Avalanches in One-Dimensional Piles with Different Types of Bases

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We perform a systematic experimental study of the influence of the type of base on the avalanche dynamics of slowly driven 1D ball piles. The control of base details allows us to explore a wide spectrum of pile structures and dynamics. The scaling properties of the observed avalanche distributions suggest that self-organized critical behavior is approached as the "base-induced" disorder at the pile profile increases.

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Avalanches in a pile of grains have been exploited to illustrate the concept of self-organized criticality (SOC) [1,2], whose eventual relevance to a wide range of natural phenomena has provoked a great amount of research in the last 13 years [3–5]. The key hypothesis of SOC is that systems containing many interacting constituents may exhibit some general behavior in which no characteristic length or time scale exists, *and this behavior does not need "tuning" from the outside to take place, i.e., the system organizes itself*. For example, if an "ideal" pile of grains is slowly driven by dropping grains at its apex, a "critical" state is reached in which each grain added is able to provoke an avalanche of any relevant size and duration. Then, the pile will show power law distributions of avalanche size and duration, "$1/f$" power spectra, and finite-size scaling of the distribution of avalanches. These fingerprints are self-organized in the sense that there is no need to fine-tune them through details of the system, such as grain shape, intergrain interaction, or type of base.

Although much theoretical work has been devoted to SOC, only a few *experimental* efforts have been carried out to check this paradigm in real piles to which grains are slowly added. Held and co-workers, for example, concluded that their 2D sandpiles followed SOC for various types of grains, even when added at different heights [6]. However, for large enough pile sizes, quasiperiodical big avalanches dominated the dynamics, in contradiction with the original SOC scheme. This was later corroborated and further examined experimentally by others [7–10]. Frette *et al.* [11] and Christensen *et al.* [12] studied the avalanches in quasi-1D rice piles for different types of grains, concluding that SOC behavior was attained only for those with relatively high aspect ratio, i.e., that guaranteeing intergrain friction strong enough to neglect inertial effects, not taken into account in the standard SOC theory. This established that at least one detail of the system (i.e., the grain geometry) is able to control the SOC behavior of a real pile. In this paper we show experimentally that the *nature* of the base is a second parameter able to modify the avalanche dynamics of slowly driven 1D piles of beads. By changing the type of base, we are able to tune the scaling properties of the avalanche distributions from "uncollapsable" to "collapsable." This tendency is paralleled with an increase of the disorder and the time-averaged layer involved in avalanche events, and of the time-averaged disorder of the pile profile.

In our setup, an acrylic strip was sandwiched between two parallel vertical glass plates 5 ± 0.2 mm apart from each other so that a horizontal surface of $5 \times L$ mm^2 (where L varied from 8 to 32 cm) was available for the formation of a quasi-1D pile of 4 ± 0.005 mm-diameter steel beads. The beads were dropped one by one from a height of 10 ± 2 cm above the apex of the pile, except otherwise stated. The extremes of the base were open, so the beads were able to fall off the system. The falling events were detected by measuring the weight variations of the pile with a digital scale. The whole setup was computer controlled in such a way that a bead was added only if the previous one had finished all its relaxational effects on the pile. The measuring software identified an avalanche of intensity n when n beads fell off the pile after dropping one bead at the apex of the pile. Avalanches of size zero (no beads falling off the pile after a dropping event) were also recorded, but not accounted for in the avalanche statistics performed later on. A typical experiment included more than 30 000 dropping events, with an average total duration of 80 h. For the avalanche statistics, the events previous to the reach of the "steady" average pile size were eliminated. To check out eventual errors due to electronic offsets, we manually counted the beads involved in avalanches for extended periods of time, and concluded that the detection system reported less than one bead in excess or in defect every 500 dropping events. We obtained images of the piles approximately every 500 dropping events by means of a digital camera.

We report the results of four types of bases, each one consisting in a row of beads stuck to the $5 \times L$ mm^2 surface with different spacing between beads: we will call Gap3, Gap0, Gap1.6, and Gapran the bases with spacing of 3, 0, 1.6 mm, and random values between 0 and 3 mm, respectively. At least two runs of each base type and length were performed to assess the repeatability of the results.

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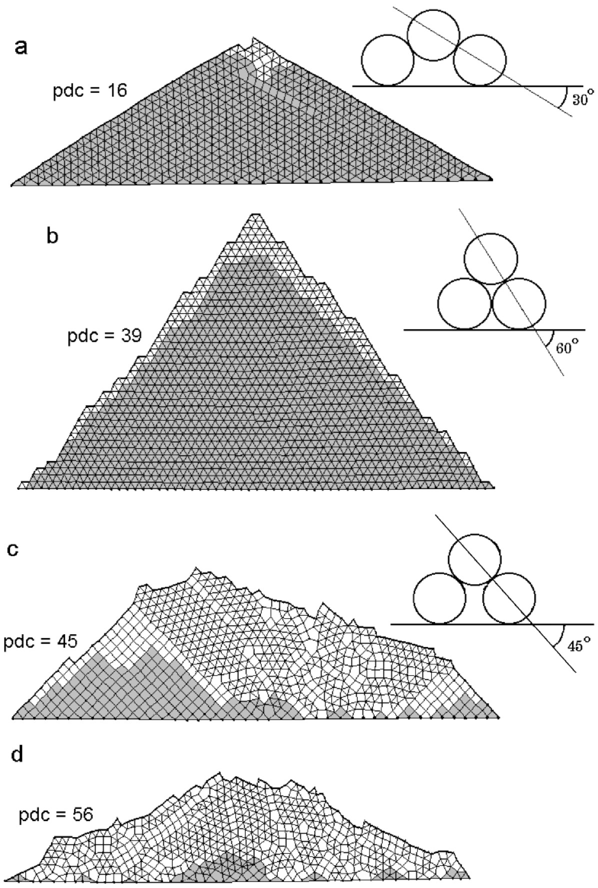

FIG. 1. Schematic representations of typical Gap3 (a), Gap0 (b), Gap1.6 (c), and Gapran (d) piles for $L = 24$ cm. The heavy dots at the base of each pile represent the centers of beads stuck to an acrylic strip. Triangulations based on the centers of the beads were performed to underline the structural features of the piles. The insets in (a)–(c) display the "ideal" structures of packed spheres. The corresponding "profile disorder coefficients" are also included.

Figure 1 displays the structure of some representative piles for a 24 cm base length. Gap3 (Fig. 1a) shows a very ordered "tilted" triangular structure in the bulk with almost perfectly linear profiles. Only close to the apex a moderate level of disorder can be identified.

In spite of its very ordered profile structure, Gap0 (Fig. 1b) shows a "kink" featured profile described by Alonso and Herrmann [13] for 1D piles formed on an infinite smooth surface. The unavoidable dispersion in the bead diameters and the lack of perfect "one dimensionality" of the structure associated with small defects in the separation and parallelism of the vertical glass plates conspires against the achievement of ordered bulk structures such as those obtained for Gap3 and Gap0 which are *key* to the exploration of the effects of different kinds of bases. These ordered structures were easily obtained by using the relatively big dropping heights stated above (10 ± 2 cm), which guaranteed a reasonably high packing density of the beads [14]. Heights of the order of 1 cm, for example, produced a looser packing and, hence, a less ordered bulk structure, particularly for Gap0.

Gap1.6 (Fig. 1c) displays the coexistence of two "structural phases": *squared* and *triangular*. Their combination provokes bulk disorder, and a quite featured profile. Finally, Gapran (Fig. 1d) shows strong bulk disorder and a heavily featured profile. We have quantified these differences by defining a "profile disorder coefficient" (PDC) in the following manner. First we trace the segments linking the centers of consecutive beads in the profile (thick lines in Fig. 1). Then, we determine the angles between one segment and the next one. This process is repeated for the whole profile. Finally, the standard deviation relative to the mean angle is calculated and averaged in 40 different profiles for each kind of base. The resulting four numbers give the PDC values shown in Fig. 1, which corroborate our qualitative descriptions (the standard deviation of the 40 PDC values for Gapran was 6, which gives an idea of the *maximum* error of the reported PDC's, since Gapran showed the widest span of PDC values).

Let us now discuss the avalanche behavior of our piles. We first approach it by graphically estimating the zone involved in avalanches for each type of pile, or *active zone*. It was determined by superimposing each image of the pile with the previous one in time, and determining which beads maintained their positions. The process was repeated for the whole time sequence of images. The sites that *never* changed positions were marked by shading their corresponding triangulation in the last image. Then, the nonshaded triangulation illustrates, at the end of the measurement, the typical proportion of "sites" participating in avalanche events, here defined as the *active zone*. The repeated observation of some "identifiable" balls during their transit through the pile, in the style of tracer particle experiments [12], confirmed the validity of our method to illustrate the qualitative differences in the active zone between the four types of bases. It is evident from Fig. 1 that the active zone increases in the order Gap3, Gap0, Gap1.6, and Gapran, i.e., in the same order as the PDC. The situation was the same for other base lengths. It should be stressed that, while Gap3 has virtually a zero-width active layer, almost *all* the positions in Gapran were involved in avalanche events. Furthermore, in the former case the width of the active layer remained close to zero for all L values, but it increased with L in the latter. This situation suggests a crossover to SOC behavior in our piles [12,15] in the order Gap3, Gap0, Gap1.6, and Gapran.

This crossover matches quite well with our avalanche size statistics, which we discuss now. In Gap3, the stability of the pile profile resulted in a great amount of very small avalanches. This behavior conceptually departs from the SOC scheme, which is further corroborated by the impossibility of obtaining finite-size scaling by applying the ansatz

$$P(s, L) = L^{-\beta} f(s/L^{\nu}) \qquad (1)$$

to the avalanche distributions shown in Fig. 2a. We also tried multifractal scaling—which corresponds to an integral over finite-size scaling forms introduced by Kadanoff *et al.* [16] to describe a large class of 1D models of SOC [17]—with negative results.

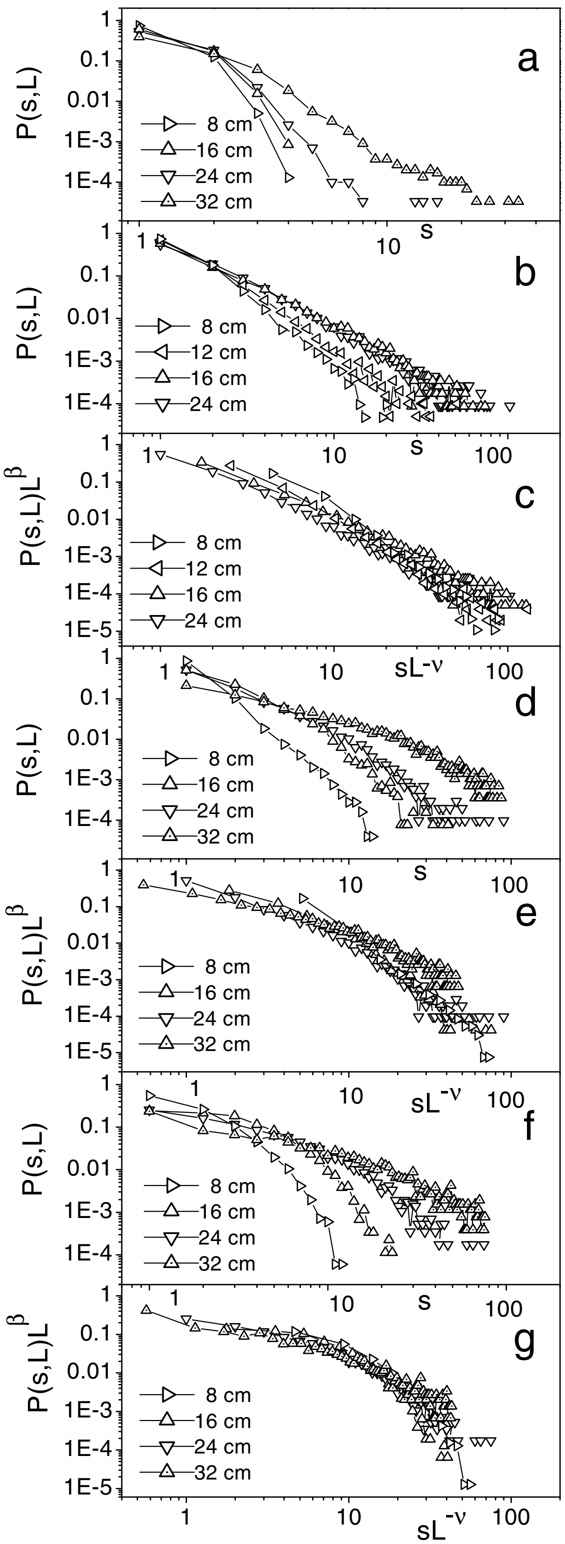

FIG. 2. Avalanche size distributions (normalized to the total number of nonzero avalanches) for (a) Gap3, (b) Gap0, (d) Gap1.6, and (f) Gapran. The scalings based in ansatz $P(s,L) = L^{-\beta} f(s/L^{\nu})$ with $\beta = \nu = 1.35$ are shown in (c) Gap0, (e) Gap 1.6, and (g) Gapran. It was not possible to obtain any reasonable collapse for Gap3 piles.

In the four base lengths under study for Gap0, we observed that the avalanches were associated with frequent changes along the pile's profile, involving the "annihilation" and "generation" of kinks, which can be defined as "unquenched" disorder. Figure 2b displays the avalanche size distributions for Gap0, and Fig. 2c displays the corresponding finite-size scaling by applying ansatz (1) with $\nu = \beta = 1.35$. This scaling relation can be obtained from the normalization condition $\langle s \rangle = \int P(s)\, ds = 1$, where $P(s)$ has been normalized to the total number of nonzero avalanches [18]. However, the collapse achieved for Gap0 is far from perfect.

Gap1.6 piles showed avalanches associated with profile changes at all length scales in quite a random fashion. For $L = 24$ cm, many of these changes were associated with transformations from squared to triangular structural domains and vice versa which included a few ball layers under the profiles, i.e., to unquenched disorder. In $L = 16$ cm and $L = 8$ cm piles, though, the square lattice tended to dominate, giving rise to kinks qualitatively similar to those observed in Gap0. Figure 2e displays the finite-size scaling of the data shown in Fig. 2d by applying ansatz (1) with $\beta = \nu = 1.35$, as for Gap0. The quality of the scaling is comparable to that corresponding to Gap0 (it should be noted that, due to experimental limitations, we were not able to try the longest base in Gap0, as we did for Gap1.6).

Even in a stronger fashion than in the case of Gap1.6, avalanches in Gapran piles were typically associated with random transformations of the profile at all length scales due to their great lack of stability. No kink-related mechanism could be associated with the avalanche formation, as in the case of Gap0 or some of the Gap1.6 piles. Sometimes beads initially very deep inside the pile contributed to an avalanche, as suggested by the active zone representation in Fig. 1 in accordance with observations by Christensen and co-workers on tracers in rice piles [12]. This picture repeated itself for the four base lengths under scrutiny. Figure 2f shows the avalanche size distributions for Gapran. Figure 2g displays the finite-size scaling of the data shown in Fig. 2f by applying ansatz (1) with $\nu = \beta = 1.35$, i.e., the same critical exponents used for Gap0 and Gap1.6. A cursory inspection of Fig. 2g shows the good quality of our scaling, pointing to SOC behavior.

Our previous descriptions strongly suggest the coexistence of different degrees of quenched and unquenched disorder in our piles, so it seems reasonable to analyze our results in the light of disordered cellular automata models reported in the literature. Puhl [19] introduces quenched disorder in his sandpile model by using a random instead of a regular lattice to which grains are centrally added. This model gives truly isotropic, conical piles, and suggests the emergence of SOC when quenched disorder is introduced. *This general trend is followed by our own experimental results* if we remember that the random base in Gapran "induces" SOC behavior, as suggested by the good critical size scaling of its avalanche distributions and depth of the active layer.

Let us now recount some of the basic results reported in the literature dealing with unquenched disorder in sandpile models. The cellular automata models proposed by Frette [20] and refined by Christensen and co-workers [12] introduced unquenched disorder by exciting the system at a single site, and treating the critical slopes as dynamic variables chosen randomly to be 1 or 2 every time a given site was toppled. The authors claimed that the randomness in this model (sometimes known as "Oslo model") is *internal*—i.e., inherent in their dynamics—in contrast to the original SOC model, in which randomness is introduced only by adding the grains at randomly chosen sites [1,2]. As a result, the 1D Oslo model exhibits SOC differently from the 1D version of the model in [1]. A different method to add unquenched disorder in 1D piles was reported by Mehta and Barker [21] by allowing the grains freedom to reorganize in the bulk as well as the ability to flow down the surface. In their model "horizontal" and "vertical" grains were introduced, randomly, onto the surface of the pile, creating regions of high and low "structural stability," respectively. The authors claimed that the addition of such unquenched disorder to the piles does not induce SOC behavior. A third approach to the introduction of unquenched disorder in sandpile models was proposed by Malthe-Sørenssen [18]. He dropped grains on a closed end of the pile and let them gain kinetic energy when bouncing down, transferring it to other particles *in a random proportion*. The grains eventually abandoned the system through an open boundary. As in the case of our experiment, Malthe-Sørenssen presents results for one-dimensional piles, excited at a single point, and both overall and off-the-edge avalanche statistics are reported. Using an avalanche distribution normalization and a scaling ansatz similar to ours, Malthe-Sørenssen obtains SOC in his model when examining their overall avalanche distribution. His off-the-edge avalanche distributions, on the other hand, scale with critical exponents $\beta = \nu = 1.35$. The off-the-edge avalanche distributions in Malthe-Sørenssen's model depicted in Fig. 8(a) of [18] qualitatively reproduce our own distributions shown in Fig. 2f for Gapran and, to a lesser extent, those shown in Fig. 2d for Gap1.6. Moreover, our good scaling of Gapran displayed in Fig. 2g was obtained with the same critical exponents reported by Malthe-Sørenssen, $\beta = \nu = 1.35$. These facts suggest the existence of SOC in our Gapran piles. As said before, this character diminishes with the quality of the scaling in the order Gap1.6, Gap0, and Gap3.

In conclusion, we have demonstrated the importance of the *nature* of the base in the avalanche dynamics of slowly driven 1D sandpiles. The scaling properties of the observed avalanche distributions suggest that SOC is displayed by piles with irregular, unstable profiles which can be reached by choosing a suitable base. The profile disorder and the depth at which the grains involved in avalanche events can be found in the pile increase with the SOC character.

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