Optical Resonance in a Narrow Slit in a Thick Metallic Screen

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Interaction of TM-polarized waves with a subwavelength thick metallic slit has been analyzed. A Fabry-Pérot-like behavior is reported. The resonance peaks, however, have very low magnitude and a systematic shift towards longer wavelengths is observed. The slit being narrow, this shift can be interpreted as the result of an aperture effect. Spectral transmission from a periodic array of such slits features the same peaks with a high increase in their magnitude, confirming that a grating acts as an amplifier of those resonances. Such a mechanism might explain the enhancement of the transmission observed in recent experiments [T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi, T. Thio, and P. A. Wolff, Nature (London) **391**, 667 (1998)].

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Recently, high transmission efficiencies from arrays of subwavelength structures have been reported [1]. It has been experimentally observed that the spectral response of bidimensional arrays of submicronic holes in a metal film exhibits high transmission of light, much more than the transmission from a single hole. If it is believed that this remarkable enhancement has important technological applications such as photolithography, the physics behind the process remains only partially explained. Indeed, recent theoretical works [2–6] involving slits instead of holes for calculation convenience have confirmed almost perfect transmission (up to 90%) for subwavelength slits, but the argument that surface plasmons are responsible for the enhancement [2,3] is not complete [4–6]. For example, it seems difficult to explain the peak shift with increasing film thickness [1] with the sole argument of surface waves.

The aim of this Letter is to prove that for thick enough conductors, light transmission from a single subwavelength slit features local maximums with very low magnitudes, which however can be identified as the resonance peaks observed when a periodic array of such slits is considered.

In the calculations, the conductor is considered perfect so that typical finite conductivity effects such as skin depth are neglected. It is assumed, however, that for not too narrow slits, the effects described here are not significantly modified by the conductivity being finite. The geometry is sketched in Fig. 1. A monochromatic wave impinges normally on a screen with thickness t. The slit, which has the same direction as the magnetic field, is in the y direction and has a rectangular cross section with width w.

An original method has been developed to study wave interaction in such a configuration: it consists of matching the cavity modes expansion [7] of the y component of the magnetic field H_y inside the slit:

$$H_y^{(\text{inside})}(x,z) = \sum_{n\geq 0} \cos\left(\frac{n\pi x}{w}\right)\left\{\alpha_n \exp\left[i\sqrt{k_0^2 - \left(\frac{n\pi}{w}\right)^2}\, z\right] + \beta_n \exp\left[-i\sqrt{k_0^2 - \left(\frac{n\pi}{w}\right)^2}\, z\right]\right\} \tag{1}$$

with the plane waves expansion above the slit:

$$H_y^{(\text{above})}(x,z) = \exp(iQ_0 x - iq_0 z) + \int_{-\infty}^{+\infty} dQ\, R(Q) \exp(iQx + iqz) \tag{2}$$

and below the slit:

$$H_y^{(\text{below})}(x,z) = \int_{-\infty}^{+\infty} dQ\, T(Q) \exp(iQx - iqz), \tag{3}$$

using the boundary conditions at $z = 0$ and $z = -t$. (Q_0, q_0) represent the (x,z) components of the incident wave vector $\mathbf{k}_0$ with wavelength λ and $R(Q)$ [respectively, $T(Q)$] corresponds physically to the reflection (respectively, transmission) coefficient. After some manipulations, it can be shown that $R(Q)$ and $T(Q)$ can be expressed in terms of Fourier transforms $W_n(Q)$ of each cavity mode:

$$R(Q) = \delta(Q - Q_0) + \sum_{n\geq 0} R_n W_n(Q)/q\,, \tag{4}$$

$$T(Q)\exp(iqt) = \sum_{n\geq 0} T_n W_n(Q)/q\,, \tag{5}$$

where δ is the Dirac impulse and

$$W_n(Q) = \int_0^w dx \exp(-iQx) \cos\left(\frac{n\pi x}{w}\right). \tag{6}$$

This important result, which is exact within the scope of Maxwell equations, reduces the original problem to a linear system thanks to the method of moments [8]. The unknown R_n and T_n are then obtained by means of a matrix inversion. Since in the present case the slit is narrow, only a few terms are necessary to achieve convergence to the solution. Finally, keeping in mind that if the single slit is

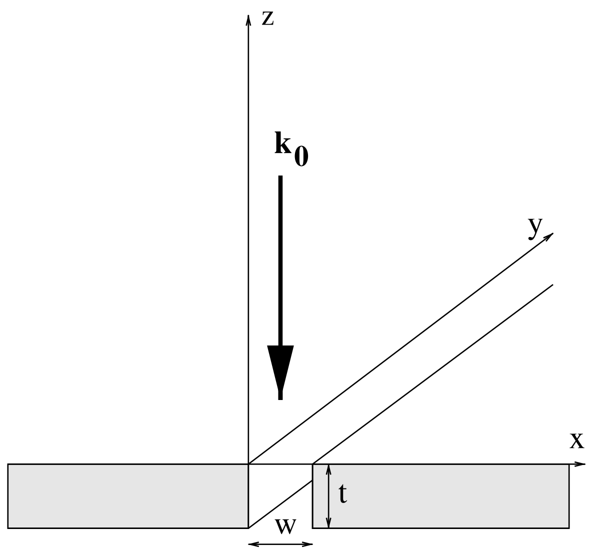

FIG. 1. Geometry to study wave interactions with a single slit in a perfect screen. The slit with width w and thickness t is in the y direction. For definition purposes, the screen is cut at $y = 0$ and the waves impinge normally from $z > 0$.

replaced by a periodic array of slits (diffraction grating), the magnetic field can be expressed in terms of a Rayleigh expansion outside the slits [7]. The method developed above then applies to gratings with only a minor modification (but it may not necessarily be the most efficient way to solve the problem): to replace the Fourier transform $W_n(Q)$ by the expansion $(2\pi/d)\sum_{p=-\infty}^{+\infty} W_n(Q_p)\delta(Q - Q_p)$, where Q_p is given by the Bragg condition:

$$Q_p = Q_0 + p\,\frac{2\pi}{d} \tag{7}$$

and d is the pitch of the grating. In what follows, the results of these investigations are given.

If it is accepted [6] that the magnetic field is sufficiently well described by the first term in the expansions of Eqs. (4) and (5) in the case of narrow slits (this means that only the fundamental mode is retained), an analytical calculation can be performed and one gets the first order estimate of T_0:

$$T_0 = \frac{2/(i\lambda)}{4(w/\lambda)\cos(k_0 t)[\ln(k_0 w/2) - 3/2] - \sin(k_0 t)}. \tag{8}$$

Equation (8) is a result of an approximation made on the moment $\langle W_0(Q)/q, \overline{W_0}(Q)\rangle$ where the zero order Hankel function of the first kind is replaced by its singularity. Analysis of the denominator leads to the argument that for w/λ small enough, $|T_0|^2$ will exhibit maximums around wavelengths where $\sin(k_0 t)$ is zero. This condition, well known in optics as the Fabry-Pérot resonance [9], has already been mentioned in Ref. [6] where the influence of geometrical parameters of a periodic array of subwavelength slits is considered. Also in Ref. [6], the authors have reported that for narrow enough slits, transmission efficiencies as functions of the slits width are flat for perfect conductors while they exhibit peaks for real ones, as a consequence of the conductivity being finite (skin effect). In the present work, the main idea is that even for a single narrow slit in a perfect conductor, if the screen is thick enough so that a Fabry-Pérot-like condition is satisfied, transmission peaks with very low magnitude can be observed. There is, however, a fundamental difference between a Fabry-Pérot and a narrow slit: in the latter case, additional wavelength dependent terms in the denominator are responsible for a shift of the resonant wavelengths, which can be estimated analytically by means of a perturbation calculation:

$$\frac{\lambda_{\mathrm{shift}}}{\lambda_{\mathrm{FP}}} = \frac{2(w/t)[\ln(\pi w/\lambda_{\mathrm{FP}}) - 3/2]}{2(w/t)[\ln(\pi w/\lambda_{\mathrm{FP}}) - 1/2] - \pi}, \tag{9}$$

where λ_{FP} is a Fabry-Pérot wavelength.

A few comments have to be made about such an estimate. In such an approximation, the relationship between λ_{shift} and λ_{FP} is locally monotonous so that there are as many peaks as Fabry-Pérot resonances. This assertion is verified by calculating rigorously a spectral transmission curve for $2t/\lambda$ smaller than unity (Fig. 2) where indeed no peak is observed. In Fig. 3, thickness t has been chosen in such a way as to include a few Fabry-Pérot wavelengths (marked by vertical continuous lines) in the scan area. The resonance peaks with very low magnitude (less than -20 dB) can be clearly identified. Analytical estimates of the wavelength shift which, according to Eq. (9), can be interpreted as an aperture effect are also drawn (dotted vertical lines): reasonable matching is observed and residual mismatch can be caused by higher orders in the perturbation calculation and by the other (evanescent) cavity modes.

If an effective index $n_e(\lambda)$ is introduced, λ_{shift} being positive for narrow slits, $n_e(\lambda)$ is always greater than unity. A physical interpretation is that a subwavelength slit behaves like a very inefficient Fabry-Pérot made out of a

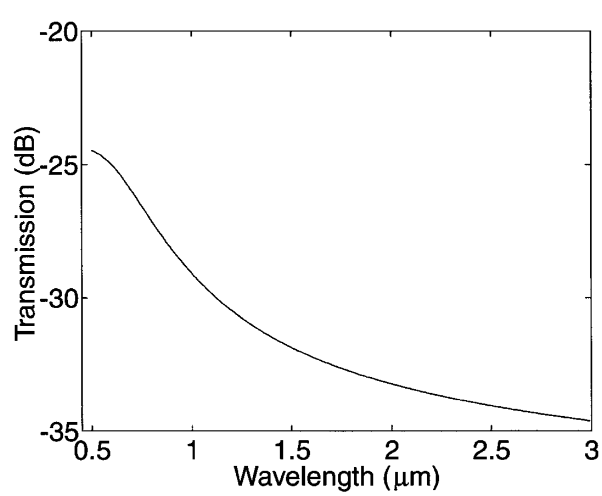

FIG. 2. Power transmitted by a slit with $w = 0.150$ μm and $t = 0.150$ μm. Since $2t/\lambda < 1$ here, no peak is observed.

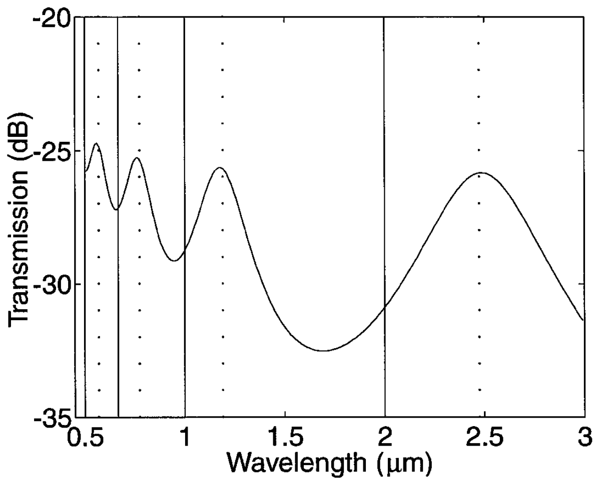

FIG. 3. Transmission spectrum of a slit with $w = 0.150$ μm and $t = 1$ μm. There are four Fabry-Pérot resonances (materialized by vertical lines) between 0.5 and 2 μm. Slit resonances have very low magnitude and are always shifted to the right of their corresponding Fabry-Pérot wavelengths. The dotted vertical lines correspond to analytical estimates of those resonances given by Eq. (9). The residual mismatch seems to be a result of higher order perturbation terms and evanescent cavity modes.

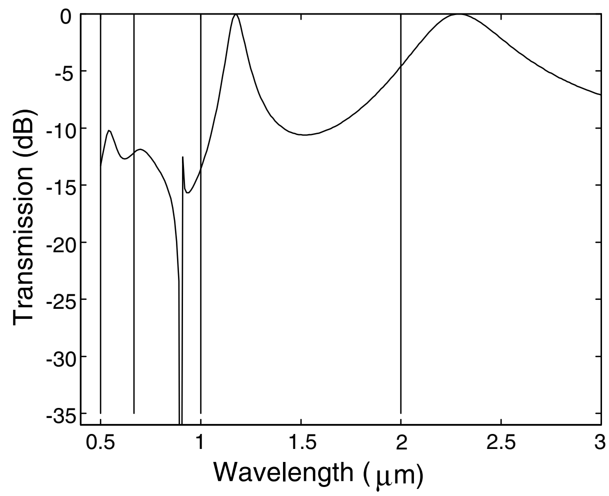

FIG. 4. Transmission efficiency of a periodic array of slits with $w = 0.150$ μm and $t = 1$ μm. The pitch of the grating is $d = 0.9$ μm and a Rayleigh-Wood anomaly is clearly visible. There are as many resonances as in Fig. 3 (vertical lines materialize Fabry-Pérot condition) but the peaks on the right of the grating period feature a significant amplification in their magnitude. For those peaks, the shift from the Fabry-Pérot wavelengths seems to be caused by the evanescent orders (surface waves).

material that would be less transparent than air. On the contrary, if an infinite number of slits are added up in such a way as to create a diffraction grating, each slit now contributes to transmit the incident radiation and since according to Eq. (9), λ_{shift} is aperture dependent, the shift must diminish because the number of apertures has been raised: this is confirmed in Fig. 4. It is also interesting to notice how the magnitude of these peaks is amplified, especially for those on the right of the grating pitch d: an almost perfect transmission is then observed. An explanation is that for such wavelengths, according to Eq. (7), there is only one diffraction order at normal incidence. All the energy is hence concentrated at this order. An approximate for the transmission efficiency $\eta_0^{(t)}$ can also be obtained for those wavelengths by keeping only the zero orders in both cavity modes and Rayleigh expansions:

$$\eta_0^{(t)} = \frac{4(d/w)^2}{4(d/w)^2 + [(d/w)^2 - 1]\sin^2(k_0 t)} . \qquad (10)$$

It is obvious that $\eta_0^{(t)}$ is maximal for $\sin^2(k_0 t) = 0$ which is indeed the Fabry-Pérot condition. Therefore, since this result is a zero-order approximation result, the redshift observed in Fig. 4 seems to be an effect that can be attributed to higher diffraction orders which are all evanescents, confirming the important influence of the surface waves (which for finite conductivity correspond to plasmons).

Putting all these arguments together and if it is accepted that finite skin depth does not fundamentally modify the process, the "extraordinary optical transmission" observed in Ref. [1] seems to be a result of a constructive interference (amplification by the grating) of the (very inefficient) Fabry-Pérot-like resonances localized in each cavity. This has been referred to as the "funnel" effect in Ref. [6].

It is expected that such resonance will be useful to design devices such as dynamic subwavelength filters (for example, by replacing the air between the slit walls by a material that can be modulated by a field [10]) or very fast optical switches.

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