Comment on "Theory of Diluted Magnetic Semiconductor Ferromagnetism"

In a recent Letter [1], a theory of carrier-induced ferromagnetism in diluted magnetic semiconductors (DMS) is proposed. By using their self-consistent spin-wave (SCSW) approximation, the authors show a nonmonotonic dependence of critical temperatures T_c on the free-carrier density in agreement with experiment. Here we emphasize that their SCSW theory is *a priori* unjustified and will lead to inaccurate results at low temperatures and near T_c. Thus we suggest another SCSW approximation to remedy these flaws.

By taking the Ising limit for the exchange coupling between magnetic ions and itinerant carriers, such that the spin-wave spectrum Ω_p is independent of momentum $\vec{p}$, one can obtain an expression for the thermal average of the impurity-spin density [1]

$$\langle S^z \rangle = \frac{1}{V} \sum_{|\vec{p}|<p_c} \{S - n(\Omega_p) + (2S + 1) \times n[(2S + 1)\Omega_p]\}, \tag{1}$$

where $n(\omega)$ is the Bose function and p_c is a Debye cutoff. ($p_c^3 = 6\pi^2 c$, c is the magnetic ion density.) The SCSW approximation used in Ref. [1] consists of extending the above formula (which is derived under the Ising limit) to the isotropic case simply by substituting the Ω_p in the isotropic case (now Ω_p is $\vec{p}$ dependent) into Eq. (1). Thus their theory can be considered phenomenological, and its validity is not guaranteed. For example, as mentioned in Ref. [1], when $T \rightarrow 0$, Eq. (1) does not yield the correct prefactor of the characteristic $T^{3/2}$ law [2]. Moreover, near T_c, where both $\langle S^z \rangle$ and the free-carrier spin density n^* approach zero, one can show that Eq. (1) leads to the following expression for T_c:

$$k_B T_c = \frac{S(S + 1)}{3} \lim_{\langle S^z \rangle, n^* \rightarrow 0} \frac{1}{V} \sum_{|\vec{p}|<p_c} \frac{\Omega_p}{\langle S^z \rangle}. \tag{2}$$

Notice that, although the low-energy spin-wave excitations do exist in the present system, this expression predicts a nonzero T_c, even for the one-dimensional (1D) and two-dimensional (2D) cases. This is incompatible with the Mermin-Wagner theorem [2] and implies that Eq. (1) does not properly capture the whole effect of spin fluctuations. Therefore, for a better SCSW theory, the contribution from spin fluctuations needs to be considered more properly, even though the predicted value of T_c may differ numerically only in the three-dimensional case.

To find another SCSW theory without these flaws, we notice that, after coarse graining as being done in Ref. [1], the Hamiltonian of DMS and the Kondo lattice model (KLM) are approximately equivalent. Therefore, by using the equation-of-motion approach under the Tyablikov decoupling scheme (random-phase-like approximation) [3] and calculating the Green function $\langle\langle S_i^+; (S_j^-)^n (S_j^+)^{n-1} \rangle\rangle$ for the KLM, another expression for $\langle S^z \rangle$ is reached [4,5], which respects the constraints on the finite dimensionality of the impurity-spin Hilbert space,

$$\langle S^z \rangle = c\left\{S - \Phi + \frac{(2S + 1)}{[(1 + \Phi/\Phi]^{2S+1} - 1}\right\}, \tag{3}$$

where the value of $\langle S^z \rangle$ in the KLM is reduced by a factor of c due to coarse graining and $\Phi = (1/cV) \times \sum_{|\vec{p}|<p_c} n(\Omega_p)$. As simple justification of Eq. (3), one can check two limiting cases: (i) when $T \approx 0$, such that $n(\Omega_p)$ and therefore Φ are vanishingly small, the last term in Eq. (3) can be dropped and Eq. (3) does lead to the correct $T^{3/2}$ law; (ii) by taking Ω_p to be $\vec{p}$ independent, Eq. (1) is recovered as it should be. Moreover, the present theory gives another expression for T_c,

$$k_B T_c = \frac{S(S + 1)/3}{\lim_{\langle S^z \rangle, n^* \rightarrow 0} (1/V) \sum_{|\vec{p}|<p_c} \langle S^z \rangle / \Omega_p c^2}. \tag{4}$$

The above formula gives $T_c = 0$ both for the 1D and 2D cases due to the fact that $\Omega_p \propto p^2$ as $p \rightarrow 0$ and therefore the integration over $\vec{p}$ diverges. It shows that the contribution of long-wavelength spin waves is treated more properly in the present method. Thus the emphasis on these low-energy excitations, especially near T_c, implied by using Eqs. (1) and (3) is different. Based on these discussions, we argue that a reasonable SCSW theory should use Eq. (3) for $\langle S^z \rangle$, rather than Eq. (1).

Note added.—After this work was completed, we noticed that an application of this approach on the KLM has been previously studied [5].

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Received 7 August 2000
DOI: 10.1103/PhysRevLett.86.5636
PACS numbers: 75.30.Ds, 75.40.Gb, 75.50.Dd

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