Isotope Effects in Underdoped Cuprate Superconductors: A Quantum Critical Phenomenon

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We show that the unusual doping dependence of the isotope effects on transition temperature and zero temperature in-plane penetration depth naturally follows from the doping driven 3D-2D crossover and the 2D quantum superconductor to insulator transition in the underdoped limit. Since lattice distortions are the primary consequence of isotope substitution, our analysis clearly reveals the strong involvement of lattice degrees of freedom in mediating superconductivity.

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The observation of an unusual isotope effect in underdoped cuprate superconductors on transition temperature [1–3] and zero temperature penetration depth [4–7] poses a challenge to the understanding of high temperature superconductivity. For some time, superconductivity in the cuprates has been believed to occur in a homogeneous system, through magnetic mechanism. However, the experimental evidence for spatial charge and spin inhomogeneities and lattice effects has been accumulating over the last years [8–14]. There is an emerging point of view that in the underdoped regime, where the transition to the insulator occurs, these inhomogeneities are the result of strong electron-electron and electron-lattice interactions. Since lattice distortions are the primary consequence of isotope substitution, the isotope effect on superconducting properties should provide unambiguous evidence for the relevance of the lattice degrees of freedom.

In this Letter we show that the anomalous isotope effect on transition temperature and penetration depth naturally follows from the doping driven 3D-2D crossover, the 2D quantum superconductor to insulator (QSI) transition in the underdoped limit and the shift of this limit upon isotope substitution. Thus, the unusual isotope effect on superconducting properties of underdoped cuprates is derived to be a 2D quantum critical phenomenon tuned by variation of the dopant concentration and lattice distortions.

Consider the empirical phase diagram of $La_{2-x}Sr_xCuO_4$ [15–23] depicted in Fig. 1. It shows that after passing the so-called underdoped limit ($x = x_u \approx 0.05$), where the QSI transition occurs [24,25], T_c rises and reaches a maximum value T_c^m at $x_m \approx 0.15$. With further increase of x, T_c decreases and finally vanishes in the overdoped limit $x_o \approx 0.3$. This phase transition line $T_c(x)$, separating the superconducting from the normal conducting phase appears to be a generic property of cuprate superconductors. In $La_{2-x}Sr_xCuO_4$, $HgBa_2CuO_{4+x}$ [27,28], and $Bi_2Sr_2CuO_{6+x}$ [29] both the underdoped and overdoped limits, corresponding to critical end points, are experimentally accessible. In other cuprates, including $Bi_2Sr_2CaCu_2O_{8+x}$ [30], $YBa_2Cu_3O_{7-x}$ [31], and $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ [32], only the underdoped and optimally doped regimes appear to be accessible. As shown in Fig. 1 for $La_{2-x}Sr_xCuO_4$ the effective mass anisotropy, measured in terms of $\gamma = \sqrt{M_c/M_{ab}}$ increases drastically by approaching the underdoped limit. This property, also observed in $HgBa_2CuO_{4+\delta}$ [28] and $YBa_2Cu_3O_{7-x}$ [31], appears to be generic and reveals the crossover from three- (3D) to two-dimensional (2D) behavior.

In our considerations the starting point is the critical end point of the phase transition line $T_c(x)$, where at $T = 0$ and $x = x_u$ the doping tuned QSI transition occurs. Close to such a critical point, low energy properties depend only on the spatial dimensionality of the system, the number of components of the order parameter and the range of the interaction. Since γ becomes very large in the underdoped regime (Fig. 1) and is expected to diverge at criticality ($x = x_u$), the bulk superconductors correspond to a stack of independent superconducting slabs of thickness d_s. The theory of quantum critical phenomena predicts that in $D = 2$ transition temperature and zero temperature in-plane penetration depths scale as [24,25,33]

$$T_c = a\delta^{z\overline{\nu}}, \qquad \frac{1}{\lambda_{ab}^2(0)} = b\delta^{z\overline{\nu}}. \tag{1}$$

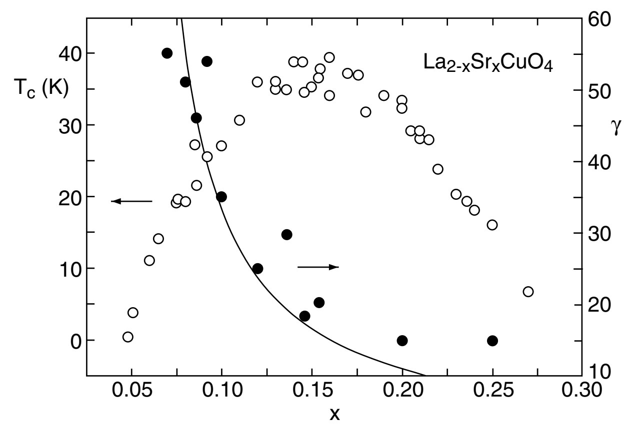

FIG. 1. T_c and γ versus x for $La_{2-x}Sr_xCuO_4$. T_c data taken from [15–27]. The solid curve corresponds to $\gamma \propto \delta^{-\overline{\nu}}$ with $\overline{\nu} = 1$, $\delta = x - x_u$, and $x_u = 0.05$.

z is the dynamic critical exponent, $\overline{\nu}$ is the exponent of the diverging length $\xi_{ab}(T=0) \propto \delta^{-\overline{\nu}}$, a and b denote nonuniversal critical amplitudes, and

$$\delta = x - x_u \tag{2}$$

measures the distance from the quantum critical point at x_u, where in cuprate superconductors the QSI transition occurs. For a complex scalar order parameter and in $D = 2$ the critical amplitudes a and b and the slab thickness d_s are not independent but related by the universal relation [24,25,33]

$$\lim_{\delta \to 0} \frac{T_c \lambda_{ab}^2(0)}{d_s} = \frac{a}{b d_s} = \frac{1}{\overline{Q}_2}\left(\frac{\Phi_0^2}{16\pi^3 k_B}\right), \tag{3}$$

where $\overline{Q}_2$ is a universal number. Although the experimental data are rather sparse close to the QSI transition, the overall picture turns out to be highly suggestive and provides consistent evidence for the QSI transition in $D = 2$ at the critical end point in the underdoped limit with critical exponents $z \approx 1$ and $\overline{\nu} \approx 1$ [24–26]. This estimate is close to theoretical predictions [34,35], from which $z = 1$ is expected for a bosonic system with long-range Coulomb interactions independent of dimensionality and $\overline{\nu} \geq 1 \approx 1.03$ in $D = 2$. In Fig. 1 it is seen that $\overline{\nu} \approx 1$ is also consistent with the doping dependence of the effective mass anisotropy $\gamma = \sqrt{M_c/M_{ab}} \propto \delta^{-\overline{\nu}}$. M_{ab} denotes the in-plane and M_c the out-of-plane effective mass of the Cooper pairs, entering the action of an anisotropic superfluid via the spatial gradient terms. This behavior is readily understood by noting that along the 3D-XY phase transition line for $T_c(x) > 0$ the universal relation, $k_B T_c = (\Phi_0^2/16\pi^3)\xi_{ab,0}^{-}/(\gamma \lambda_{ab,0}^2)$, holds [36]. $\xi_{ab,0}^{-}$ and $\lambda_{ab,0}$ are the finite temperature critical amplitudes of in-plane correlation length and penetration depth, respectively. Matching with the quantum behavior (1) requires $\xi_{ab,0}^{-} \propto \xi_{ab}(T=0) \propto \delta^{-\overline{\nu}}$ and $\lambda_{ab,0}^{-2} \propto \lambda_{ab}^{-2}(0) \propto \delta^{z\overline{\nu}}$ so that $\gamma \propto \delta^{-\overline{\nu}}$.

Given the evidence for a generic 2D-QSI transition in underdoped cuprate superconductors [24–26], we are now prepared to explore the implications on the isotope effects on T_c and $1/\lambda_{ab}^2(0)$. From the definition of the isotope coefficient

$$\beta_{T_c} = -\frac{m}{T_c}\frac{dT_c}{dm}, \tag{4}$$

and Eq. (1) we obtain the scaling expression

$$\beta_{T_c} = \beta_a + \beta_\delta, \tag{5}$$

where

$$\beta_a = -\frac{m}{a}\frac{da}{dm}, \qquad \beta_\delta = \frac{z\overline{\nu}}{\delta}\overline{\beta}_\delta, \qquad \overline{\beta}_\delta = -m\frac{d\delta}{dm}, \tag{6}$$

and m denotes the isotope mass. Since in the doping regime of interest, isotope substitution lowers the transition temperature [1,3], while the dopant concentration x remains nearly unchanged [6], there is a positive shift of the underdoped limit x_u, and $\overline{\beta}_\delta$ reduces to

$$\overline{\beta}_\delta = -m\frac{d\delta}{dm} \approx \overline{\beta}_{x_u} = m\frac{dx_u}{dm} > 0. \tag{7}$$

Thus, by approaching the QSI transition β_δ diverges as $\beta_\delta \propto \delta^{-1}$ [Eq. (6)], and provided that β_a remains finite, β_{T_c} is predicted to tend to β_δ so that

$$\frac{1}{\beta_{T_c}} \to \frac{1}{\beta_\delta} = \overline{r}\left(\frac{T_c}{T_c^m}\right)^{1/z\overline{\nu}}, \qquad \overline{r} = \frac{1}{z\overline{\nu}\,\overline{\beta}_{x_u}}\left(\frac{T_c^m}{a}\right)^{1/z\overline{\nu}}. \tag{8}$$

Here we expressed δ in terms of T_c [Eq. (1)] and rescaled T_c by T_c^m, the transition temperature at optimum doping, to reduce variations of T_c between different materials [2,3].

To confront this prediction with experiment we show in Fig. 2 $1/\beta_{T_c}$ versus T_c/T_c^m for $La_{1.85}Sr_{0.15}Cu_{1-x}Ni_xO_4$ [37], $YBa_{2-x}La_xCu_3O_7$ [38], and $Y_{1-x}Pr_xBa_2Cu_3O_7$ [39]. As predicted, approaching the QSI transition ($T_c/T_c^m = 0$), the data tend to collapse on a straight line, consistent with the expected exponents $z\overline{\nu} \approx 1$. It also confirms that β_a is finite and $\overline{\beta}_\delta \approx \overline{\beta}_{x_u} > 0$. In this context it is important to recognize that the 2D-QSI critical point exhibits an unexpectedly large critical region. Indeed, the linear relationship between T_c and $1/\lambda_{ab}^2(0)$ [Eq. (3)] holds reasonably well up to $T_c/T_c^m \lessapprox 0.5$ [25,26]. Thus, a 2D-OSI transition accounts remarkably well for the unusual doping dependence of β_{T_c}. Moreover, the resulting relation for the maximum transition temperature, $T_c^m \approx \overline{r} a \overline{\beta}_{x_u} = \overline{r} a m\, dx_u/dm$ ($\overline{r} \approx 6$), reveals the strong involvement of lattice degrees of freedom in determining the maximum transition temperature.

Another important element brought by the QSI transition in $D = 2$ is the universal relation given in Eq. (3).

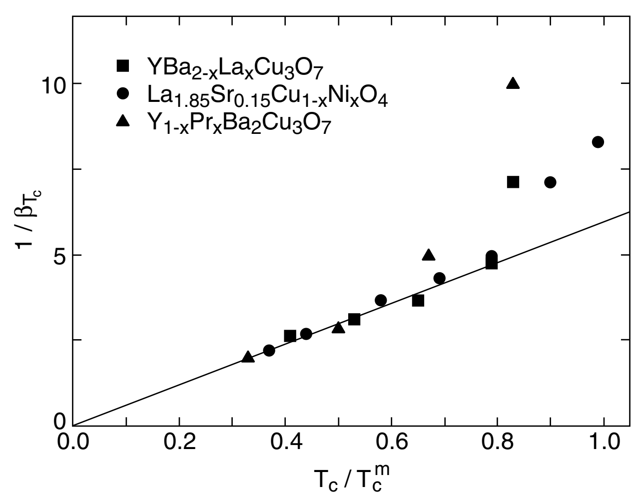

FIG. 2. Inverse isotope coefficient $1/\beta_{T_c}$ versus T_c/T_c^m for $La_{1.85}Sr_{0.15}Cu_{1-x}Ni_xO_4$ [37], $YBa_{2-x}La_xCu_3O_7$ [38], and $Y_{1-x}Pr_xBa_2Cu_3O_7$ [39]. The straight line corresponds to Eq. (8) with $z\overline{\nu} = 1$ and $\overline{r} = 6$.

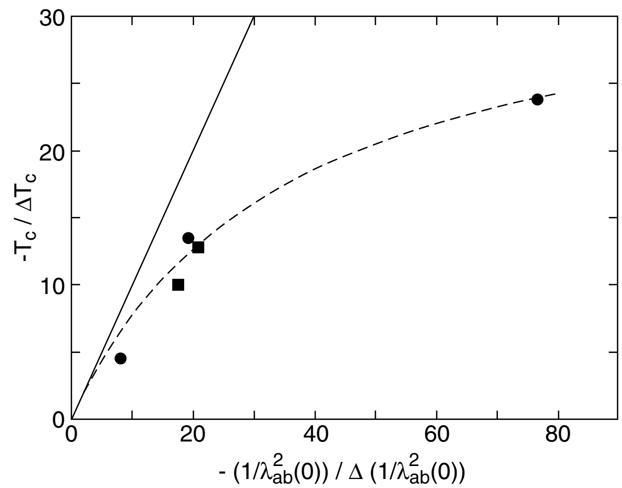

FIG. 3. Oxygen isotope effect for underdoped $La_{2-x}Sr_xCuO_4$ [6,7] in terms of $-T_c/\Delta T_c \propto 1/\beta_{T_c}$ versus $-(1/\lambda^2_{ab}(0))/\Delta(1/\lambda^2_{ab}(0)) \propto \beta_{1/\lambda^2_{ab}}$. The dashed line indicates the approach to the asymptotic behavior marked by the straight line [Eq. (10)]. ●: taken from [6]; and ■: taken from [7].

It implies with Eq. (5) that the isotope coefficients of T_c, $1/\lambda^2_{ab}$, critical amplitudes a and b, and slab thickness d_s are related by

$$\beta_{T_c} = \beta_{1/\lambda^2_{ab}} + \beta_{d_s}, \qquad \beta_a = \beta_b + \beta_{d_s}, \tag{9}$$

where $\beta_F = -\frac{m}{F}\frac{dF}{dm}$ and $F = T_c$, $1/\lambda^2_{ab}$, d_s, a, and b. Noticing that β_a was confirmed to be bounded, this is also true for β_b and β_{d_s}. Since β_{T_c} diverges as $\beta_{T_c} = \beta_\delta = (T_c/T_c^m)^{-1/z\overline{\nu}}/\overline{r}$ [Eq. (8)], $\beta_{1/\lambda^2_{ab}}$ is predicted to approach

$$\beta_{T_c} = \beta_{1/\lambda^2_{ab}}, \tag{10}$$

close to the QSI transition. Although the experimental data for $\beta_{1/\lambda^2_{ab}}$ and β_{T_c} on identical samples are rather sparse, the results shown in Fig. 3 for the oxygen isotope effect in $La_{2-x}Sr_xCuO_4$ [6,7] reveal considerable consistency with a crossover to the predicted 2D-QSI criticality. Indeed, as the underdoped limit is approached, the data points tend to the solid line, marking $1/\beta_{T_c} = 1/\beta_{1/\lambda^2_{ab}}$.

In conclusion, we have shown that a strong doping dependence of the isotope effects on transition temperature and zero temperature in-plane penetration depth in underdoped cuprates naturally follows from the doping driven 3D-2D crossover and the 2D-QSI transition in the underdoped limit. As the quantum superconductor to insulator transition is approached, the isotope coefficient of transition temperature β_{T_c} and penetration depth $\beta_{1/\lambda^2_{ab}}$ tend to the coefficient of the relative dopant concentration $\beta_\delta = \overline{\beta}_{x_u}/\delta$. Its divergence sets the scale, controlled by the shift of the underdoped limit $[\beta_{x_u} = m(dx_u/dm)]$. Although the experimental data shown in Figs. 2 and 3 are rather sparse and do not include the critical regime, they are fully consistent with the crossover to 2D-QSI criticality. Given the previous evidence for a 2D-QSI transition [24–26,36] and noting that isotope substitution induces lattice distortions but does not change the dopant concentration, the resulting empirical relation for the transition temperature at optimum doping, $T_c^m \approx \overline{r} a \overline{\beta}_{x_u}$ ($\overline{r} \approx 6$), clearly reveals the strong involvement of lattice degrees of freedom in determining the maximum transition temperature. Finally we hope that this novel point of view about the isotope effects in cuprate superconductors will stimulate further experimental work to obtain new data to confirm or refute our predictions.

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