Rotations in Isospace: A Doorway to the Understanding of Neutron-Proton Superfluidity in $N = Z$ Nuclei

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(Received 13 October 2000)

The $T = 2$ excitations in even-even $N = Z$ nuclei are calculated within the isospin cranked mean-field approach. The response of pairing correlations to rotation in isospace is investigated. Whereas the isovector pairing rather modestly modifies the single-particle moment of inertia in isospace, the isoscalar pairing strongly reduces its value. This reduction of the isomoments of inertia with respect to its rigid body value is a strong indicator of collective isoscalar pairing correlations. These results are further generalized yielding beautiful analogies between the role of isovector pairing for the case of spatial rotations and the role of isoscalar pairing for the case of isorotations.

DOI: 10.1103/PhysRevLett.86.4488 PACS numbers: 21.60.–n, 21.10.–k, 74.20.–z

The ground state of most nuclei can be characterized in terms of a superfluid condensate of Cooper pairs which are formed by nucleons moving in time reversed orbits [1]. Nuclei with the identical number of protons and neutrons, $N = Z$, exhibit an additional symmetry, related to the similarity of proton and neutron wave functions at the Fermi surface. In such a case protons and neutrons occupying identical spatial orbitals may form two fundamentally different Cooper pairs of either isovector ($T = 1$) or isoscalar ($T = 0$) type [2]. The question whether isoscalar pairing may form a condensate similar to the well-established isovector pairing has gained considerable interest in recent time.

Already early on it was noticed that the understanding of excitation energies of the isobaric analog states yield important information on the effective nuclear force, in particular, also on its pairing component [3,4]; see also recent works [5,6]. Therefore, we analyze the $T = 2$ excitations in even-even $N = Z$ nuclei by means of the cranking approximation in isospace. This approximation has been tested within an exactly solvable model by Chen *et al.* [7] where it was concluded that, in conjunction with number projection, it offers a reliable approximation to the exact solutions. Another motivation to apply the cranking approximation stems from the formal analogy between spatial and isospin rotations, following $a_I I(I+1)$ and $a_T T(T+1)$ patterns, respectively [8]. The crucial quantity of our investigation is the inertia parameter in isospace, a_T (reciprocal of the isomoment of inertia $\Im_T$). Indeed, the study of the rotational spectra was crucial in establishing evidence for superfluidity in atomic nuclei [9]. Similarly, our microscopic calculations of the nuclear inertia parameter a_T in isospace show that it strongly depends on the short range pairing correlations. However, in contrast to spatial rotations it is shown that a_T is rather insensitive to isovector but extremely sensitive to isoscalar proton-neutron pairing correlations.

Before entering the details of our model, let us consider a single-particle (sp) Routhian $\hat{H}^\omega = \hat{H}_{sp} - \hbar\omega\hat{t}_x$. For simplicity, let us also assume that the spectrum of $\hat{H}_{sp}$ is equidistant $[e_i = i\delta e]$ and isosymmetric. Hence, at $\hbar\omega = 0$ each eigenstate of $\hat{H}^\omega$ is fourfold degenerate. The isocranking term, $-\hbar\omega\hat{t}_x$, lifts the isospin but not Kramers degeneracy resulting in $|-\rangle = \frac{1}{\sqrt{2}}(|n\rangle - |p\rangle)$ and $|+\rangle = \frac{1}{\sqrt{2}}(|n\rangle + |p\rangle)$ doublets, with isoalignment of $\langle\pm|t_x|\pm\rangle = \pm 1/2$, respectively (Fig. 1). Clearly, the ground state configuration changes stepwise at the crossing frequencies: $\hbar\omega_c^{(n)} = \delta e, 3\delta e, 5\delta e, \ldots, (2n-1)\delta e$. At each crossing

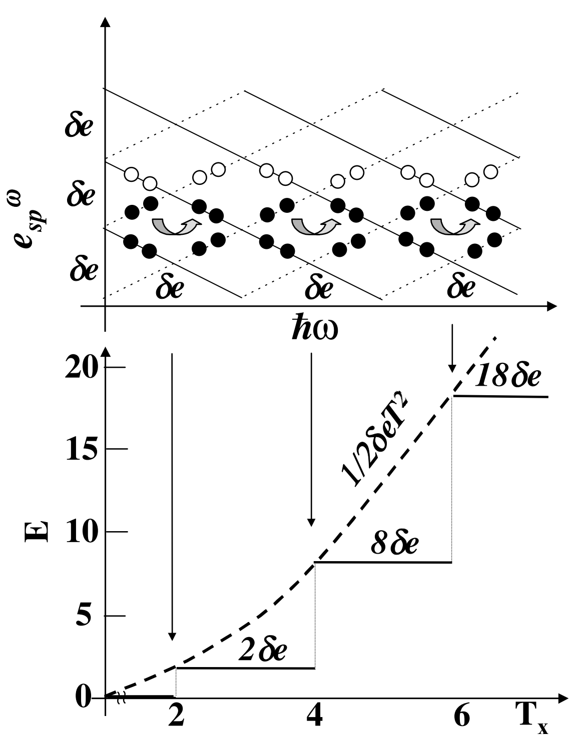

FIG. 1. The single-particle Routhians (upper panel) versus the isocranking frequency for the equidistant level model. Solid (dashed) lines depict $\tau_x = +1/2$ ($\tau_x = -1/2$) sp states, respectively. At each crossing frequency (indicated by arrows) the configuration changes, and hence excitation energy and isoalignment (lower panel).

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frequency two isoscalar pairs ($|+\rangle|-\rangle$) are emptied and two isovectors ($|+\rangle|+\rangle$) become occupied. Hence, the total isoalignment, $\langle \hat{t}_x \rangle \equiv T_x$, changes in steps of $\Delta T_x = 2$. Since at the same time $T_z = T_y \equiv 0$, i.e., $\Delta T_x \equiv \Delta T$, the stepwise increase corresponds to *noncollective isorotation*. The ground state band (gsb) consists of only even isospin states, $T = 0, 2, 4, \ldots, 2n$ in complete analogy to the even spin sequence in the gsb of spatially rotating even-even nuclei. Exploring further this analogy, one can show that the even-T sequence in the gsb is a consequence of isosignature symmetry, $\hat{R}_\tau = \exp(-i\pi\hat{t}_x)$, similar to the signature symmetry conservation, $\hat{R} = \exp(-i\pi\hat{j}_x)$, for rotational motion. It is of importance to underline that *odd-T states can be reached only by the proper particle-hole excitation at $\hbar\omega = 0$*.

Once the crossing frequencies are calculated, it is straightforward to compute the excitation energy E_T (with respect to the gs) spectrum of the isorotational gsb band:

$$E_T = E^\omega + \hbar\omega T_x = 2\sum_{i=1}^{T_x/2} \hbar\omega_c^{(i)} = \frac{1}{2}\,\delta e T_x^2 . \quad (1)$$

The schematic sp model leads to the classical rotational formula $\sim T_x^2$ with an inertia parameter proportional to the single-particle splitting at the Fermi energy.

Let us now briefly investigate how the sp model is affected by the presence of isovector pairing correlations. To study this issue we have performed a series of Lipkin-Nogami calculations for selected $N = Z$ nuclei using the deformed Woods-Saxon (WS) potential as a mean-field (at fixed deformation) and standard isovector seniority-type pairing interaction [10]. A representative example reflecting the generic results of the study is illustrated in Fig. 2. The major modification introduced by isovector pairing correlations is the smooth increase of the isoalignment with isocranking frequency; see Fig. 2a. The isovector pairing introduces a kind of *collectivity* on top of the sp model but does not affect the bulk properties. The value of the inertia parameter depends on the shell structure at the Fermi energy but in a modest and intuitively understandable way. Namely, as compared to the sp estimate, it decreases (increases) when shell gaps are present (absent). It is also interesting to note that the smoothing effect depends only weakly on the strength of isovector pairing correlations, $G_{T=1}$. The alignment as well as the excitation energy almost does not change, even when the isovector pairing strength, $G_{T=1}$, is strongly reduced. Henceforth, the excitation energy of the T states will be computed at the standard isocranking constraint $T_x = \sqrt{T(T+1)}$.

The main objective of our study is to clarify the role played by isoscalar pairing correlations, in particular, we show that one simultaneously (i) can recover the Wigner energy as it was shown in our previous work [10,11] and (ii) determine the excitation energies of the $T = 2$ states in even-even $N = Z$ nuclei. In addition, our subsequent

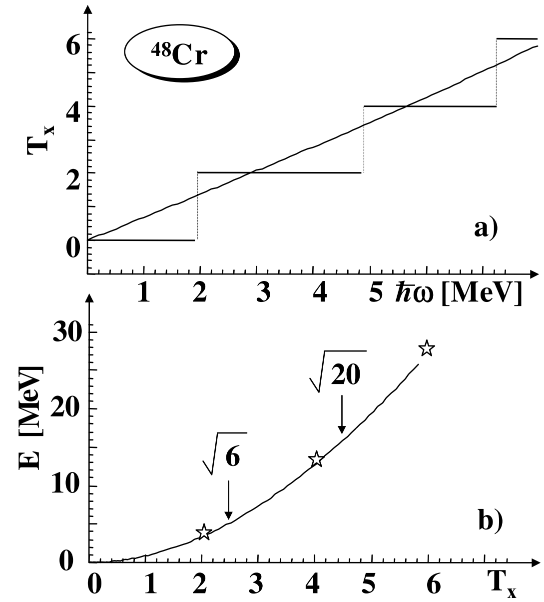

FIG. 2. Alignment versus frequency (a) and excitation energy versus alignment (b) for the sp model (discrete step function and ⋆) and for a model including isovector pairing correlations. Calculations have been performed for ^{48}Cr at fixed deformation $\beta_2 = 0.25$.

publication will show that the mean-field model incorporating isoscalar pairing correlations also is able to explain on the same footing the excitation energies of $T = 1$ states in even-even $N = Z$ nuclei as well as the competition of $T = 0$ and $T = 1$ states in odd-odd $N = Z$ nuclei [12]. Our Hamiltonian is based on the deformed mean-field potential of a WS type [13]. The two-body residual interaction contains both isovector and isoscalar seniority pairing:

$$\hat{H}^\omega = \hat{h}_{\rm WS} + G_{T=1}\hat{P}_1^\dagger \hat{P}_1 + G_{T=0}\hat{P}_0^\dagger \hat{P}_0 - \hbar\omega\hat{t}_x , \quad (2)$$

where $\hat{P}_1^\dagger$ and $\hat{P}_0^\dagger$ create isovector and isoscalar pairs, respectively. The Hamiltonian (2) is solved using the Hartree-Fock-Bogoliubov equations with the Lipkin-Nogami method. The model is very similar to the one described in detail in Ref. [11]. However, different from Ref. [11], we now employ the most general Bogoliubov transformation. It allows us to fully explore the isoscalar pairing channel without any symmetry induced restrictions, i.e., to include simultaneously $\boldsymbol{\alpha\alpha}$ and $\boldsymbol{\alpha\tilde{\alpha}}$ isoscalar pairs. In the present study, where we confine to $I = 0$ states in even-even nuclei, it is sufficient to consider $\boldsymbol{\alpha\tilde{\alpha}}$ $T = 0$ correlations. Moreover, since this study aims at a qualitative description, we have assumed near spherical deformation, $\beta_2 = 0.05$, for all nuclei. A drawback of our model is the lacking response of the isovector particle-hole (ph) field to rotations in isospace. To fully investigate in a quantitative way the interplay of isoscalar pairing and rotations in isospace, requires self-consistent Hartree-Fock-Bogoliubov calculations with realistic isovector and isoscalar two-body interactions in both ph and pairing channels. This, however, is clearly beyond our present approach.

The isovector pairing strength, $G_{T=1}$, is computed using the average gap method of Ref. [14] where the number of proton (and neutron) WS states retained for the pairing calculations is consistently put to $A/2$. To compute the strength of the isoscalar pairing correlations, $G_{T=0}$, we follow the prescription given in Ref. [11]. This method is based on the assumption that, within the mean-field model, the Wigner energy is predominantly due to the $T = 0$ pairing correlations. In other words we fit $G_{T=0}$ to reproduce roughly the Wigner energy strength $W(A) \approx 47/A$ MeV using the technique provided in Ref. [15]. The result of these calculations is shown in Fig. 3a. Interestingly, to obtain the proper value of $W(A)$, the mass scaling of isovector and isoscalar pairing strengths has to be different. The ratio $x^{T=0} = G_{T=0}/G_{T=1}$ necessary to reproduce the empirical trends decreases smoothly from $x^{T=0} \approx 1.65$ at the beginning of the sd shell to $x^{T=0} \approx 1.40$ in the $f_{7/2}$ subshell as shown in the inset. The calculated and empirical excitation energies of the lowest $T = 2$ states, $\Delta E_{T=2}$, for even-even $N = Z$, $20 \leq A \leq 56$, nuclei are displayed in Fig. 3b. Calculations including isoscalar pairing correlations (•) are in excellent agreement with the empirical data (⋆). In contrast, calculations including only the isovector pairing field (◇) account for roughly half of the empirical excitation energy. It is interesting to notice that although generally $\Delta E_{T=2}$ decreases as a function of A, it clearly rises at closed (sub)shells, particularly for $N = Z = 20$. This nicely reflects the dominant role played by the sp substructure $\Delta E_{T=2} \propto \delta e$ [see Eq. (1)] and the reduced role of the pairing correlations (in particular, the $T = 0$ pairing) at shell closure. Note also the pronounced smoothing effect of isoscalar pairing on the $\Delta E_{T=2}$ excitations at shell closure. Last but not least, it is important to remember that the Wigner energy and the $T = 2$ excitations are calculated in a totally different manner.

There is an appealing correspondence between the role of isoscalar and isovector correlations. The binding energy of even nuclei is lowered with respect to their odd neighbors. Similarly, the presence of isoscalar pairing correlations lowers the ground state of even-even $N = Z$ nuclei and accounts for the missing binding energy commonly known as the Wigner energy [10]. Analogous, the generalized blocking of isoscalar pairing results in reduced binding when moving from the $N = Z$ line. Isovector pairing is weakened at high angular momenta due to the Coriolis effect that tends to align the angular momenta of the nucleons along the rotational axis. Similarly, because the $T = 0$ pairs have isospins coupled antiparallel, rotations in isospace tend to destroy these correlations. In contrast, isovector pairs have their isospins coupled parallel and are hardly affected by isorotations. Pairing correlations as a function of rotational frequency in either real space or

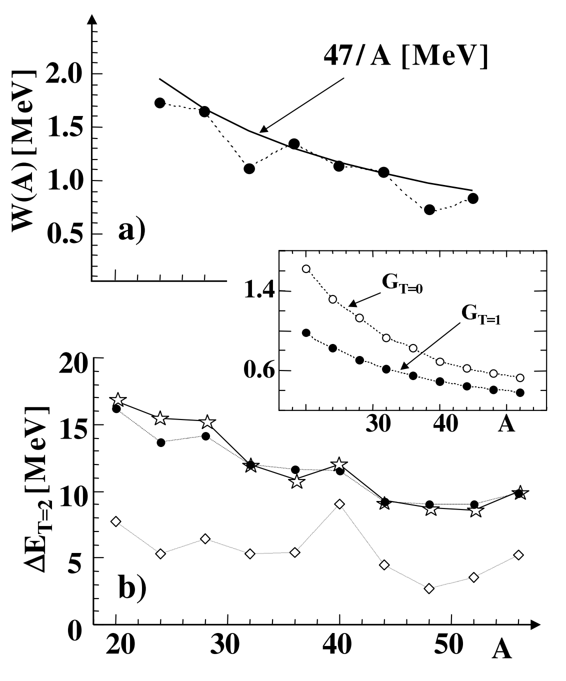

FIG. 3. Calculated Wigner energy strength $W(A)$ (a) in $N = Z$ nuclei. The inset shows the isovector and isoscalar pairing strength parameters used to reproduce the smooth empirical trend of the Wigner energy strength $47/A$ MeV. The lower panel (b) shows the excitation energy of the $T = 2$, $I = 0$ states. The experimental data values are marked by (⋆); the calculations including both $T = 0$ and $T = 1$ pairings are marked with (•) and the calculations with $T = 1$ pairing only are labeled with (◇).

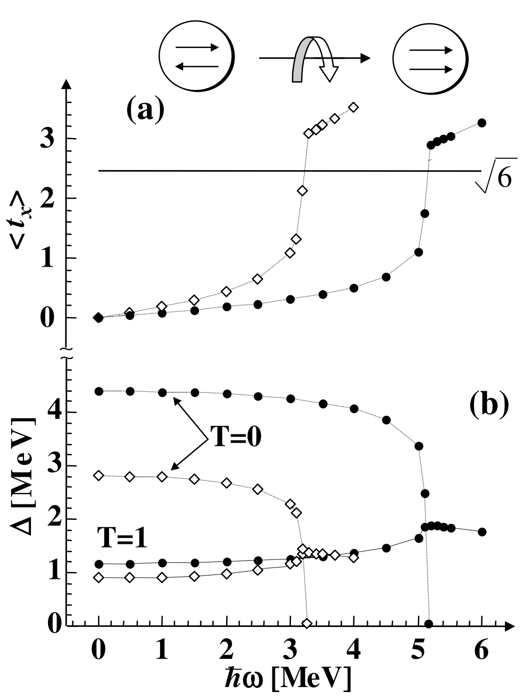

FIG. 4. Alignment (a) and isoscalar and isovector gap parameters (b) versus isocranking frequency calculated for ^{24}Mg (•) and ^{48}Cr (◇). The figure illustrates the phase transition leading to the disappearance of isoscalar $T = 0$ pairing correlations (lower panel) once the alignment reaches a value of $\sqrt{6}$ (upper panel).

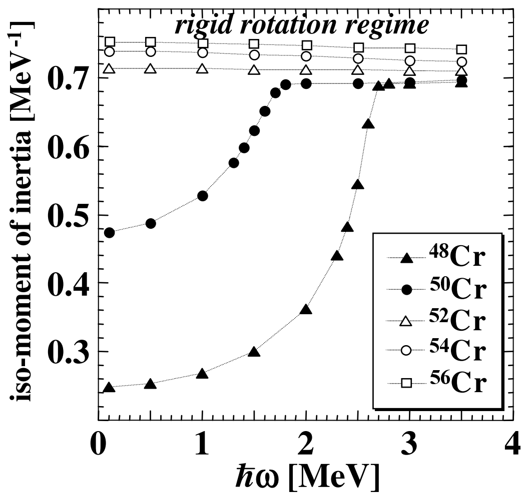

FIG. 5. Isomoment of inertia (MoI), $\Im_T^{(x)} \equiv T_x/\hbar\omega$, for a sequence of $N - Z = 0$, 2, 4, 6, 8 Cr isotopes at constant deformation ($\beta_2 = 0.25$) versus rotational frequency. The figure clearly illustrates the pronounced effect of the $T = 0$ pair correlations in $N - Z = 0$ and 2 isotopes and the phase transition raising the MoI to the "rigid body" value at high frequencies.

isospace are quenched in a similar fashion like the magnetic field destroys the electronic Cooper pairs in metallic superconductors. Hence, with increasing isocranking frequency one does expect a bulk phase transition similar to the well-known Meissner effect [16].

The *phase transition* discussed above on general grounds indeed occurs systematically in our calculations. Two representative cases are depicted in Fig. 4. The phase transition (cf. lower panel of Fig. 4) takes place almost exactly at, or just before the isoalignment (cf. upper panel of Fig. 4) reaches the value of $T_x = \sqrt{6}$, corresponding to the $T = 2$ state. Evidently, once we reach the $T = 2$ states, isoscalar pairing correlations have essentially dropped to zero. At the same time, isovector pairing correlations are still strong. The quenching of isoscalar pairing in the ground state of $|N - Z| = 4$ nuclei is a general feature of most known calculations, independent of the interaction [10,11,17]. These findings are therefore consistent with the isobaric symmetry which demands that the structure (and hence, also the excitation energy) of $T = 2$, $T_z = 0$ states to be similar to the structure of $T = 2$, $T_z = \pm 2$ members of the $T = 2$ quintuplet. The $T = 2$, $T_z = \pm 2$ states are just the ground states of the $|N - Z| = 4$ nuclei, since, by the rule, the ground states of even-even nuclei are the states of minimum isospin: $T = |T_z| = |N - Z|/2$.

The dependence of the isomoments of inertia (MoI), $\Im_T^{(x)} = T_x/\hbar\omega$ as a function of the isocranking frequency and $N - Z$ for a sequence of Cr isotopes is further illustrated in Fig. 5. At small frequencies the $T = 0$ pairing phase is present only in the $T_z = 0$, 1 isotopes, where it strongly lowers the MoI. At high frequency a *phase transition* takes place resulting in a rapid increase of the MoI to the "rigid body" value, i.e., the value corresponding to $\Im_T^{(x)}$ for the $T = 0$ unpaired system.

In summary, the response of pairing correlations to rotations in isospace is investigated within a simple model. The present calculations show that on a qualitative level, the mean-field method is capable to account for both mass excess in $N = Z$ nuclei and the MoI in isospace if and only if the short range correlations take into account isoscalar pairing. Pairing correlations of isovector and isoscalar types respond totally different to rotations in isospace. The presence of isoscalar pairing strongly reduces the MoI in isospace, but only for low values of T. With increasing isocranking frequency, isospin starts to align, and isopairs become broken, resulting eventually in the quenching of isoscalar pairing and a rapid increase of the MoI. In the regime of large isospin, no isoscalar pairing is present. These results are in beautiful analogy to spatial rotations.

This work was supported by the Göran Gustafsson Foundation, the Swedish Natural Science Research council (NFR), and the Polish Committee for Scientific Research (KBN) under Contract No. 2 P03B 040 14.

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