Behavior of Boundary String Field Theory Associated with Integrable Massless Flow

A. Fujii* and H. Itoyama†
Department of Physics, Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan
(Received 23 December 2000)

We put forward an idea that the boundary entropy associated with integrable massless flow of thermodynamic Bethe ansatz (TBA) is identified with tachyon action of boundary string field theory. We show that the temperature parametrizing a massless flow in the TBA formalism can be identified with tachyon energy for the classical action at least near the ultraviolet fixed point, i.e., the open string vacuum.

DOI: 10.1103/PhysRevLett.86.5235 PACS numbers: 11.25.Sq, 04.70.Dy

A lot of effort to reveal the tachyon condensation mechanism has been made in an attempt to find a stable vacuum both in bosonic and in supersymmetric string theory. According to Sen's conjecture, the closed string vacuum is realized after an annihilation mechanism of an open string is completed by the cancellation between tensions of D-branes and energy of the tachyon [1]. In string field theory, the main ways to analyze this problem have been through Witten's cubic string field theory [2] and the boundary string field theory (BSFT) [3,4].

In the latter context with a special choice of the tachyon profile, some evidence to support Sen's conjecture has recently been provided in an exact manner [5–8]. In this Letter we take this latter approach, BSFT. In the Batalin-Vilkovisky formalism used for BSFT [3], the space-time string action S is conjectured to satisfy the differential equation of the form [5–8]

$$\frac{\partial S}{\partial \lambda_i} = \mathcal{G}^{ij}\beta_j\,. \tag{1}$$

Here λ_i are the coupling constants of the boundary operators, β_j are the β functions, and $\mathcal{G}^{ij}$ is the metric in the space of coupling constants.

On the other hand, ground state degeneracy (g-function) [9] of the world sheet theory with a boundary perturbation is also expected to satisfy a differential equation of the same form as (1). Thus, we expect that string action S will be identified with g-function. To investigate this correspondence, we analyze this problem using the boundary sine-Gordon model (BSG) [10] and attendant thermodynamic Bethe ansatz (TBA). TBA is intended to obtain thermodynamic quantities at finite temperature and has also been used to extract information on the g-function in some of the exactly solvable models [11].

We consider the process in which single D25-brane decays into a D24-brane by tachyon condensation. In the context of TBA, temperature in one-dimensional soliton gas is the renormalization group (RG) scale, which is regarded as an order parameter of the tachyon condensation. The sine-Gordon parameter in the boundary term can be identified with the inverse of the compactification radius in BSFT. From this fact, the boundary entropies at two ends of the flow have been shown to give an exact ratio of the brane tensions [5]. In this Letter we find that the temperature in TBA can be identified with energy of the classical solution of the tachyon action in BSFT. We provide evidence in support of this correspondence by comparing the behavior of TBA and the behavior of the classical solution in BSFT. This correspondence is confirmed by a numerical calculation, too. We propose that integrable massless flows generated by TBA provide a description of the open string action even away from the fixed points.

Massless TBA and g-function as string action.—To utilize the soliton picture, we begin with the action of the sine-Gordon model on a segment $\sigma \in [0, L]$ at finite temperature θ

$$S = \frac{1}{4\pi\alpha'}\int_0^{1/\theta} dt \times \int_0^L d\sigma\left[[\partial_\mu X(\sigma,t)]^2 + G\cos\frac{4\pi}{R}X(\sigma,t)\right] + \zeta\int_0^{1/\theta} dt\cos\frac{2\pi}{R}X(\sigma=0,t)\,. \tag{2}$$

This system is shown to possess an infinite number of conserved currents and hence is integrable [12]. This action permits the field $X(\sigma,t)$, namely, X_{25}, to be compactified: $X \sim X + R$. We impose the Dirichlet boundary condition at $\sigma = L$ and pay attention to the boundary at $\sigma = 0$. We assume that $\lambda = R^2/4\pi^2\alpha' - 1$ is a non-negative integer. The strength G gives a mass scale of the soliton/antisoliton and ζ gets traded with boundary temperature θ_B [11], which plays a similar role to that of the Kondo temperature in the Kondo problem. Because we are interested in models with conformal invariance in the bulk $\sigma \in (0, L)$, the massless limit $G \to 0$ is taken after TBA formalism is set up.

The free energy of this model in the $L \to \infty$ limit should be

$$\frac{F}{L} = f_{\rm bulk} - \frac{\theta}{L}\ln g + O(1/L^2)\,, \tag{3}$$

where g is the ground state degeneracy of the system with the boundary at $\sigma = 0$. We focus on this g-function.

The TBA procedure gives us an equation for the g-function in terms of the hole energy functions ϵ_r ($r = 1, 2, \ldots, \lambda + 1$) [11] (strictly speaking, we are going to consider just the difference of the g-functions at two different temperatures):

$$\ln g = \sum_{r=1}^{\lambda+1} \int_{-\infty}^{\infty} \frac{dv}{2\pi} \kappa_r[v - \ln(\theta/\theta_B)] \ln(1 + e^{-\epsilon_r(v)}), \tag{4}$$

where κ_r are the kernels whose Fourier transforms are

$$\begin{aligned} \tilde{\kappa}_n(y) &= \frac{\sinh y}{2 \sinh y \cosh \lambda y}, \\ \tilde{\kappa}_\lambda(y) &= \frac{\sinh(\lambda - 1)y}{2 \sinh 2y \cosh \lambda y}, \\ \tilde{\kappa}_{\lambda+1}(y) &= \tilde{\kappa}_\lambda(y) + \frac{1}{2\cosh y}, \\ \tilde{\kappa}_r(y) &= \int_{-\infty}^{\infty} \frac{dv}{2\pi} e^{i2\lambda v y/\pi} \kappa(v). \end{aligned} \tag{5}$$

The hole energies $\epsilon_r(v)$ satisfy the following TBA equation:

$$\epsilon_r(v) = \sum_{s=1}^{\lambda+1} a_{rs} \int_{-\infty}^{\infty} \frac{dv'}{2\pi} \frac{1}{\cosh(v - v')} \ln(1 + e^{\epsilon_s(v')}), \tag{6}$$

where a_{rs} is the incidence matrix of the $D_{\lambda+1}$-type Dynkin diagram;

$$\begin{aligned} a_{ij} &= \delta_{i,j+1} + \delta_{i,j-1} \qquad (i, j = 1, 2, \ldots, \lambda - 1), \\ a_{\lambda,j} &= \delta_{j,\lambda-1}, \qquad a_{\lambda+1,j} = \delta_{j,\lambda-1}. \end{aligned} \tag{7}$$

In the two limits, $\theta/\theta_B = 0$ and $\theta/\theta_B = \infty$, the above TBA equation can be solved analytically [11]. We call the former limit infrared (IR) and the latter ultraviolet (UV). The difference of the boundary entropies in these two limits is

$$\frac{g_{\rm UV}}{g_{\rm IR}} = \frac{R}{2\pi\sqrt{\alpha'}}. \tag{8}$$

As is conjectured by the g-theorem [9], g decreases along the RG flow from UV to IR if $R > 2\pi\sqrt{\alpha'}$; i.e., the boundary perturbation is relevant. g's in the two limits have been identified with the respective values of the tachyon actions [5,13]. We can compare the tensions of D25-branes and D24-branes, τ_{25} and τ_{24}, respectively. In this view, we should set $g_{\rm UV} = \tau_{25} R$ and $g_{\rm IR} = \tau_{24}$. Thus, we get the well-known relation $\tau_{24} = 2\pi\sqrt{\alpha'}\,\tau_{25}$.

Even at general θ, g is obtained numerically by means of TBA. We expect that this g will give the string action even in an intermediate region between the open string vacuum and the closed string one. The quantity $\ln(\theta/\theta_B)$ is identified with the RG scale. As an example, let us calculate the g-function for the $\lambda = 2$ (i.e., $R = 2\sqrt{3\alpha'}\,\pi$) case explicitly. The plot $\ln(\theta/\theta_B)$- $\ln g$ is shown in Fig. 1. As is seen in Fig. 1, g becomes stationary both as $\ln(\theta/\theta_B) \to -\infty$ and as $\ln(\theta/\theta_B) \to +\infty$. Having this aspect of the g-function in mind, we would like to gain more insight into the RG behavior of the tachyon condensation.

Let us consider the behavior of the g-function at large θ/θ_B in field theory analysis [9]. Let the dimension of the boundary term be Δ. We see that the β function of the coupling ζ is

$$\beta(\zeta) = \frac{d\zeta}{d\ln|x|} = (1 - \Delta)\zeta + O(\zeta^2), \tag{9}$$

where $|x|$ is the inverse of the momentum cutoff at the boundary and equal to the ratio, $(\theta/\theta_B)^{-1}$. Thus, we see the relation $\zeta \sim (\theta/\theta_B)^{-(1-\Delta)}$ for large θ/θ_B. Upon taking (1) into account, we obtain the asymptotic behavior

$$g \sim g_{\rm UV} - c_0(\theta/\theta_B)^{-2(1-\Delta)} \tag{10}$$

for small ζ, where c_0 is a constant. We compare this with the tachyon action later.

Tachyon field and its energy.—Let us consider the tachyon field with the codimension one, i.e., the case in which the tachyon field depends on just one coordinate $X^{25}(= x)$. The action obtained in [6,7] is

$$S = \tau_{25} V_{25} \int_{-\infty}^{+\infty} [\alpha' e^{-T} \dot{T}^2 + V(T)]\, dx, \tag{11}$$

where $V(T) = e^{-T}(T + 1)$ is the tachyon potential, $\dot{T} = dT/dx$, and V_{25} is the volume of 25-dimensional spacetime. Here we have ignored the higher derivative corrections. Let us set $\tau_{25} V_{25}$ to be $1/2\pi$ for simplicity and $\alpha' = 1$. Let us consider classical solutions of this action (11). The equation of motion obtained from (11) is

$$2\ddot{T} - \dot{T}^2 - e^T V'(T) = 0, \tag{12}$$

which can be integrated once to give

$$\dot{T} = \pm e^{T/2}\sqrt{E + V(T)}. \tag{13}$$

Here E is a constant that can be regarded as energy. If $-1 \le E < 0$, the tachyon field $T(x)$ looks like a classical

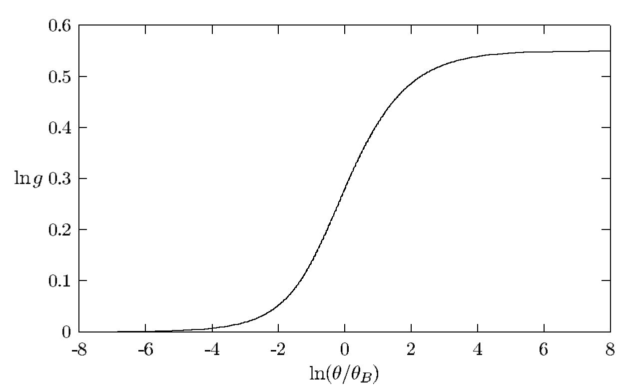

FIG. 1. Boundary entropy for $R = 2\sqrt{3\alpha'}\,\pi$.

lump. That is, $T(x)$ oscillates between T_i and T_f. The constants T_i and T_f are, respectively, the negative and positive solutions of the equation

$$E + V(T) = 0, \tag{14}$$

and we set $T(0) = T_i$ (see Fig. 2). For example, setting $E = -1$, we see $T_i = T_f = 0$; thus, $T(x) = 0$ for all $x \in (-\infty, +\infty)$, which is on the UV fixed point and regarded as the tachyonic open string vacuum. Another example is $E = 0$ which has $T_i = -1$ and $T_f = +\infty$. In this case, (13) is easily integrated to give

$$T(x) = -1 + x^2/4. \tag{15}$$

This form is already used in [7].

Let us evaluate (11) for given energy ($-1 \le E \le 0$). Because (11) naively diverges, it is regularized by introducing the cutoff like

$$S(E,R) = \int_{-R/2}^{R/2} \frac{dx}{2\pi} [e^{-T}\dot{T}^2 + V(T)]. \tag{16}$$

This cutoff R is identified with that in (2). Using (13), we see

$$S(E,R) = -\frac{RE}{2\pi} + 4\oint_{T_i}^{T_R} \frac{dT}{2\pi} e^{-T/2}\sqrt{E + V(T)}, \tag{17}$$

where we have set $T(R/2) = T(-R/2) = T_R$. From the form of (17), we conclude $S(-1,R) = R/2\pi$ and $S(0,R) = e/\sqrt{\pi} + O(e^{-R^2}/R)$. We should note that the precise value of $S(0,\infty)$ should be 1 as is seen in (8). In order to get this precise value, a more careful treatment to include the higher derivative terms, which we have ignored in (11), is necessary [7].

Let us concentrate on the region near the UV fixed point, namely, the region where the condensation is forming. Let $E = -1 + \epsilon^2$ ($0 < \epsilon \ll 1$). Because $|T_i|, |T_R|, T_f \ll 1$ there, we approximate $V(T) \cong 1 - T^2/2$. Thus, we see $-T_i \cong T_f \cong \sqrt{2}\,\epsilon$. After some elementary calculation, we obtain, for example,

$$S(-1 + \epsilon^2, R) = R/2\pi - \epsilon^2\sqrt{2}\sin(R/\sqrt{2}) + O(\epsilon^3) \quad \text{for } 2\sqrt{2}\,\pi < R < 3\sqrt{2}\,\pi. \tag{18}$$

Let us compare the action (18) with the g-function (10) of the BSG model with TBA. Comparing the scaling of g with θ/θ_B and the scaling of S with ϵ, we find

$$\epsilon \propto (\theta/\theta_B)^{-(1-\Delta)}. \tag{19}$$

Then we expect $S(-1 + \epsilon^2, R) = g(\theta/\theta_B, R)$ after fixing the constant c_0 in (10) appropriately. Let us consider the case where the cutoff is $R = 2\sqrt{3}\,\pi$. We have already shown the flow of the g-function in Fig. 1. The string action S can also be calculated numerically by using (14) and (17). Because, in this case, the boundary interaction has the dimension $\Delta = 1/3$, the two scaling parameters E and θ/θ_B should be related as $E + 1 \sim (\theta/\theta_B)^{-4/3}$. The plots are shown in Fig. 3, where we put $\epsilon \cong 5.4(\theta/\theta_B)^{-2/3}$. Figure 3 indicates the numerical agreement of the scalings for g and S for $\sqrt{E + 1} \lesssim 0.2$. When approaching the IR fixed point, $\theta/\theta_B \to 0$ and $E \to 0$, the higher derivative correction for (11) must become important in order that the relation between S and g holds in this region as well.

In this paper we have argued that the behavior of the boundary entropy g as a function of temperature can be identified with that of the tachyon action as energy, taking the boundary sine-Gordon model as an example. Let us finally argue that this correspondence is generic, not tied to the particular model studied here.

In Euclidean quantum field theory, inverse temperature is the size of the circle in the time direction. This is the information on the energy scale of g which lies at the boundary time circle. If g is to be identified with the value of the tachyon action (with all the higher derivatives included), which we have assumed in this paper, the energy scale of g should translate into that of the tachyon action. The only possibility is that the constant of motion with time obtained from the tachyon equation of motion is, in fact, temperature in the g side.

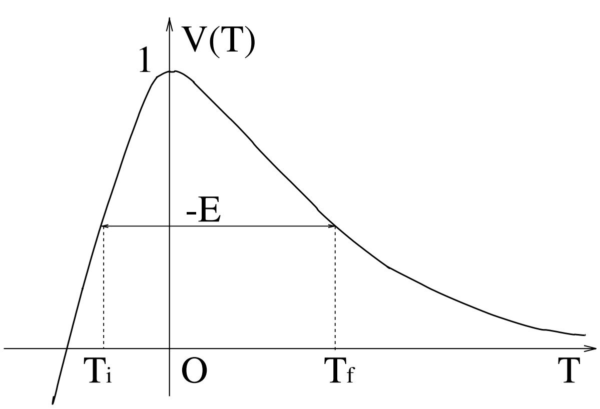

FIG. 2. Classical solution.

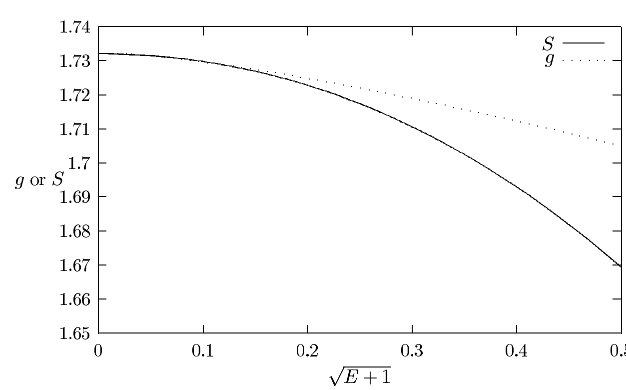

FIG. 3. g-function and tachyon action for $R = 2\sqrt{3}\,\pi$.

The authors thank T. Nakatsu, N. Ohta, K. Okuyama, and T. Suyama for helpful discussions.

*Present address: Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan.
Email address: fujii@het.phys.sci.osaka-u.ac.jp
†Email address: itoyama@funpth.phys.sci.osaka-u.ac.jp

[1] A. Sen, Int. J. Mod. Phys. A **14**, 4061 (1999).
[2] E. Witten, Nucl. Phys. **B268**, 253 (1986).
[3] E. Witten, Phys. Rev. D **46**, 5467 (1992); **47**, 3405 (1993).
[4] S. Shatashvili, Phys. Lett. B **311**, 83 (1993); hep-th/9311177.
[5] J. A. Harvey, D. Kutasov, and E. Martinec, hep-th/0003101.
[6] A. A. Gerasimov and S. L. Shatashvili, J. High Energy Phys. **0010**, 034 (2000).
[7] D. Kutasov, M. Mariño, and G. Moore, J. High Energy Phys. **0010**, 045 (2000); hep-th/0010108.
[8] A. A. Tseytlin, hep-th/0011033.
[9] I. Affleck and A. W. Ludwig, Phys. Rev. Lett. **67**, 161 (1991); Phys. Rev. B **48**, 7297 (1993).
[10] C. G. Callan and I. R. Klebanov, Phys. Rev. Lett. **72**, 1968 (1994); C. G. Callan, I. R. Klebanov, A. W. W. Ludwig, and J. Maldacena, Nucl. Phys. **B422**, 417 (1994).
[11] P. Fendley, H. Saleur, and N. P. Warner, Nucl. Phys. **B430**, 577 (1994).
[12] I. V. Cherednik, Theor. Math. Phys. **61**, 977 (1984); S. Ghoshal and A. B. Zamolodchikov, Int. J. Mod. Phys. A **9**, 3841 (1994).
[13] J. A. Harvey, S. Kachru, G. Moore, and E. Silverstein, J. High Energy Phys. **0003**, 001 (2000).