Chiral Symmetry and the Intrinsic Structure of the Nucleon

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Understanding hadron structure within the framework of QCD is an extremely challenging problem. In order to solve it, it is vital that our thinking should be guided by the best available insight. Our purpose here is to explain the model-independent consequences of the approximate chiral symmetry of QCD for two famous results concerning the structure of the nucleon. We show that both the apparent success of the constituent quark model in reproducing the ratio of the proton to neutron magnetic moments and the apparent success of the Foldy term in reproducing the observed charge radius of the neutron are coincidental. That is, a relatively small change of the current quark mass would spoil both results.

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The chiral properties of QCD have been the subject of considerable attention, from chiral quark models [1,2,3] to the less ambitious, but more systematic, approach of chiral perturbation theory [4]. Most recently one has begun to realize the importance of chiral symmetry in describing the dependence of hadron properties such as masses [5] and magnetic moments [6] on quark mass. This is vital if one is to compare lattice QCD calculations, which are presently confined to current quark masses, $\bar{m}$, of order 40–80 MeV or higher, with experimental data.

For our purposes the essential point is that chiral symmetry is dynamically broken. The resulting Goldstone bosons enter the calculation of hadron properties through loops which lead to a characteristic dependence on $\bar{m}$ which is not analytic. Indeed for the magnetic moment of the nucleons one finds a leading nonanalytic behavior proportional to $m_q^{1/2}$. In the chiral limit $m_\pi^2 \propto m_q$ and

$$\begin{aligned} \mu^p &= \mu_0^p - \alpha m_\pi + \mathcal{O}(m_\pi^2), \\ \mu^n &= \mu_0^n + \alpha m_\pi + \mathcal{O}(m_\pi^2). \end{aligned} \tag{1}$$

It is a crucial property of the leading nonanalytic (LNA) coefficient, α, that it is entirely determined by the axial charge of the nucleon and the pion decay constant (both in the chiral limit):

$$\alpha = \frac{g_A^2 M_N}{8\pi f_\pi^2}. \tag{2}$$

Taking the one-loop value of $g_A (= F_1 + D_1 = 0.40 + 0.61)$ from chiral perturbation theory [7] we find $\alpha = 4.41$. [Note that all magnetic moments will be in nuclear magnetons (μ_N) and all masses in GeV.]

Clearly the LNA term is large, of order 0.6 μ_N, at the physical pion mass. This is one-third of the magnetic moment of the neutron. Provided the $\mathcal{O}(m_\pi^2)$ terms are small at the physical pion mass we can use Eq. (1) to extract the proton and neutron magnetic moments in the chiral limit:

$$\begin{aligned} \mu_0^p &\cong \mu^p + \alpha m_\pi^{\rm phys}, \\ \mu_0^n &\cong \mu^n - \alpha m_\pi^{\rm phys}. \end{aligned} \tag{3}$$

One then finds a model-independent expression for the dependence of the proton to neutron magnetic moment ratio on the pion mass:

$$\frac{\mu^p}{|\mu^n|} = \frac{\mu_0^p}{|\mu_0^n|}\left(1 + \left[\frac{1}{|\mu_0^n|} - \frac{1}{|\mu_0^p|}\right]\alpha m_\pi\right) + \mathcal{O}(m_\pi^2). \tag{4}$$

Constraining the chiral expansions to reproduce the experimental proton moment μ^p and the experimental ratio $\mu^p/|\mu^n|$ provides

$$\mu_0^p = 3.41\ \mu_N, \qquad \frac{\mu_0^p}{|\mu_0^n|} = 1.37, \tag{5}$$

and

$$\frac{\mu^p}{|\mu^n|} = 1.37 + 0.09\frac{m_\pi}{m_\pi^{\rm phys}} + \mathcal{O}(m_\pi^2). \tag{6}$$

As a consequence of Eq. (6), we see that the ratio of the p to the n magnetic moments varies from 1.37 to 1.55 (a variation of order 13%) as m_π varies from 0 to $2m_\pi^{\rm phys}$. In terms of the underlying quark mass, such a variation corresponds to a current quark mass variation from 0 to just 20 MeV. Within the constituent quark model this ratio would remain constant at 3/2, independent of the change of quark mass.

A study by Leinweber *et al.* [6] suggests a new method for describing the mass dependence of baryon magnetic moments which satisfies the chiral constraints imposed by QCD. We briefly summarize the main results of that analysis. A series expansion of $\mu_{p(n)}$ in powers of m_π is not a valid approximation for m_π larger than the physical mass. On the other hand, the simple Padé approximant

$$\mu^{p(n)} = \frac{\mu_0^{p(n)}}{1 \pm \frac{\alpha}{\mu_0^{p(n)}} m_\pi + \beta^{p(n)} m_\pi^2}, \tag{7}$$

has the correct leading nonanalytic (LNA) behavior of chiral perturbation theory

$$\mu^{p(n)} = \mu_0^{p(n)} \mp \alpha m_\pi,$$

and also builds in the expected behavior at large m_π. At heavy quark masses we expect that the magnetic moment should fall off as the Dirac moment

$$\mu = \frac{e_q}{2m_q} \propto \frac{1}{m_\pi^2}$$

as m_π becomes moderately large. (Note that this form is valid provided $m_\pi^2 \propto \bar{m}$, which seems to be true for m_π up to at least 1 GeV within lattice simulations.) A fit of the Padé approximant to lattice QCD data [8,9] leads to predictions of the magnetic moments of 2.90(20) and $-1.79(21)$ μ_N to be compared with 2.793 and -1.193 μ_N for p and n, respectively. The smooth transition between the chiral and heavy quark regimes provided by the Padé approximant models the lattice QCD results well.

Figure 1 shows a similar fit to the lattice data, this time constrained to pass through the experimental moments, and providing the solid curve in Fig. 2 for the p/n ratio of magnetic moments.

The Padé approximate fit parameters are $(\mu_0, \beta) = (3.33, 0.527)$ and $(-2.41, 0.427)$, for p and n, respectively.

Figure 2 also shows the result of the constituent quark model (dashed line) and the variation of the ratio predicted by the leading nonanalytic behavior of chiral perturbation theory in Eq. (6) (dotted line). The importance of the terms of order m_π^2 and higher are revealed by the ratio calculated using the Padé approximant of Eq. (7) (solid curve). The values of $\mu_0^{p(n)}$ vary slightly in the chiral expansion and the Padé due to these small higher order corrections at the physical pion mass. However, it is important to note that the slopes of the curves agree exactly in the chiral limit, as demanded by chiral perturbation theory.

The key point is that the ratio displays a significant quark mass dependence. It is roughly linear in m_π until m_π is of order $2m_\pi^{\mathrm{phys}}$. It is amusing to imagine the excitement had the pion mass been 100 MeV heavier at 240 MeV where the Padé crosses the constituent quark model prediction of $3/2$. However the constituent quark model prediction really corresponds to the $m_\pi \to \infty$ limit, and Fig. 2 suggests this limit is approached rather slowly.

The surprising consequences of chiral symmetry for this famous ratio naturally lead us to reconsider the neutron charge radius. The squared charge radius of the neutron ($\langle r^2 \rangle_{\mathrm{ch}}^n$) is obtained from the slope of the neutron electric form factor, $G_{En}(Q^2)$ as $Q^2 \to 0$:

$$\langle r^2 \rangle_{\mathrm{ch}}^n = -6 \frac{d}{dQ^2} G_E(Q^2)|_{Q^2=0}\,. \tag{8}$$

The Sachs electric and magnetic form factors can be written in terms of the covariant vertex functions F_1 and F_2 as

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2)\,, \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2)\,. \end{aligned} \tag{9}$$

Note that for a neutral charge particle $F_1(Q^2 = 0)$ vanishes and hence $F_2(Q^2 = 0)$ is simply the magnetic moment of the particle. Now the charge radius squared of the neutron can be written as

$$\langle r^2 \rangle_{\mathrm{ch}}^n = -6 \frac{d}{dQ^2} F_{1n}(Q^2)|_{Q^2=0} + \frac{3}{2} \frac{\mu^n}{M_N^2}\,. \tag{10}$$

Experimentally $\langle r^2 \rangle_{\mathrm{ch}}^n = -0.113 \pm 0.003 \pm 0.004$ fm^2 [10], while the last term in Eq. (10), the Foldy term [11], is numerically -0.126 fm^2.

The close agreement between the Foldy term and the observed mean square charge radius of the neutron has

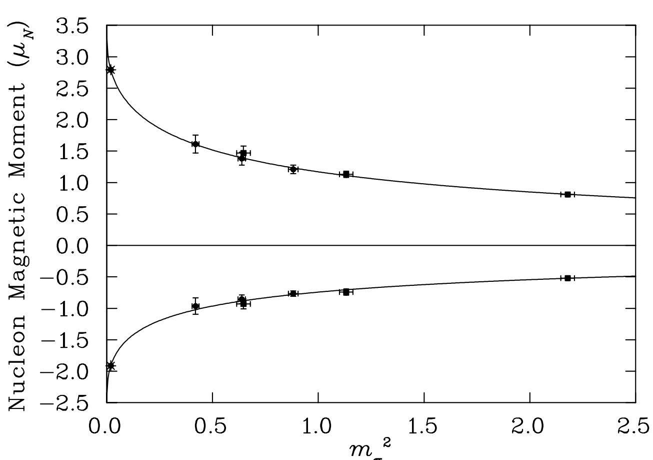

FIG. 1. Extrapolation of lattice QCD magnetic moments (•, LDW [8]; ■, WDL [9]) for the proton (upper curve) and neutron (lower curve) to the chiral limit. The curves are constrained to pass through the experimentally measured moments which are indicated by asterisks.

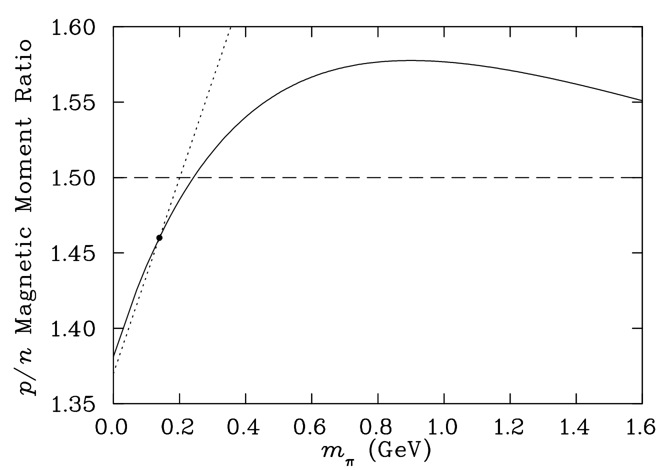

FIG. 2. Ratio of the magnitudes of the proton to neutron magnetic moments. The solid curve describes the predictions of the Padé approximant while the dashed line denotes the constituent quark model prediction of 3/2. The dotted line is the leading nonanalytic behavior of chiral perturbation theory. The experimental measurement is indicated by the solid point.

led to considerable controversy. It has been argued that the difference, namely, the term involving the Dirac form factor (F_{1n}), should be interpreted as the true indication of the intrinsic charge distribution of the neutron. Clearly this would be quite insignificant. On the other hand, decades of modeling the structure of the nucleon have suggested that the neutron must have a nontrivial intrinsic charge distribution. Pre-QCD it was clear that the long-range tail must be negative, corresponding to the emission of a negative pion ($n \to p\pi^-$), but old-fashioned meson theory was incapable of describing the interior of the neutron. Post-QCD this was resolved in the cloudy bag model [3,12], where the convergence of an expansion in numbers of pions was assured—provided the quark confinement region was fairly large and the decuplet states [in this case the $\Delta(1232)$] was included on the same footing as the nucleon [13]. The neutron charge distribution then originated mainly from the Fock component of its wave function consisting of a π^- cloud and a positive core of confined quarks. Alternatively, within the constituent quark model, it was proposed that the repulsive gluon exchange interaction between the two d quarks would tend to force them to the exterior of the neutron—again yielding a positive core and a negative tail [14].

In view of these expectations of an internal charge distribution, the interpretation of $\langle r^2\rangle_{\rm ch}^n$ in terms of the Foldy term has been controversial. Isgur [15] has recently shown that a careful treatment of relativistic corrections for the calculation of $\langle r^2\rangle_{\rm ch}^n$, in a quark-di-quark model, leads to a recoil contribution that cancels the Foldy term exactly, hence restoring the interpretation in terms of an intrinsic charge distribution—see also [16]. We now show that the study of the chiral behavior of $\langle r^2\rangle_{\rm ch}^n$ and μ^n supports this idea, establishing in a model-independent way that the observed similarity between the experimental value and the Foldy term is purely accidental.

It is a little appreciated consequence of the approximate chiral symmetry of QCD that the mean square charge radius of the nucleon has a leading nonanalytic term proportional to $\ln m_\pi$ [17]:

$$\langle r^2\rangle_{\rm ch}^{p(n)}|_{\rm LNA} = \mp \frac{1 + 5g_A^2}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi}{\Lambda}\right), \tag{11}$$

where the upper and the lower sign correspond to p and n, respectively. As a result, the charge radii of both p and n diverge logarithmically as the quark mass tends to zero. Physically this is easy to understand; as $m_\pi \to 0$ the Heisenberg uncertainty principle allows the pion cloud, and therefore the charge density, to extend to infinite distance. For the magnetic moment, on the other hand, there is no divergence—indeed the neutron magnetic moment increases in magnitude by about 30% as the pion mass moves from its physical value to zero. (Loosely speaking, even though the pion may be at a large distance it moves slowly; its angular momentum is constrained to one by angular momentum conservation.)

To summarize, whereas a change of order 5 MeV in the light quark mass leads to a 30% change in the Foldy term, the neutron charge radius $\langle r^2\rangle_{\rm ch}^n$ becomes infinite. Hence, the similarity of $\langle r^2\rangle_{\rm ch}^n$ and the Foldy term is purely an accident. A small change in the quark mass leads to completely different values. This physics is not captured in the constituent quark model where a 5 MeV change in the light quark mass corresponds to a change in the constituent quark mass from roughly 340 to 335 MeV. In this case the neutron charge radius originates in the one-gluon-exchange interaction which is proportional to the inverse square of the constituent quark mass and therefore $\langle r^2\rangle_{\rm ch}^n$ would change by only 3%.

In summary, chiral perturbation theory provides model-independent constraints on the quark mass dependence of nucleon magnetic moments and charge radii which compel one to conclude that the apparent success of the constituent quark model to predict the p/n magnetic moment is accidental. Had the pion mass been lighter than the observed value, the p/n ratio would drop further from the constituent quark model prediction of $3/2$, the latter corresponding to the $m_\pi \to \infty$ limit. The coincidence of the Foldy term and the observed neutron charge radius is also accidental. Here a small change in the quark mass to the chiral limit increases the neutron moment by about 30% while the charge radius becomes infinite. These results, which are a rigorous consequence of the chiral symmetry of QCD, cannot be simulated in conventional constituent quark models.

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[1] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[2] G. E. Brown and M. Rho, Phys. Lett. **82B**, 177 (1979).
[3] S. Théberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D **22**, 2838 (1980); A. W. Thomas, Adv. Nucl. Phys. **13**, 1 (1984).
[4] S. Weinberg, Physica (Amsterdam) **96A**, 327 (1979); J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984); E. Jenkins, M. Luke, A. V. Manohar, and M. J. Savage, Phys. Lett. B **302**, 482 (1993); V. Bernard, N. Kaiser, and U.-G. Meißner, Int. J. Mod. Phys. E **4**, 193 (1995).
[5] D. B. Leinweber, A. W. Thomas, K. Tsushima, and S. V. Wright, Phys. Rev. D **61**, 074502 (2000).
[6] D. B. Leinweber, D. H. Lu, and A. W. Thomas, Phys. Rev. D **60**, 034014 (1999); E. J. Hackett-Jones, D. B. Leinweber, and A. W. Thomas, Phys. Lett. B **489**, 143–147 (2000).
[7] E. Jenkins, M. Luke, A. V. Manohar, and M. J. Savage, Phys. Lett. B **302**, 482 (1993).
[8] D. B. Leinweber, T. Draper, and R. M. Woloshyn, Phys. Rev. D **46**, 3067 (1992).
[9] W. Wilcox, T. Draper, and K. Liu, Phys. Rev. D **46**, 1109 (1992).
[10] S. Kopecky *et al.,* Phys. Rev. Lett. **74**, 2427 (1995).

[11] L. L. Foldy, Phys. Rev. **87**, 688 (1947).
[12] S. Theberge, G. A. Miller, and A. W. Thomas, Can. J. Phys. **60**, 59 (1982).
[13] L. R. Dodd, A. W. Thomas, and R. F. Alvarez-Estrada, Phys. Rev. D **24**, 1961 (1981).
[14] R. Carlitz, S. D. Ellis, and R. Savit, Phys. Lett. **64B**, 85 (1976); N. Isgur, G. Karl, and R. Koniuk, Phys. Rev. Lett. **41**, 1269 (1978).
[15] N. Isgur, Phys. Rev. Lett. **83**, 272 (1999).
[16] M. Bawin and S. A. Coon, nucl-th/0101005 [Nucl. Phys. A (to be published)].
[17] M. A. B. Bég and A. Zepeda, Phys. Rev. D **6**, 2912 (1972); J. Gasser, M. E. Sainio, and A. Svarc, Nucl. Phys. **B307**, 779 (1988); D. B. Leinweber and T. D. Cohen, Phys. Rev. D **47**, 2147 (1993).