Fast Formation of Magnetic Islands in a Plasma in the Presence of Counterstreaming Electrons

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(Received 7 February 2001)

With the help of 2D-3V (two dimensional in space and three dimensional in velocity) Vlasov simulations we show that the magnetic field generated by the electromagnetic current filamentation instability develops magnetic islands due to the onset of a fast reconnection process that occurs on the electron dynamical time scale. This process is relevant to magnetic channel coalescence in relativistic laser plasma interactions.

DOI: 10.1103/PhysRevLett.86.5293 PACS numbers: 52.38.Fz, 52.35.Qz, 52.65.Ff

Magnetic field generation at the expense of the "thermal" energy of an anisotropic electron population and the conversion of magnetic energy into electron energy in the presence of inhomogeneous currents represent two basic features of the dynamics of a magnetic field in a plasma. The first process is related to the well-known Weibel instability [1] and can occur under various conditions that include the case where the role of anisotropy is taken by the presence of two counterstreaming (cold) populations. In this latter case we refer to the so-called electromagnetic current filamentation instability (EMCFI) [2] and the magnetic field generation can be interpreted as resulting from the separation in space of the two counterstreaming populations due to the repulsion of oppositely directed currents. The second process is related to the well-known magnetic field line reconnection instabilities that have been studied extensively in the context of laboratory and of astrophysical plasmas in dissipative and in collisionless regimes. In the context of the present work, the reconnection processes occur on the electron dynamical time scales and are related to the collisionless instability studied in Refs. [3–6] (see also references therein). As in the case of magnetohydrodynamic (MHD) reconnection, this instability leads to the formation of magnetic islands and, as shown for collisionless kinetic reconnection in the MHD frequency range [7,8] anisotropy can either suppress it (large temperature component parallel to the magnetic field) or enhance it (large perpendicular temperature).

Recently, the electromagnetic current filamentation instability has been thoroughly investigated [9–11] in the context of the generation of a quasistatic magnetic field in the wake of an ultrashort, ultraintense laser pulse propagating in an underdense plasma [12] and of electron transport in overdense plasmas [13]. In this case the two oppositely directed currents consist of the fast electrons, accelerated by the laser plasma interaction in the direction of the laser pulse propagation, and of the return (lower energy) electron current needed to maintain plasma neutrality. This magnetic field is quasistatic on the time scale of the oscillation period of the laser light and evolves on the electron dynamical time scale, say on time intervals of the order or longer than the Langmuir wave period, so that its evolution is fast on the ion and on the MHD time scales.

In Refs. [10,11] a two-dimensional, Cartesian, spatial geometry was assumed, defined by the plane containing the two counterstreaming electron populations and the wave vector of the EMCFI. In such geometry, the magnetic field is generated along the symmetry direction perpendicular to the plane, and magnetic reconnection processes are automatically suppressed. In this geometry the EMCFI and the electrostatic two-stream instability, with wave number along the direction of the electron streams, are coupled. The nonlinear evolution of these coupled instabilities was examined in the fluid approximation in [10] and their kinetic saturation was investigated with a Vlasov code in [11]. The quasistatic magnetic field generated by the EMCFI is spatially inhomogeneous and the current separation that is at the basis of the instability mechanism leads to strong current gradients that are prone to the development of reconnection-type processes. In order to study the combined development of these processes, a fully three-dimensional description of the plasma is needed [14]. The minimal description of the combined magnetic field generation and reconnection is actually obtained in a 2D-3V (two dimensional in space and three dimensional in velocity) configuration where all vector fields are three dimensional, but are independent of the coordinate x along which the streams propagate. In this description, which is complementary to the one used in Refs. [9–11], the (electrostatic) plasma dynamics along the stream direction is suppressed.

The aim of the present paper is to show that the quasistatic magnetic field generated by the EMCFI, which we take to be oriented along z, develops magnetic islands on a fast electron time scale because of the combined effect of the current gradients and of the anisotropy in the y-z plane induced by the deflection of the streams along y caused by the EMCFI magnetic field itself. This eventually leads to a 3D isotropization of the electron distribution.

The concept of magnetic field line reconnection in the frequency range considered, where ions can be taken as immobile, arises in the context of the conservation of the generalized vorticity $\mathbf{B}_e \equiv \mathbf{B} + (m_e c/e)\nabla \times \mathbf{u}_e$, where $\mathbf{u}_e$ is the electron fluid velocity. When the electrons are either cold or isotropic and obey a barotropic law, $\mathbf{B}_e$ is frozen in the electron fluid [15] for processes that occur on time scales longer than the Langmuir period such that charge separation effects can be neglected. Then, for phenomena with spatial scales larger than the electron inertial skin depth $d_e \equiv c/\omega_{pe}$, with ω_{pe} the Langmuir frequency, the generalized vorticity reduces simply to the magnetic field $\mathbf{B}$, which is thus frozen in the electron fluid. Under these conditions, in the absence of dissipation, the reconnection processes discussed in Refs. [3,4,6] can arise due to the formation of current sheets, with size of the order of or smaller than d_e, that effectively decouple the conserved vorticity from the reconnecting magnetic field.

Under the conditions examined in the present paper, no true freezing constraint can be invoked for the magnetic field produced by the EMCFI, because of the intrinsically kinetic and anisotropic nature of the electron dynamics [16]. Yet we will show that the structure and the time evolution of the magnetic islands and of the flow patterns that are formed after the nonlinear phase of the EMCFI conform to those produced by magnetic field line reconnection instabilities.

The results presented in this paper are based on 2D-3V kinetic simulations of the nonlinear electron dynamics. These results are obtained with a Vlasov code that is an extension of the 2D-2V code used in Ref. [11] and documented in [17]. These simulations are extremely costly in terms of memory requirements and computational time. We integrate the self-consistent Vlasov-Maxwell system of equations in the (y, z, v_x, v_y, v_z) phase space of electrons while ions are taken as a fixed neutralizing background. We normalize quantities to the electron mass m_e, to a characteristic particle density $\bar{n}$, to the speed of light c, and, consequently, to the electron plasma frequency $\omega_{pe} = (4\pi\bar{n}e^2/m_e)^{1/2}$ and to the characteristic electric and magnetic fields $\bar{E} = \bar{B} = mc\omega_{pe}/e$. The normalized electron skin depth equals unity. In our simulations the electromagnetic fields have three components.

We consider a spatially homogeneous initial state with zero electric and magnetic fields and with two symmetric counterstreaming electron populations modeled as the sum of two Maxwellians, with densities $n_1 = n_2 = 1/2$ and equal thermal velocity $v_{th} = 5 \times 10^{-3}$. The two distributions are centered around the velocities $v_1 = -v_2 = 0.4$ such that $n_1 v_1 + n_2 v_2 = 0$. Periodic boundary conditions are used both in the y and in the z directions with box size 2π. The ratio of the box size $L = 2\pi$ to the electron skin depth is such that it allows the fastest EMCFI (corresponding to $k_y d_e \approx 1$) [9] to grow. In contrast, in terms of the standard Δ' parameter of reconnection theory and in the absence of anisotropy effects, we would expect the reconnection mode to be (marginally) stable. Note that Δ' is well defined only in the asymptotic limit $d_e/L_B \ll 1$, with L_B the inhomogeneity scale of the magnetic field, but it provides qualitatively correct information [6] on the plasma stability also when $d_e \sim L_B$.

At $t = 0$ a perturbed magnetic field of the form

$$\mathbf{B} = B_o \sin(y)\mathbf{e}_z + \delta\mathbf{B}(y, z) \tag{1}$$

is added to the system. Here $B_o = 10^{-3}$, $\mathbf{e}_z$ is the unit vector along z, and $\delta\mathbf{B}(y, z)$ is a divergence-free random noise perturbation $\sim 10^{-2}$ smaller. This initialization allows the system to develop first a coherent y-dependent magnetic field along z due to the EMCFI seeded by B_o. The random noise perturbation seeds the subsequent development of the reconnection process.

Here we report the results of two runs with identical plasma parameters, but different resolution corresponding to $32^2 \times 61^3$ and to $64^2 \times 61^3$ grid points, respectively. In both runs the EMCFI develops with a growth rate $\gamma_{\mathrm{CF}} \approx 0.28$ leading to the formation of a dipolar magnetic field $B_z(y)$ with amplitude $B_{\max} \approx 0.27$ at saturation, as shown in Fig. 1 at $t = 38.2$. A local decrease of the electron density $\delta n_e \approx -0.4$ occurs where B_z^2 is largest. The occurrence of charge separation in the nonlinear development of the EMCFI, even in the absence of coupling with the two-stream instability, was already noted in Refs. [9,10].

The nonlinear development of the EMCFI leads to steep current density profiles and to the magnetic deflection of the electron streams in the y direction. In terms of an effective stress tensor measuring the spatially averaged kinetic energy T_j in the jth direction, this leads to an increase of T_y. While initially we had $T_x \gg T_y = T_z$, in this phase we find $T_\perp \equiv T_x \gtrsim T_y \gg T_z$, as shown in Fig. 2. These conditions lead to the onset of the reconnection process shown in Figs. 3 and 4 characterized by the exponential growth of the $k_z = 1$ Fourier component of the magnetic

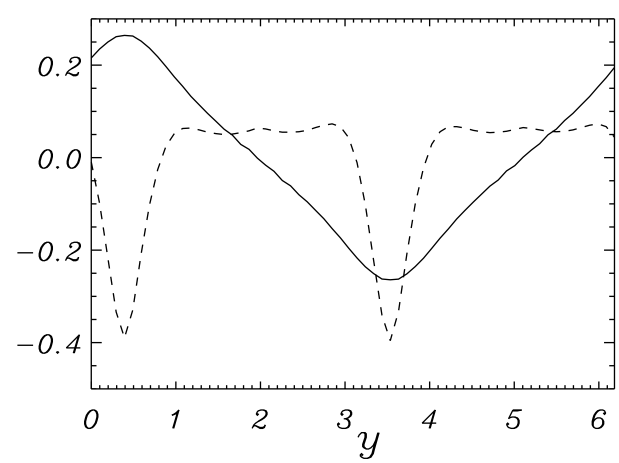

FIG. 1. Spatial profile of the magnetic field $B_z(y)$ (solid line) and of the density perturbation $\delta n_e(y)$ (dashed line) produced by the EMCFI at $t = 38.2$.

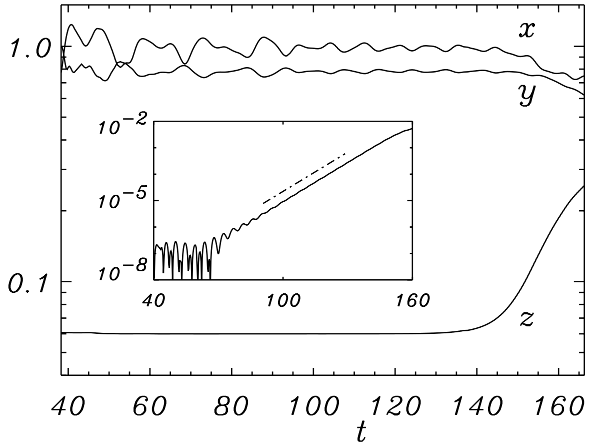

FIG. 2. Time behavior of the normalized temperatures T_j, $j = x, y, z$ after the EMCFI has saturated. In the small frame the time evolution of the amplitude of the reconnecting field δB_y is shown.

field $\delta \mathbf{B}(y, z)$ with growth rate $\gamma_R \approx 0.11$ (see Fig. 2, small frame). This value is largely independent of the resolution adopted, which demonstrates that reconnection is not caused by numerical dissipation in the code.

In the literature considerable work has been devoted to the study of the collisionless reconnection instability (in the MHD frequency range) in a neutral sheet configuration [18], and it has been shown that its growth rate γ is strongly affected by the temperature anisotropy $(T_\perp/T_z - 1)$ relative to the magnetic field [7]. This effect is due to the coupling of the reconnecting mode with the Weibel instability, i.e., in our case, with the secondary Weibel instability induced by the y deflections of the streams. The growth rate γ_R is given by

$$\frac{\gamma_R \sqrt{\pi}}{k_z v_{the,z}} \frac{T_\perp}{T_z} = \frac{d_e^2}{\delta_e} (\Delta' + \Delta'_0), \qquad (2)$$

where $\delta_e \sim L^{1/2} T_\perp^{1/4} / (2^{1/2} B_{\max})^{1/2}$ is the scale of the so-called Parker orbits [19] and $\Delta'_0 = (\delta_e/d_e^2)(T_\perp/T_z - 1)$. From the y profile of the EMCFI magnetic field and from the temperature ratios given in Fig. 2, we estimate $\Delta' \simeq -1.3$ and $\Delta'_0 \simeq 9.0$, so that the condition $\Delta' + \Delta'_0 > 0$ is well satisfied. The numerical growth rate γ_R is also consistent with the value given by Eq. (2) which arises from electron phase-space effects, i.e., from a resonant term in the plasma dispersion function. This picture is confirmed by the numerical value of the ratio $\delta B_y / \delta B_x \simeq 4$.

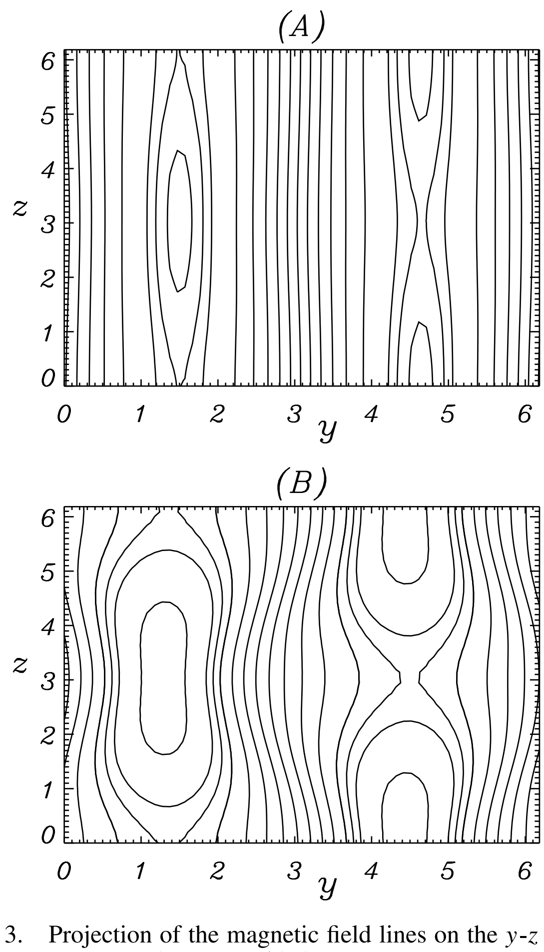

FIG. 3. Projection of the magnetic field lines on the y-z plane at $t = 128.8$ (a) and at $t = 165$ (b).

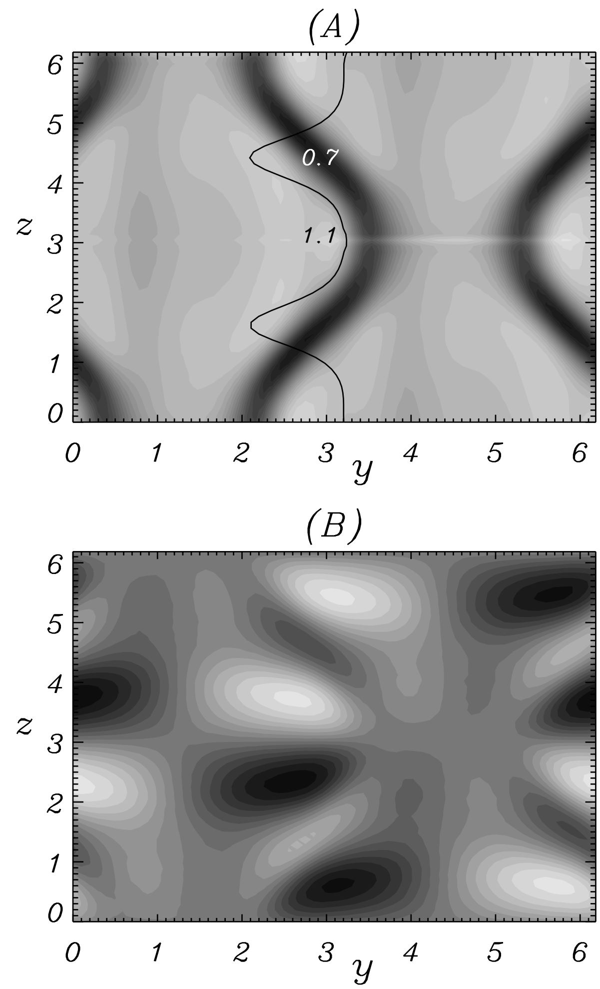

FIG. 4. Density (a) and velocity (b) distributions in the y-z plane at $t = 165$. The black curve in frame (a) gives the profile at $y = 3$ of the density which varies between 0.7 and 1.1.

We recall that in the case of magnetic reconnection on the electron time scales [4,6] the perturbed magnetic field component δB_x, in the geometry of the present paper, plays a role analogous to that of the stream function ϕ (i.e., of the plasma fluid displacement ξ) in the standard theory of MHD reconnection. In the case when resonant phase-space effects are dominant, the reconnecting component δB_y is much larger than the stream-function component δB_x, in full analogy with the corresponding MHD result [18].

The development of the reconnection instability leads to the formation of magnetic islands in the y-z plane [see Fig. 3(a), at $t = 128.8$] which eventually fill the simulation box, as shown in Fig. 3(b) at $t = 165$. At this point the simulation is interrupted. The island structure is reproduced in the electron density spatial distribution and in the flow pattern shown in Figs. 4(a) and 4(b). Charge separation effects on electron reconnection instabilities have been studied in Ref. [20], where they have been found to lead to a reduction of the growth rate. In the nonlinear phase of the reconnection instability the temperature T_z increases while $T_{x,y}$ decrease (see Fig. 2), thus indicating a tendency toward full isotropization of the electron distribution.

The results presented above show that the field lines of the magnetic field generated by the EMCFI reconnect on a fast electron time scale. The reconnection growth rate found in our simulations is smaller than that of the EMCFI only by a factor of ≈ 3. The main source of free energy for both instabilities is essentially the initial anisotropy of the two counterstreaming electron populations.

The reconnection of magnetic field lines is of great importance for the understanding of the 3D dynamics of the magnetic plasma channels that are formed in the wake of an ultraintense, ultrashort laser pulse in a plasma [12]. In particular, magnetic reconnection allows magnetic channels to coalesce [21], as shown recently, e.g., in 3D particle-in-cell simulations in overdense plasmas in Ref. [22]. To be compared directly with these physical conditions, the present results must be generalized so as to include relativistic electron kinematics and asymmetric ($n_1 \neq n_2$) electron streams, as is the case in laser plasma interactions where the fast electrons are a minority relativistic population and the density of the electrons in the return current is close to the plasma density.

One of us, F. C., is glad to acknowledge A. Mangeney for interesting discussions and C. Cavazzoni for numerical support about the parallelization strategies of the code. This work was supported by the INFM Parallel Computing Initiative and by MURST.

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