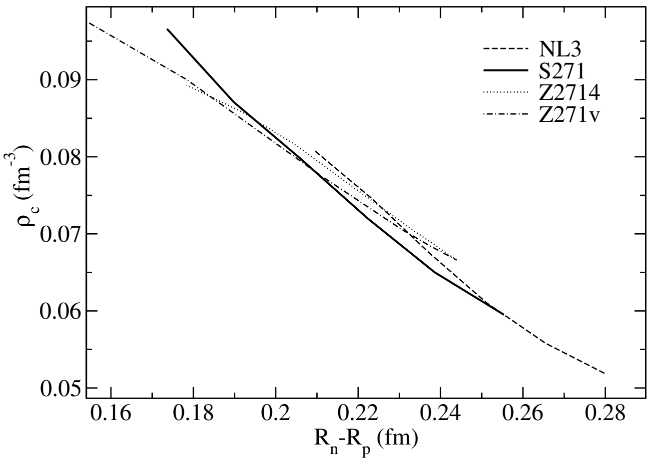

FIG. 1. Estimate of the transition density from nonuniform to uniform neutron-rich matter versus neutron-minus-proton radius in ^{208}Pb. The curves are for the four parameter sets described in the text.

density [15]. We start with the longitudinal dielectric function ϵ_L, as defined in Eq. (68) of Ref. [16], evaluated at an energy transfer $q_0 = 0$ and at an arbitrary momentum transfer q. That is,

$$\epsilon_L(q_0 = 0, q) = \det(1 - D_L \Pi_L). \quad (2)$$

Here Π_L is a longitudinal polarization matrix describing particle-hole excitations of a uniform system of protons, neutrons, and electrons in beta equilibrium, as given in Eq. (56) of Ref. [16]. The matrix D_L, describing meson and photon propagation, follows from Eq. (57) of Ref. [16]—but includes additional terms to account for the nonlinear nature of the meson self-couplings [17]. We estimate the transition density ρ_c by computing the largest density at which $\epsilon_L(0, q) < 0$ for any given q.

In Fig. 1 we display the transition density for various parameter sets (see Table II) as a function of the predicted difference in the root-mean-square neutron and proton radii $R_n - R_p$ in ^{208}Pb. The curves are parametrized by different values of Λ_v, as shown in Table I. The NL3 parameter set saturates nuclear matter with a relatively small value of the nucleon effective mass: $M^* \equiv M - g_s\phi = 0.59M$. The parameter set S271 saturates nuclear matter as NL3 but with $M^* = 0.70M$. This set also has $\zeta = 0$. The two remaining curves in the figure are for parameter sets having $\zeta = 0.06$ and both saturate nuclear matter with $M^* = 0.80M$. (Set Z271v has a nonzero Λ_v, while set Z2714 uses a nonzero Λ_4.) Note that the scalar mass m_s for parameter sets S271, Z271v, and Z2714 is adjusted to reproduce the proton radius in ^{208}Pb as computed with NL3. Figure 1 displays a clear inverse correlation between the transition density and the neutron-skin thickness $R_n - R_p$. The transition density expressed in fm^{-3} is about

$$\rho_c \approx 0.16 - 0.39(R_n - R_p), \quad (3)$$

with the skin thickness expressed in fm. Moreover, this correlation seems to be insensitive to M^* or to using Λ_4 or Λ_v to change $R_n - R_p$. These results suggest that a measurement of the neutron radius in ^{208}Pb will provide considerable information on the transition density.

Note that Fig. 1 shows only our results. Yet all other calculations that we are aware of also give consistent results. For example, the nonrelativistic microscopic equation of state of Friedman and Pandharipande has a transition density of $\rho_c = 0.096$ fm^{-3} according to Lorenz *et al.* [9]. For this equation of state Brown finds $R_n - R_p = 0.16 \pm 0.02$ fm [11]. These numbers are in excellent agreement with Eq. (3).

Brown also finds a linear relation between $R_n - R_p$ and the derivative of the energy of neutron matter versus density $dE/d\rho$ evaluated at $\rho = 0.1$ fm^{-3} [11]. He considers a large variety of nonrelativistic Skyrme interactions. Our results for $dE/d\rho$ versus $R_n - R_p$ are completely consistent. We expect these common $dE/d\rho$ values to give similar ρ_c values consistent with Eq. (3) for these Skyrme interactions. This is because $dE/d\rho$ is related to the pressure while ρ_c depends on the density dependence of the pressure.

Finally, for the relativistic interaction TM1 of Sugahara and Toki [18], we calculate from Eq. (2) $\rho_c \approx 0.059$ fm^{-3} and $R_n - R_p = 0.27$ fm. The numbers are again in good agreement with Eq. (3).

In Fig. 2 we show the electron fraction per baryon Y_e versus density for uniform neutron-rich matter in beta equilibrium. We include results only for the S271 parameter set as all other sets yield similar results. The different curves are for different values of Λ_v which predict the indicated $R_n - R_p$ values. The curves start near the transition densities displayed in Fig. 1. The electron fraction Y_e is determined by the symmetry energy while $R_n - R_p$ is sensitive to the density dependence of the symmetry energy. Therefore a measurement of $R_n - R_p$ constrains the growth of Y_e with density. If $R_n - R_p$ is greater than about 0.24 fm, Y_e becomes large enough to allow the direct URCA process, of neutron followed by proton beta decays [19], to cool down a 1.4 solar mass neutron star.

TABLE II. Model parameters used in the calculations. The parameter κ and the scalar (m_s) and vector (m_v) masses are given in MeV. The nucleon and rho masses are kept fixed at $M = 939$ and $m_\rho = 763$ MeV, respectively.

Model	g_s^2	g_v^2	κ	λ	ζ	m_s	m_v
NL3	104.387	165.585	3.860	−0.015 91	0	508.194	782.5
S271	81.103	116.766	6.68	−0.015 78	0	505	783
Z271	49.440	70.669	6.17	0.156 34	0.06	465	783

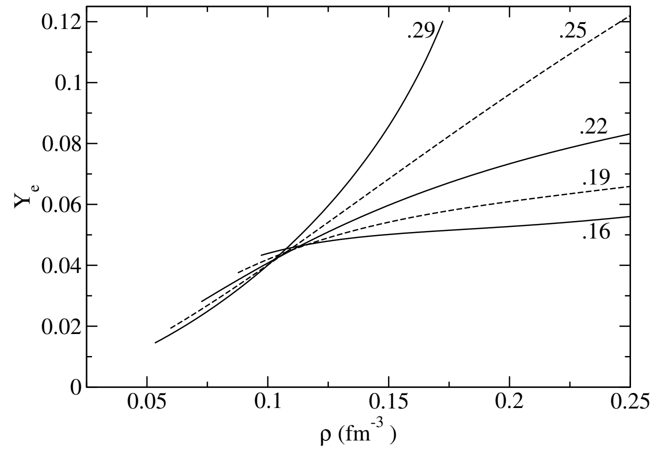

FIG. 2. Electron fraction Y_e versus baryon density for uniform neutron-rich matter in beta equilibrium using the S271 parameter set (other sets yield similar results). The curves are for different values of Λ_v that predict the indicated values of $R_n - R_p$ for ^{208}Pb, in fm.

The relationship between the radius of a 1.4 solar mass neutron star and $R_n - R_p$ will be discussed in future work.

We conclude the following:

(1) It is possible to fit nuclear observables—such as charge densities, binding energies, and single particle spectra—with effective field theories that predict a range of neutron-skin thicknesses. This can be done by adding nonlinear couplings between isoscalar and isovector meson fields or, in general, by adding interactions that modify the density dependence of the symmetry energy. We conclude that the neutron-skin thickness is not tightly constrained by these observables. Yet a measurement of the skin thickness will constrain the density dependence of the symmetry energy.

(2) The density dependence of the symmetry energy is adjustable in our relativistic effective field models while still reproducing nuclear-matter properties and other ground-state observables. Indeed, our models can provide a Lorentz-covariant extrapolation for the high-density equation of state with a symmetry energy that rises slower with density relative to earlier relativistic mean-field models.

(3) The electron fraction Y_e of neutron-rich matter in beta equilibrium is correlated with the neutron-skin thickness in ^{208}Pb. The thicker the neutron skin, the faster Y_e rises with density. In our models a neutron-skin thickness of the order of 0.24 fm or larger suggests that Y_e will become large enough to allow a direct URCA process to cool down a $1.4 M_\odot$ neutron star.

(4) We have found an inverse correlation between the neutron-skin thickness and the density of a phase transition from nonuniform to uniform neutron-rich matter. In our models the transition density obeys the approximate relation: $\rho_c \approx 0.16 - 0.39(R_n - R_p)$, with $R_n - R_p$ in fm and ρ_c in fm^{-3}. This suggests that a neutron skin measurement in ^{208}Pb can provide important information on the thickness and other properties of the crust of a neutron star.

(5) Microscopic calculations of the energy of neutron matter constrain the density dependence of the symmetry energy and hence the neutron-skin thickness in ^{208}Pb. Therefore a neutron-skin measurement may provide an important observational check on such calculations that is not provided by other observables. Moreover, a neutron-skin measurement may constrain three-body forces in neutron-rich matter.

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[1] J. M. Lattimer and M. Prakash, Astrophys. J. **550**, 426 (2001).
[2] D. Pines and M. A. Alpar, Nature (London) **316**, 27 (1985).
[3] B. Link, R. I. Epstein, and J. M. Lattimer, Phys. Rev. Lett. **83**, 3362 (1999).
[4] G. Ushomirsky, L. Bildsten, and C. Cutler, Los Alamos Archive, astro-ph/0001129; L. Bildsten, Astrophys. J. **501**, L89 (1998).
[5] See, for example, A. D. Kaminker *et al.,* Astron. Astrophys. **343**, 1009 (1999).
[6] Jefferson Laboratory Experiment E-00-003, Spokespersons R. Michaels, P. A. Souder, and G. M. Urciuoli.
[7] C. J. Horowitz, S. J. Pollock, P. A. Souder, and R. Michaels, Phys. Rev. C **63**, 025501 (2001).
[8] L. Ray, G. W. Hoffmann, and W. R. Coker, Phys. Rep. **212**, 223 (1992).
[9] B. Friedman and V. R. Pandharipande, Nucl. Phys. **A361**, 502 (1981); C. P. Lorenz, D. G. Ravenhall, and C. J. Pethick, Phys. Rev. Lett. **70**, 379 (1993).
[10] B. S. Pudliner *et al.,* Phys. Rev. Lett. **76**, 2416 (1996).
[11] B. Alex Brown, Phys. Rev. Lett. **85**, 5296 (2000).
[12] H. Müller and B. D. Serot, Nucl. Phys. **A606**, 508 (1996).
[13] G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C **55**, 540 (1997).
[14] B. Frois *et al.,* Phys. Rev. Lett. **38**, 152 (1977).
[15] C. J. Pethick, D. G. Ravenhall, and C. P. Lorenz, Nucl. Phys. **A584**, 675 (1995); F. Douchin and P. Haensel, Phys. Lett. B **485**, 107 (2000).
[16] C. J. Horowitz and K. Wehrberger, Nucl. Phys. **A531**, 665 (1991).
[17] C. J. Horowitz and J. Piekarewicz (to be published).
[18] Y. Sugahara and H. Toki, Nucl. Phys. **A579**, 557 (1994).
[19] C. J. Pethick and V. Thorsson, in *Lives of the Neutron Stars,* edited by A. Alpar *et al.,* NATO Advanced Study Institutes, Ser. C, Vol. 450 (Kluwer, Dordrecht, 1994).