Coulomb Interaction and Quantum Transport through a Coherent Scatterer

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(Received 31 October 2000)

An interplay between charge discreteness, coherent scattering, and Coulomb interaction yields nontrivial effects in quantum transport. We derive a real-time effective action and an equivalent quantum Langevin equation for an arbitrary coherent scatterer and evaluate its current-voltage characteristics in the presence of interactions. Within our model, at large conductances G_0 and low T (but outside the instanton-dominated regime), the interaction correction to G_0 saturates and causes conductance suppression by a universal factor which depends only on the type of the conductor.

DOI: 10.1103/PhysRevLett.86.4887 PACS numbers: 72.15.–v, 72.70.+m, 73.40.Gk

Coulomb effects in mesoscopic tunnel junctions have recently received a great deal of attention [1–3]. One of the remarkable features of such systems is that charge quantization (and, hence, Coulomb blockade) persists even for junctions with low resistances $1/G_t \ll R_Q = h/e^2 \approx 25.8\ \mathrm{k}\Omega$. In this limit an effective Coulomb gap $\tilde{E}_C$ for a junction with the "bare" charging energy E_C suffers exponential renormalization [4]

$$\tilde{E}_C/E_C \propto \exp(-G_t R_Q/2), \tag{1}$$

but remains finite even at very large values of $G_t R_Q$. Equation (1) was confirmed in several later studies both analytically [5] and numerically [6]. Experiments [7] clearly demonstrated the existence of charging effects for the values of G_t as large as $G_t R_Q \approx 33$.

Recently another interesting prediction was made by Nazarov [8], who argued that features of charge quantization may also persist in arbitrary conductors including, e.g., disordered metallic wires with $g = G_0 R_Q \gg 1$. Here and below $G_0 \equiv 1/R = (2e^2/h)\sum_n T_n$ is the conductance of an arbitrary scatterer and T_n are the transmissions of its conducting modes. If one accounts for the spin degeneracy, the renormalized Coulomb energy for a general conductor derived in [8] takes the form

$$\tilde{E}_C/E_C \propto \prod_n R_n, \tag{2}$$

where $R_n = 1 - T_n$. In particular, for diffusive conductors, similarly to Eq. (1), one finds [8,9] $\tilde{E}_C/E_C \propto \exp(-\pi^2 g/8)$. The same result (2) follows from the effective action derived in [2,10] for metallic contacts within the quasiclassical Green functions technique. Hence, one can expect the effective actions [2,10] and [8] to be equivalent, perhaps up to some unimportant details.

Equation (2) sets an important energy scale for the problem in question: at temperatures below an exponentially small value $\tilde{E}_C$ a conductor with $g \gg 1$ should show *insulating* behavior due to Coulomb effects. On the other hand, at larger temperatures/voltages this insulating behavior should not be pronounced. Furthermore, according to (2) Coulomb blockade is destroyed completely ($\tilde{E}_C \equiv 0$) even at $T = 0$ if at least one of the conducting channels is fully transparent $R_n = 0$ [11].

In this Letter we analyze an interplay between Coulomb effects and quantum transport at energies larger than $\tilde{E}_C$ (2). We derive a real-time effective action and formulate a quantum Langevin equation for an arbitrary (albeit relatively short) conductor. At temperatures or voltages above $\tilde{E}_C$ we obtain a complete I-V curve at large enough g. We demonstrate that Coulomb interaction leads to (partial) conductance suppression with respect to its "noninteracting" value G_0. This suppression effect is controlled by the parameter

$$\beta = \frac{\sum_n T_n(1 - T_n)}{\sum_n T_n}, \tag{3}$$

well known in the theory of shot noise [12]. The parameter β (3) equals one for tunnel junctions and $1/3$ for diffusive conductors. In contrast to $\tilde{E}_C$ (2), it vanishes only if *all* the conducting channels are fully transparent.

We identify four different regimes for the interaction correction to G_0. Let us display our results for a linear conductance $G(T)$. At $T/E_C \gg \max(1, g)$ perturbation theory in E_C (or in $1/T$) is sufficient. It yields

$$\frac{G}{G_0} \simeq 1 - \beta\left\{\frac{E_C}{3T} - \left(\frac{3\zeta(3)}{2\pi^4}g + \frac{1}{15}\right)\left(\frac{E_C}{T}\right)^2\right\}. \tag{4}$$

Here $\zeta(3) \simeq 1.202$ and g needs not to be necessarily large. For $g \gg 1$ there exist three further nonperturbative in the interaction regimes. At intermediate temperatures $gE_C \exp(-g/2) \ll T \ll gE_C$ we have

$$\frac{G}{G_0} \simeq 1 - \frac{2\beta}{g}\left[\gamma + 1 + \ln\left(\frac{gE_C}{2\pi^2 T}\right)\right], \tag{5}$$

where $\gamma \simeq 0.577$. Here energy relaxation plays an important role turning the power law dependence (4) into a much slower one (5). At even lower temperatures $T < gE_C \exp(-g/2)$ (but $T > \tilde{E}_C$) relaxation processes yield saturation of $G(T)$:

$$G/G_0 \simeq 1 - \beta + O(\beta/g). \tag{6}$$

It is remarkable that the result (6) does not depend on the charging energy E_C at all. In the tunneling limit (all

$T_n \ll 1$) the regime (6) does not exist. Two other regimes are already known for tunnel junctions: by setting $\beta = 1$ in Eqs. (4) and (5) we recover the results [13,14]. Finally, at $T < \tilde{E}_C$ instanton effects [4,5,8] become important, and the conductance G should vanish at $T = 0$. If, however, the instanton effects are suppressed, then $\tilde{E}_C = 0$ and Eq. (6) remains valid down to $T = 0$.

The model and effective action.—Now let us proceed with the derivation of the above results and the I-V curve. We consider an arbitrary scatterer between two big reservoirs. Similarly to Ref. [8] the scatterer length is assumed to be shorter than dephasing and inelastic relaxation lengths, so that phase and energy relaxation may occur only in the reservoirs and not during scattering. Coulomb effects in the scatterer region are described by an effective capacitance C. The charging energy $E_C = e^2/2C$, temperature T, as well as other energy scales are assumed to be smaller than the typical inverse scattering time (e.g., the Thouless energy in the case of diffusive conductors).

Quantum dynamics of our system is fully described by the evolution operator on the Keldysh contour. The kernel of this operator J may be represented as a path integral over the fermionic fields. Performing a standard Hubbard-Stratonovich decoupling of the interacting term in the Hamiltonian enables one to integrate out fermions. Then the kernel J acquires the form of the path integral over the Hubbard-Stratonovich fields on the forward (V_1) and backward (V_2) parts of the Keldysh contour

$$J = \int \mathcal{D}V_1\, \mathcal{D}V_2 \exp(iS[V]), \qquad (7)$$

where $S[V]$ is the effective action defined as

$$iS[V] = 2\,\mathrm{Tr}\ln\hat{\mathbf{G}}_V^{-1} + i\,\frac{C}{2}\int_0^t dt'\,(V_{LR1}^2 - V_{LR2}^2), \quad (8)$$

where $V_{LRi} \equiv V_{Li} - V_{Ri}$ are the voltage drops between the reservoirs. The Green-Keldysh matrix $\hat{\mathbf{G}}_V(X_1, X_2)$ [here $X = (t, \boldsymbol{r})$] obeys the 2×2 matrix equation

$$\left[i\frac{\partial}{\partial t_1}\hat{\mathbf{1}} - \hat{H}_0(\boldsymbol{r}_1)\hat{\mathbf{1}} + e\hat{\mathbf{V}}(X_1)\right]\hat{\mathbf{G}}_V = \delta(X_1 - X_2)\hat{\boldsymbol{\sigma}}_z, \qquad (9)$$

where $\hat{H}_0(\boldsymbol{r})$ is a free electron Hamiltonian for the system "scatterer + reservoirs," $\hat{\mathbf{V}}$ is the diagonal 2×2 matrix with the elements $\mathbf{V}_{ij} = V_i\delta_{ij}$, and $\hat{\boldsymbol{\sigma}}_z$ is the Pauli matrix. In the last term of Eq. (8) we already made use of our model and assumed that the fields $V_{1,2}$ do not depend on the coordinates inside the reservoirs, i.e., for the left (right) reservoir we put $V_j(t', \boldsymbol{r}) \equiv V_{L(R)j}(t')$.

In order to proceed we make use of the quantum Langevin equation approach [15]. In the case of metallic tunnel junction this approach was developed in Refs. [13,16,17]. Let us define $\varphi_{1,2}(t) = \int_0^t dt'\, eV_{1,2}(t')$ and $\varphi^+ = (\varphi_1 + \varphi_2)/2$, $\varphi^- = \varphi_1 - \varphi_2$. The key step is to treat quantum dynamics of the V fields within the quasiclassical approximation, i.e., to assume that fluctuations of $\varphi^-(t)$ are sufficiently small at all times. This assumption allows us to expand the exact effective action in powers of φ^- while keeping the full nonlinear dependence on the "center-of-mass" field φ^+. This approximation is known [13,17] to be particularly useful in the limit $g \gg 1$.

Expanding $\mathrm{Tr}\ln\hat{\mathbf{G}}_V^{-1}$ up to the second order in φ^- we obtain

$$2\,\mathrm{Tr}\ln\hat{\mathbf{G}}_V^{-1} \simeq 2\,\mathrm{Tr}\ln\hat{\mathbf{G}}^{-1}|_{\varphi^-=0} + iS_R - S_I, \qquad (10)$$

where

$$iS_R = \mathrm{Tr}[(\hat{G}_{11} + \hat{G}_{22})\hat{\dot{\varphi}}^-],$$
$$S_I = \mathrm{Tr}[\hat{G}_{12}\hat{\dot{\varphi}}^-\hat{G}_{21}\hat{\dot{\varphi}}^-], \qquad (11)$$

and $\hat{\dot{\varphi}}^-$ is the diagonal matrix with the elements $\dot{\varphi}_i^-\delta_{ij}$. The zero order term in the expansion (10) vanishes. The elements of the Green-Keldysh matrix $\hat{G}_{ij}$ depend only on V^+ (or φ^+) and can be expressed as follows:

$$\hat{G}_{11}(t_1, t_2) = -i\theta(t_1 - t_2)\hat{U}(t_1, t_2) + i\hat{U}(t_1, 0)\hat{\rho}_0\hat{U}(0, t_2),$$
$$\hat{G}_{21}(t_1, t_2) = -i\hat{U}(t_1, 0)\,(\hat{1} - \hat{\rho}_0)\hat{U}(0, t_2), \qquad (12)$$

and similarly for $\hat{G}_{12}$ and $\hat{G}_{22}$. Here and below integration over the spatial coordinates is implied in the products of operators. In (12) we have defined

$$\hat{U}(t_1, t_2) = \hat{T}\exp\left[-i\int_{t_1}^{t_2} dt'\,[\hat{H}_0 - eV^+(t', \boldsymbol{r})]\right] \quad (13)$$

as the evolution operators and $\hat{\rho}_0$ is the electron density matrix at equilibrium.

Next we define the conducting channels in a standard manner. They are just the transverse quantization modes in the reservoirs. Describing the longitudinal motion within one channel quasiclassically we define the free electron Hamiltonian in the reservoirs as follows:

$$\hat{H}_{0,mn} = -iv_m\delta_{mn}\frac{\partial}{\partial y}, \qquad (14)$$

where m, n are the channel indices and v_m is the channel velocity. In every channel the coordinate y runs from $-\infty$ to 0 for the incoming waves, and from 0 to $+\infty$ for the outgoing ones. The scattering matrix $\hat{S}$, which is assumed here to be energy independent, relates the amplitudes of incoming and outgoing modes as follows:

$$\psi_m(y = +0) = \sum_n S_{mn}\sqrt{v_n/v_m}\,\psi_n(y = -0), \quad (15)$$

where S_{mn} are the elements of the scattering matrix $\hat{S}$ defined in the basis $\psi_{0,m} = e^{iky}/\sqrt{v_m}$. The factor $\sqrt{v_n/v_m}$ appears in (15) since we work in the basis of the eigenfunctions of (14) $\psi_{0,m} = e^{iky}$. Finally, the matrix elements of the fluctuating voltages $V_j(t)$ [and, analogously, phases $\varphi_j^{\pm}(t)$] are $V_{j,mn}(t) = V_{j,m}(t)\delta_{mn}$, where $V_{j,m}(t) = V_{Lj}(t)$ for the left channels and $V_{j,m}(t) = V_{Rj}(t)$ for the right ones.

With the aid of (14) and (15) the evolution operators (13) can be evaluated exactly. Solving the corresponding Schrödinger equation and introducing a new coordinate $\tau = y/v_n$ we obtain

$$\hat{U}(t_2t_1;\tau_2\tau_1) = \delta(\tau_2 - \tau_1 - t_2 + t_1)e^{i\hat{\varphi}^+(t_2)}\{\hat{1} + \theta(\tau_2)\times\theta(-\tau_1)e^{-i\hat{\varphi}^+(t_2-\tau_2)}[\hat{S} - \hat{1}]e^{i\hat{\varphi}^+(t_1-\tau_1)}\}e^{-i\hat{\varphi}^+(t_1)}, \quad (16)$$

where $e^{i\hat{\varphi}^+}$ is the diagonal matrix with the elements $e^{i\varphi_n^+}$.

Now we are in a position to derive the effective action. Evaluating the first order term iS_R (11) with the aid of Eqs. (12) and (16), one finds

$$iS_R = -\frac{ig}{2\pi}\int_0^t dt'\,\varphi^-(t')\dot{\varphi}^+(t'). \quad (17)$$

Here $g = 2\,\mathrm{tr}[\hat{t}^+\hat{t}]$ is the dimensionless conductance of the scatterer expressed in terms of the transmission matrix $\hat{t}$. An analogous calculation of the second order term S_I yields

$$S_I = -\frac{g}{4\pi^2}\int_0^t dt'\int_0^t dt''\,\alpha(t'-t'')\varphi^-(t')\varphi^-(t'') \times\{\beta\cos[\varphi^+(t')-\varphi^+(t'')] + 1 - \beta\}, \quad (18)$$

where $\beta g = 2\,\mathrm{tr}[\hat{t}^+\hat{t}(1-\hat{t}^+\hat{t})]$ and we have defined $\alpha(t) = (\pi T)^2/\sinh^2[\pi T t]$. The function $\alpha(t)$ is directly related to the equilibrium density matrix $\hat{\rho}_0$. Combining the results (17) and (18) with the last term of Eq. (8) we arrive at the final expression for the effective action

$$iS = i\int_0^t dt\left[\frac{C}{e^2}\dot{\varphi}^+\dot{\varphi}^- + \frac{I_x}{e}\varphi^-\right] + iS_R - S_I. \quad (19)$$

In (19) we also included the term which accounts for an external current bias I_x. The action (17)–(19) has the same form as one derived within the same approximation for an effective system of a tunnel junction with the conductance βG_0 shunted by an Ohmic conductor $(1-\beta)G_0$. Note that this formal analogy allows us to reconstruct the result (6) without any calculation, since at low T electron tunneling across the junction βG_0 should be blocked due to Coulomb effects and the total conductance of such an effective system should approach $(1-\beta)G_0$.

Quantum Langevin equation.—Equations (17)–(19) are equivalent to the Langevin equation

$$\frac{C}{e}\ddot{\varphi}^+ + \frac{1}{eR}\dot{\varphi}^+ - I_x = \xi_1\cos\varphi^+ + \xi_2\sin\varphi^+ + \xi_3, \quad (20)$$

where the terms on the right-hand side account for the current noise. Here $\xi_j(t)$ are Gaussian stochastic variables defined by the correlators

$$\langle|\xi_{1,2}|^2_\omega\rangle = \frac{\beta}{R}\,\omega\coth\frac{\omega}{2T}, \qquad \langle|\xi_3|^2_\omega\rangle = \frac{1-\beta}{R}\,\omega\coth\frac{\omega}{2T}. \quad (21)$$

In the small transparency limit Eqs. (20) and (21) reduce to those derived before for metallic tunnel junctions [16,17]. If we decompose $\varphi^+(t) = eVt + \delta\varphi^+$ (V is the average voltage across the conductor), neglect the fluctuating part of the phase $\delta\varphi^+$, and define the total fluctuating current $\delta I(t) = \xi_1\cos eVt + \xi_2\sin eVt + \xi_3$, we immediately reproduce the well known result for the current noise in mesoscopic conductors [12]:

$$\langle|\delta I|^2_\omega\rangle = \frac{e^2}{\pi}\left\{\omega\coth\frac{\omega}{2T}\sum\nolimits_n T_n^2 + \frac{1}{2}\left[(\omega + eV)\coth\frac{\omega+eV}{2T} + (\omega - eV)\coth\frac{\omega-eV}{2T}\right]\sum\nolimits_n T_n(1-T_n)\right\}.$$

This observation as well as the dependence of the interaction correction to the conductance on the parameter β (3), $\delta G = G_0 - G \propto \beta$, makes a close relation between noise and interaction effects particularly transparent. Interaction effects are mostly pronounced for tunnel junctions $\beta \to 1$ and vanish completely for ballistic noiseless systems $\beta \to 0$.

I-V curve.—In order to study the influence of Coulomb effects on the current-voltage characteristics for an arbitrary scatterer we make use of the exact identity

$$\int \mathcal{D}\varphi^+\mathcal{D}\varphi^-\,i\,\frac{\delta S[\varphi^+,\varphi^-]}{\delta\varphi^-(t)}\,e^{iS[\varphi^+,\varphi^-]} \equiv 0. \quad (22)$$

Evaluating this path integral we set $\cos[\varphi^+(t') - \varphi^+(t'')] = \cos[eV(t'-t'')]$ in the exponent of (22) but retain the full nonlinearity in $\delta S/\delta\varphi^-$. This approximation works well provided either $g \gg 1$ or $\max(T, eV) \gg E_C$. A straightforward calculation then yields

$$I_x = \frac{V}{R} - \frac{e\beta}{\pi}\int_0^{+\infty} dt\,\alpha(t)e^{-F(t)}(1 - e^{-\frac{t}{RC}})\sin[eVt], \quad (23)$$

$$F(t) = -\frac{1}{g}\int_{-\infty}^{+\infty} dt'\,\alpha(t')\,(\beta\cos[eVt'] + 1 - \beta) \times[|t'-t| - |t'| + RC(e^{-|t'-t|/RC} - e^{-|t'|/RC})]. \quad (24)$$

Equations (23) and (24) represent the central result of this paper. This result can also be derived directly from the Langevin equation (20).

Single scatterer.—In the limit $g \gg 1$ and $\max(eV, T) \gg gE_C\exp(-g/2)$ the integral in (23) converges at times for which $F(t)$ is still small and can be neglected. In this limit Eq. (23) yields

$$I_x = \frac{V}{R} - e\beta T\,\mathrm{Im}\left[w\Psi\left(1+\frac{w}{2}\right) - iv\Psi\left(1+\frac{iv}{2}\right)\right], \quad (25)$$

where $\Psi(x)$ is the digamma function, $w = u + iv$, $u = gE_C/\pi^2 T$, and $v = eV/\pi T$. At $T \to 0$ from (25) we obtain

$$R\frac{dI_x}{dV} = 1 - \frac{\beta}{g}\ln\left(1 + \frac{1}{(eVRC)^2}\right), \qquad (26)$$

while in the limit $eV/E_C \gg \max(1, g)$ we find

$$RI_x = V - \beta e/2C. \qquad (27)$$

For $\beta = 1$ the result (26) reduces to that derived in Ref. [18] for tunnel junctions. Equation (27) demonstrates that at large V the I-V curve of *any* relatively short conductor should be offset by the value $\beta e/2C$ due to Coulomb effects. For instance, in disordered conductors this offset is expected to be only 3 times smaller than for a tunnel junction with the same E_C. At $V \to 0$ from (25) we get

$$G/G_0 = 1 - \frac{2\beta}{g}\left[\gamma + \Psi\left(1 + \frac{u}{2}\right) + \frac{u}{2}\Psi'\left(1 + \frac{u}{2}\right)\right], \qquad (28)$$

which yields Eqs. (4) and (5). [The term with $1/15$ in (4) is recovered from more general Eqs. (23) and (24).]

In the limit $\max(eV, T) < gE_C\exp(-g/2)$ the integral (23) converges at very long times and the function F (24) cannot be disregarded. Evaluating (24) at $t \gg 1/RC$ we find $F(t) \simeq (2/g)[\ln(t/RC) + \gamma]$ and performing the integral in (23) for $g \gg 1$ we arrive at the result (6) $G = (2e^2/h)\sum_n T_n^2$. Hence, at very low T the conductance $G(T)$ *saturates* due to Coulomb and relaxation effects. For diffusive conductors Eq. (6) yields $G/G_0 \simeq 2/3$.

In order to better understand this effect let us recall that for tunnel junctions with $g \gg 1$ the interaction term is known to nearly fully (up to terms $\sim 1/g$) compensate G_0 at $T \sim gE_C\exp(-g/2)$ (Coulomb blockade). At lower T instanton effects [4,5] gain importance and eventually turn a conductor into an insulator at $T = 0$. For a general scatterer with $\beta < 1$ and $g \gg 1$ the interaction effects are reduced by the factor β. Hence, at $T \lesssim gE_C\exp(-g/2)$ no compensation can occur and G can become smaller than G_0 only by the factor $1 - \beta$. On the other hand, instanton effects [8] (causing further conductance suppression) are only important at $T \lesssim \tilde{E}_C$ well below $gE_C\exp(-g/2)$. Thus, at $\tilde{E}_C \lesssim T \lesssim gE_C\exp(-g/2)$ one has $G/G_0 = 1 - \beta$ in agreement with our result (6).

Two scatterers.—The effects discussed here can be conveniently measured, e.g., in the "SET transistor" configuration [1,3] of two scatterers connected by a small metallic island. Each of these scatterers serves as an effective environment for the other. With simple modifications our results hold for such two scatterer systems as well. For instance, in Eqs. (26) and (28) R is now the sum of two resistances $R_1 + R_2$, $u \to (g_1 + g_2)E_C/\pi^2 T$ and

$$\beta/g \to (\beta_1 g_2 + \beta_2 g_1)/(g_1 + g_2)^2. \qquad (29)$$

The I-V curve is offset at high voltages as in Eq. (27) with $\beta \to \beta_1 + \beta_2$ and C being the total capacitance of the device. Gate modulation and environmental effects are treated the same way as it was done in Refs. [13,17].

In summary, we studied the effect of Coulomb interaction on the I-V curve of a coherent scatterer. At low T its conductance is suppressed by the universal factor (6). Our results emphasize a direct relation between noise and interaction effects in mesoscopic conductors.

We thank Yu. V. Nazarov and G. Schön for useful discussions.

Note added.—After this Letter had already been submitted more works on the subject appeared [19–21]. The results (5) and (26) fit well to the experimental data [19] for short diffusive conductors with $g \approx 2000$. In the experiments [20] the case $g \approx 10$ was realized. The data confirm our results (26) and (6); both fits yield $\beta \approx 1/3$. In a theoretical paper [21] the effect of a linear environment on the electron transport through a single channel scatterer was treated perturbatively in the interaction. The result [21] agrees with our Eqs. (26) and (29) at $g_2 \gg g_1 \gtrsim 1$ if we identify R_Q/g_2 with the environment resistance R_s.

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