Comment on "Invalidation of the Kelvin Force in Ferrofluids"

In a recent Letter [1], Odenbach and Liu claim that their experimental results for the force on a container filled with ferrofluid in an inhomogeneous external magnetic field invalidate the standard Kelvin expression $\mathbf{f} = \mu_0(\mathbf{M} \cdot \nabla)\mathbf{H}$ for the magnetic force density in a magnetizable medium. It is the purpose of this Comment to point out that the described experiment measuring the *total* force on a magnetizable body cannot verify or falsify different expressions for the magnetic force *density* without taking into account the corresponding surface contributions.

The magnetic force on a magnetizable body in thermodynamic equilibrium in an external magnetic field can be determined in a simple and unambiguous way. The change of the free energy of the body due to variations of the external field is well-known to be [2]

$$\delta F = -\mu_0 \int_V d^3r\, \mathbf{M}(\boldsymbol{r}) \cdot \delta \mathbf{H}_0(\boldsymbol{r}), \qquad (1)$$

where the integral is over the volume of the body, $\mathbf{M}(\boldsymbol{r})$ is its local magnetization, and $\mathbf{H}_0(\boldsymbol{r})$ denotes the external field in the absence of the body. If the change in the field is due to a displacement of the body by an infinitesimal vector $\delta\boldsymbol{r}$, we have $\delta\mathbf{H}_0 = (\delta\boldsymbol{r} \cdot \nabla)\mathbf{H}_0$. At the same time, the corresponding change in free energy is given by $\delta F = -\mathbf{F} \cdot \delta\boldsymbol{r}$, where $\mathbf{F}$ is by definition the total force on the body. Using $\nabla \times \mathbf{H}_0 = 0$, we find

$$\mathbf{F} = \mu_0 \int_V d^3r (\mathbf{M} \cdot \nabla)\mathbf{H}_0 . \qquad (2)$$

This is a generally valid expression, subject only to the constraint of thermodynamic equilibrium. In particular, it does correctly describe the experimental findings reported in [1].

By formal manipulations, expression (2) can be decomposed into a surface and a volume part in various ways. Besides the decomposition advocated in [1], there is the standard possibility to use the Kelvin force density $\mathbf{f} = \mu_0(\mathbf{M} \cdot \nabla)\mathbf{H}$ in the volume and a surface integral over $\mu_0 M_n^2/2$, with M_n denoting the normal component of the magnetization [3]. If consistently used, all these decompositions, including the latter one using the Kelvin force density, are equivalent to (2), and therefore describe the experimental findings equally well.

The expression for the force suggested in [1] [their Eq. (6)] differs from (2) by a factor $(1 + \chi)/(1 + D\chi)$ with D denoting the demagnetization factor. This is probably due to the fact that at the same time where demagnetization effects are taken into account also contributions from the surface integral (in their case involving M_t^2) matter. Since in the experiment $D \cong 0.9694$, the difference between their result (6) and the correct expression (2) is too small to cause noticeable differences with the experiment.

In conclusion, the main aim of the Letter, namely, to invalidate the Kelvin force on the basis of experimental facts, was not accomplished. Moreover, the variant expression (6) offered as an alternative to describe the experiment is incomplete due to the neglect of surface contributions.

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