Destabilization of Internal Kink Modes at High Frequency by Energetic Circulating Ions

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A theoretical model is proposed to interpret the high-frequency fishbone instability observed in tangential neutral-beam-injection discharges in a tokamak. It is shown that, when the beam ion beta exceeds a critical value, energetic circulating ions can indeed destabilize the internal kink mode through circulation resonance at a high frequency comparable to the circulation frequency of the energetic ions. The critical beta value of the energetic ions, the real frequency, and the growth rate of the mode are in general agreement with the high-frequency fishbone instability observed in experiments.

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It is of particular interest to investigate the fishbone instability in tokamak fusion research [1–5]. Experiments have shown that a mode with dominant toroidal and poloidal wave numbers $n = 1$ and $m = 1$ can be strongly destabilized with the presence of energetic ions produced by neutral beam injection. The frequency of the dominant mode is around the toroidal precession frequency of the trapped energetic ions for the perpendicular injection case [1] or around the plasma toroidal rotation frequency for the tangential injection case [3]. Such a low-frequency fishbone instability is sometimes accompanied by high-frequency events in the perpendicular injection case [2]; it is always accompanied by the high-frequency events in the tangential injection case [3]. The high-frequency fishbone burst results in a few percent of beam ion loss [3,4]. Currently, it is understood that the low-frequency fishbone instability is due to the precession resonance of trapped energetic ions in the perpendicular injection case [6,7] and the crossing resonance (arising from the effect of finite radial drift of energetic circulating ions) of energetic circulating ions in the tangential injection case [8]. However, understanding the high-frequency fishbone is still an open issue. In this paper, we propose a theoretical model to interpret the high-frequency fishbone instability observed in the tangential injection case. We show that when the beta value of the energetic ions is high enough, energetic circulating ions can indeed destabilize the internal kink mode at a high frequency comparable to the circulation frequency of the energetic ions. Here the beta value is the ratio between particle and magnetic pressures. The critical beta value of the energetic ions, the real frequency, and the growth rate of the mode are in general agreement with the experimental observations.

We consider a large-aspect-ratio tokamak plasma consisting of core and hot components. The inverse of the aspect ratio, $\varepsilon \equiv a/R \ll 1$, with a and R the minor radius and the major radius, respectively. Since we are interested in the high-frequency fishbone, we make the ordering, $\omega/\omega_A \sim \mathcal{O}(\varepsilon)$, with ω the mode frequency and ω_A the usual shear Alfvén frequency [6,7]. We also make the ordering $\omega/\omega_c \sim \mathcal{O}(\varepsilon)$, with ω_c the gyrofrequency of the energetic ions. The core plasma toroidal beta value is ordered as $\beta_c \sim \mathcal{O}(\varepsilon^2)$. In typical tangential injection discharges in PBX [3,4], $\beta_c \approx 1.3\%$, and the toroidal beta value of the energetic ions is estimated as [4] $\beta_h \approx 0.4\%$. Note that these are volume-averaged beta values. Since we lack the detailed experimental data on the energetic ion pressure profile in PBX, we assume that it is similar to DIII-D [5] and PLT [9], where similar neutral beam injection discharges were carried out. Therefore, we take the beta value of the energetic ions at the magnetic axis, $\beta_{h,0} \approx 8\beta_h$ (we assume that the peaking factor of beam ion pressure in PBX is similar to that in DIII-D [5]). This can be justified by the fact that most of the fusion reactions in PBX ($\sim$90%) occur in the center of the plasma (inside the half minor radius) and the fusion reactions in PBX are dominated by beam-target reactions [4]. We make the ordering $\beta_{h,0} \sim \mathcal{O}(\varepsilon^2)$, $\beta_h/\beta_{h,0} \sim \mathcal{O}(\varepsilon) \sim \beta_h/\beta_c$. Note that in the above orderings, only in the region very near to the magnetic axis, the beam ion pressure is comparable to the core plasma pressure; in most regions of the plasma, the beam ion pressure is much smaller than the core pressure.

The stability analysis is carried out by following the generalized variational principle [6,7]. The energy functional is given by

$$D(\omega) = \delta I + \delta W_{\mathrm{MHD}} + \delta W_k, \tag{1}$$

where δI is the usual MHD kinetic energy functional. δW_{MHD} is the usual MHD potential energy functional. The last term is the nonadiabatic contribution of the energetic ions. The perturbation is assumed to be in the form $\sim \exp[-i(\omega t - \zeta + \theta)]$, with ζ and θ the toroidal angle and the poloidal angle, respectively. The nonadiabatic part of the perturbed distribution of the hot component, δf, is given by the drift kinetic equation

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$$[(\omega - \dot{\zeta}) + i\dot{\theta}\partial_\theta]\delta f = -(\omega - \omega^*)H_1\partial_E F, \quad (2a)$$

$$\omega^* = \frac{q\partial_r F}{\omega_c r \partial_E F}, \quad (2b)$$

$$H_1 = 2E\left(1 - \frac{3}{2}\lambda B\right)\kappa \cdot \xi_\perp - E\lambda B \nabla \cdot \xi_\perp, \quad (2c)$$

where $\xi_\perp$ is the usual fluid displacement. q is the MHD safety factor; r is the coordinate of the minor radius; $E = v^2/2$; $\lambda = \mu/E$, $\mu = v_\perp^2/2B$, with $v_\perp$ ($v_\parallel$) the perpendicular (parallel) velocity and B the equilibrium magnetic field. κ is the curvature of the equilibrium magnetic field. $F(r, E, \lambda)$ is the equilibrium distribution of the energetic ions. In Eq. (2), we have dropped the effects of finite radial drift of energetic ions, which is related to the crossing resonance inducing the low-frequency fishbone instability in the tangential injection case [8] and is beyond the scope of this paper.

$$\delta W_k = 2^{3/2} m_h \pi^2 RB \int r\,dr \oint d\theta \int d\lambda \int dE \times \left(\frac{E}{1 - \lambda B}\right)^{1/2} H_1^* \delta f, \quad (3)$$

with m_h the mass of the beam ion.

The usual minimization of δW_{MHD} gives [6,7]

$$\xi_\perp = \xi_0 H(r_s - r)(\mathbf{e}_r - i\mathbf{e}_\theta)\exp[-i(\omega t - \zeta + \theta)], \quad (4)$$

$$\nabla \cdot \xi_\perp + 2\kappa \cdot \xi_\perp = 0, \quad (5)$$

$$\delta \hat{W}_{\mathrm{MHD}} \equiv \delta W_{\mathrm{MHD}} \Big/ \left[2\pi R \xi_0^2 \left(\frac{r_s B}{2R}\right)^2\right] = \mathcal{O}(\varepsilon^2), \quad (6)$$

$$\delta \hat{I} \equiv \delta I \Big/ \left[2\pi R \xi_0^2 \left(\frac{r_s B}{2R}\right)^2\right] = -i\frac{\omega}{\omega_A}, \quad (7)$$

where ξ_0 is a constant. $H(x)$ is the Heaviside step function. r_s is the minor radius of the singular surface where $q = 1$. $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the radial unit vector and the poloidal unit vector, respectively.

Before solving the drift kinetic equation, in addition to neglecting the effects of finite radial drift, we assume that $v_\perp \ll v_\parallel$; this is reasonable for the tangential injection case [8]. With these approximations, substituting Eqs. (4) and (5) into Eq. (2c), we found that

$$H_1 = -2E\frac{\xi_0}{R}H(r_s - r)\exp[-i(\omega t - \zeta)]. \quad (8)$$

In obtaining Eq. (8), we have used $\kappa = (\mathbf{e}_\theta \sin\theta - \mathbf{e}_r \cos\theta)/R$.

Since we are interested in the mode with frequency around the circulation frequency and the weak instability case, we search the solution satisfying

$$(\omega - \dot{\zeta}) \sim (\omega - \omega^*) \ll \dot{\theta}. \quad (9)$$

The solution can be readily found:

$$\delta f = \frac{\omega^* - \omega}{\omega - v/R}H_1\partial_E F, \quad (10)$$

where we have used the approximation $\langle\dot{\zeta}\rangle \approx v/R$, with $\langle x\rangle \equiv (1/\tau_b)\oint d\theta\,(x/\dot{\theta})$, and $\tau_b = \oint d\theta\,(1/\dot{\theta})$, since $v_\perp \ll v_\parallel$, $v_\parallel \approx v$.

To proceed, we adopt the model slowing-down equilibrium distribution for the purely circulating ($v_\perp \ll v_\parallel$, $v_\parallel \approx v$) energetic ions

$$F = c_0(r)E^{-3/2}\delta(\lambda)H(E_0 - E), \quad (11a)$$

$$c_0(r) = p_h(r)/(2^{3/2}\pi m_h B E_0), \quad (11b)$$

where $E_0 = v_0^2/2$ is the birth energy of the beam ions and $p_h(r)$ is the beam ion pressure.

Substituting Eq. (8), Eq. (10), and Eq. (11), respectively, into Eq. (3), we found that

$$\delta \hat{W}_k \equiv \delta W_k \Big/ \left[2\pi R \xi_0^2 \left(\frac{r_s B}{2R}\right)^2\right] = \delta \hat{W}_{k,d} + \delta \hat{W}_{k,s}, \quad (12)$$

$$\begin{aligned}\delta \hat{W}_{k,d} &= \frac{2}{\varepsilon_s^2 \Omega_c}\left[\frac{1}{3} + \frac{1}{2}\Omega + \Omega^2 + \Omega^3 \log\left(1 - \frac{1}{\Omega}\right)\right]\frac{-8\pi}{B^2}\int_0^{r_s} dr\, q\frac{d}{dr}p_h(r) \\ &\approx \frac{2\beta_{h,0}}{\varepsilon_s^2 \Omega_c}\left[\frac{1}{3} + \frac{1}{2}\Omega + \Omega^2 + \Omega^3 \log\left(1 - \frac{1}{\Omega}\right)\right], \end{aligned} \quad (13)$$

$$\delta \hat{W}_{k,s} \approx \sqrt{2}\,\beta_h^s \frac{\Omega^2}{\Omega - 1}, \quad (14)$$

where β_h^s is the beam ion beta value volume averaged within the singular surface. $(\Omega, \Omega_c, \Omega_A) = (\omega, \omega_c, \omega_A)/\omega_{\zeta 0}$, $\omega_{\zeta 0} = v_0/R$. $\varepsilon_s = r_s/R$. In writing Eq. (14), we have dropped a few higher order terms. Note that the Ω dependence of $\delta \hat{W}_k$ found here for the tangential injection case is different from the perpendicular injection case [6,7]. This difference results from the different $\langle\dot{\zeta}\rangle$.

Now, since $\delta \hat{W}_{\mathrm{MHD}}$ is a higher order term comparing to $\delta \hat{W}_k$ [$\delta \hat{W}_{\mathrm{MHD}}/\delta \hat{W}_k \sim \mathcal{O}(\varepsilon)$], it can be dropped out of the energy functional, and we obtained

$$D(\omega) = -i\frac{\Omega}{\Omega_A} + \delta \hat{W}_k. \quad (15)$$

Consider the weak unstable case, $\Omega = \Omega_r + i\Omega_i$, $\Omega_i \ll \Omega_r$. The real and imaginary parts of the dispersion relation are obtained by setting $D(\omega) = 0$,

$$\frac{\Omega_i}{\Omega_A} + \frac{2\beta_{h,0}}{\varepsilon_s^2\Omega_c}\left(\frac{1}{3} + \frac{1}{2}\,\Omega_r + \Omega_r^2 + \Omega_r^3 \log\left|1 - \frac{1}{\Omega_r}\right|\right) + \sqrt{2}\,\beta_h^s\,\frac{\Omega_r^2}{\Omega_r - 1} = 0\,, \tag{16}$$

$$\frac{\Omega_r}{\Omega_A} - \frac{2\pi\beta_{h,0}}{\varepsilon_s^2\Omega_c}\,\Omega_r^3 - \Omega_i\left[\left(\frac{1}{2} + 2\Omega_r + 3\Omega_r^2 \log\left|1 - \frac{1}{\Omega_r}\right| + \frac{\Omega_r^2}{\Omega_r - 1}\right)\frac{2\beta_{h,0}}{\varepsilon_s^2\Omega_c} + \sqrt{2}\,\beta_h^s\,\frac{\Omega_r^2 - 2\Omega_r}{(\Omega_r - 1)^2}\right] = 0\,. \tag{17}$$

Equation (17) indicates that instability can be found only when $\beta_{h,0}$ exceeds a critical value given by

$$\beta_{h,0}^{\text{crit}} = \frac{\varepsilon_s^2\Omega_c}{2\pi\Omega_A\Omega_r^2}\,. \tag{18}$$

The real frequency is found through setting $\Omega_i = 0$ and replacing $\beta_{h,0}$ by $\beta_{h,0}^{\text{crit}}$ in Eq. (16),

$$\frac{1}{3} + \frac{1}{2}\,\Omega_r + \Omega_r^2 + \Omega_r^3 \log\left|1 - \frac{1}{\Omega_r}\right| + \sqrt{2}\,\pi\beta_h^s\Omega_A\,\frac{\Omega_r^4}{\Omega_r - 1} = 0\,. \tag{19}$$

Writing $\beta_{h,0} = \beta_{h,0}^{\text{crit}} + \Delta\beta_{h,0}$, we found from Eq. (17) the growth rate

$$\Omega_i = \frac{\pi\Omega_r^3\Delta\beta_{h,0}/\beta_{h,0}^{\text{crit}}}{\frac{1}{\Omega_r} + 1 + \Omega_r + \frac{\Omega_r^2}{1-\Omega_r} + \sqrt{2}\,\pi\beta_h^s\Omega_A\Omega_r^2\,\frac{2\Omega_r^2-\Omega_r}{(\Omega_r-1)^2}}\,. \tag{20}$$

Generally, Eqs. (18)–(20) determine the critical central beam ion beta, the real frequency, and the growth rate of the internal kink mode destabilized by energetic circulating ions through circulation resonance at a high frequency comparable to the circulation frequency of the beam ions. For PBX parameters, $B = 0.84T$, $\omega_{\zeta 0}/2\pi = 190$ kHz, $m_i = m_h = 2m_p$, $n_i = 1.7 \times 10^{13}$ cm^{-3}, $R = 130$ cm, $\varepsilon_s \approx 1/9$. The central beta value of the beam ions is estimated as $\beta_{h,0} \approx 3.2\%$, and the beam ion beta value volume averaged within the singular surface is estimated as $\beta_h^s \approx 1.4\%$; the magnetic shear at the singular surface is estimated as $s \approx 0.4$. We found from Eq. (18) that $\beta_{h,0}^{\text{crit}} \approx 3.2\%$, which is approximately the central beam ion beta likely attained in PBX. From Eq. (19), we found that $\Omega_r \approx 0.83$ ($\omega_r/2\pi \approx 160$ kHz), which is in good agreement with the experimental data ($\omega_r/2\pi \sim 150$ kHz) [3]. Since the neutron rate reduction induced by the high-frequency fishbone in PBX is 1%–6%, and the fusion reactions in PBX are dominated by the beam-target reactions [3,4], we may take $\Delta\beta_{h,0}/\beta_{h,0}^{\text{crit}} \approx 3\%$. Then, from Eq. (20) we obtained $\Omega_i \approx 6 \times 10^{-3}$ [$\omega_i \approx 1/(140\ \mu$sec)], which is in good agreement with the experimental data [$\omega_i \sim 1/(125\ \mu$sec)] [3].

In summary, we have shown that energetic circulating ions can destabilize the internal kink mode at a high frequency comparable to the circulation frequency of the energetic ions when the central beam ion beta exceeds a threshold value. Such an instability induced by the circulation resonance has a threshold value of central beam ion beta, real frequency, and growth rate in general agreement with the high-frequency fishbone instability observed in experiments [3].

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