BPS States in M Theory and Twistorial Constituents

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We provide a complete algebraic description of Bogomol'nyi-Prasad-Sommerfield (BPS) states in M theory in terms of primary constituents that we call BPS preons. We argue that any BPS state preserving k of the 32 supersymmetries is a composite of $(32-k)$ BPS preons. In particular, the BPS states corresponding to the basic $M2$ and $M5$ branes are composed of 16 BPS preons. By extending the M algebra to a generalized $D=11$ conformal superalgebra $osp(1|64)$ we relate the BPS preons with its fundamental representation, the $D=11$ supertwistors.

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(1) *Introduction.*—The dynamical description of the eleven-dimensional M theory [1,2], which should unify all fundamental interactions including gravity, is not known. It is characterized by a set of (conjectured) duality symmetries and by the low energy limit, $D=11$ supergravity [3]. Relevant information is provided by the $D=11$ Poincaré superalgebra recently called M algebra [4]

$$\{Q_\alpha, Q_\beta\} = Z_{\alpha\beta}, \qquad [Q_\alpha, Z_{\alpha\beta}] = 0\,,$$
$$Z_{\alpha\beta} \equiv \Gamma^\mu_{\alpha\beta}P_\mu + i\Gamma^{\mu\nu}_{\alpha\beta}Z_{\mu\nu} + \Gamma^{\mu_1\ldots\mu_5}_{\alpha\beta}Z_{\mu_1\ldots\mu_5}\,, \tag{1}$$

where $\Gamma^\mu_{\alpha\beta} = (\Gamma^\mu)_{\alpha.}{}^{\gamma}C_{\gamma\beta}$, etc., Q_α is a 32-component real Majorana spinor of supercharges, and the symmetric 32×32 generator $Z_{\alpha\beta}$ of tensorial (central with respect to Q_α) charges extends the eleven momentum components P_μ to a set of $528 = 11 + 55 + 462$ generators. The additional 517 charges characterize the basic M branes. Assuming that the M algebra is valid for all energies, the information about the spectrum of states in M theory can be deduced from the representation theory of the algebra (1). Of special importance is the notion of Bogomol'nyi-Prasad-Sommerfield (BPS) states. A BPS state $|k\rangle$ can be defined as an eigenstate with eigenvalue $z_{\alpha\beta}$ of the "generalized momentum" generator, $Z_{\alpha\beta}|k\rangle = z_{\alpha\beta}|k\rangle$, such that $\det z_{\alpha\beta} = 0$:

$$\frac{k}{32} - \text{BPS state:} \quad \{\text{rank } z_{\alpha\beta} = 32-k, 32 > k \geq 1\}. \tag{2}$$

Moreover [5], Eq. (2) implies that the BPS state $|k\rangle$ preserves a fraction $\nu = \frac{k}{32}$ of supersymmetries.

Without a knowledge of the fundamental dynamics of M theory, it is difficult to determine which representations of (1) are primary and which are composite. A point of view based on the study of solitonic solutions of $D=11$ supergravity [10,11] considers as the most elementary ones the $\frac{1}{2}$-BPS states describing the $M2$ and $M5$ branes, the $M9$ brane and the M-KK6 brane ($D=11$ Kaluza-Klein monopole), as well as the M wave ($M0$ brane). By considering superpositions of these $D=11$ elementary objects (intersecting branes and branes ending on branes) one can construct $\frac{k}{32}$-BPS $D=11$ supergravity solitons with $k \leq 16$ ($\nu \leq \frac{1}{2}$) [5,12]. Despite the fact that exotic BPS states with $\nu = \frac{k}{32} > \frac{1}{2}$ can also be treated algebraically as a kind of superposition of branes and antibranes [12,13], solitonic solutions are known only for BPS states that preserve a fraction $\nu \leq \frac{1}{2}$ of supersymmetries.

In this paper we propose another algebraic scheme aimed to describe the representations of M algebra, with a different choice of primary and composite objects. Equation (2) suggests that the most elementary component permitting one to construct *all* BPS states as composites corresponds to tensorial charges with rank $z_{\alpha\beta} = 1$, or $k = 31$. We shall call *BPS preons* the hypothetical objects carrying these "elementary values" of $z_{\alpha\beta}$. Thus, a BPS preon may be characterized by the following choice of central charges matrix:

$$z_{\alpha\beta} = \lambda_\alpha\lambda_\beta, \qquad \alpha,\beta = 1,\ldots,32\,, \tag{3}$$

where λ_α is a real (Majorana) SO(1,10) bosonic spinor. We notice that Eq. (3) can be looked at as an extension of the Penrose formula (see, e.g., [14]) expressing a massless $D=4$ four-momentum as a bilinear of a Weyl spinor π_A

$$p^\mu = \frac{1}{2}(\sigma^\mu)_{A\dot{B}}\pi^A\bar{\pi}^{\dot{B}}, \qquad A,\dot{B} = 1,2\,, \tag{4}$$

to the case of $D=11$ generalized momenta (with Abelian addition law). Following the spirit of the twistor approach [14–16] we generalize Eq. (3) to the general case of $\frac{k}{32}$-BPS states by

$$z_{\alpha\beta} = \Sigma^n_{i=1}\lambda^i_\alpha\lambda^i_\beta, \qquad n = 32-k, \qquad 32 > k \geq 1\,, \tag{5}$$

with n linearly independent λ's. Thus, the $\frac{k}{32}$-BPS state may be regarded as composed of $n = (32-k)$ BPS preons.

Group theory methods often compensate the lack of knowledge about dynamical mechanisms. At this stage our aim is modest, and begins by pointing out an interesting structure of the representation theory of the M algebra that arises when the tensorial charges are introduced as bilinears of spinors. Further, to conjecture the dynamics of BPS preons we may follow the considerations in [17,18] and enlarge the M algebra to the superconformal one $osp(1|64)$ [6,19]. In such a dynamical picture a BPS preon (a fundamental $\frac{31}{32}$-BPS state) is described by a $D = 11$ supertwistor $(\lambda_\alpha, \omega^\alpha, \xi)$, with ξ fermionic, i.e., by the fundamental representation of $OSp(1|64)$.

We stress that our method provides a *complete* algebraic classification of all BPS states (note also that, although we consider here the $D = 11$ case, our approach applies to any D). In particular, we shall describe in the language of n BPS preons the special choices of $z_{\alpha\beta}$ describing $M2$ branes and $M5$ branes ($n = 16$), two orthogonal M branes ($n = 24$) and the example [13] of an exotic BPS state with $\nu = \frac{3}{4}$. Because of the lack of $D = 11$ solutions corresponding to exotic BPS states, one can conjecture that branes that can be described in $D = 11$ spacetime should have $n \geq 16$ in Eq. (5).

(2) *Arbitrary BPS states as composites of BPS preons.*—We show now that any $\frac{k}{32}$-BPS state $|k\rangle$ can be characterized by Eq. (5). $GL(32,\mathbb{R})$ is the maximal automorphism group of the algebra (1) [20,21,26]. As the matrix $z_{\alpha\beta} = \langle k|Z_{\alpha\beta}|k\rangle$ is symmetric and positive-definite, $z_{\alpha\beta}x^\alpha x^\beta = 2\Sigma_{l=1}^{32}|\langle k|x^\alpha \hat{Q}_\alpha|l\rangle|^2 \geq 0$, it can be diagonalized by a $GL(32,\mathbb{R})$ matrix $G^{\alpha}_{.\beta}$,

$$z_{\alpha\beta} = G^{\gamma}_{.\alpha} z^{(0)}_{\gamma\delta} G^{\delta}_{.\beta} \,. \tag{6}$$

Moreover, the diagonal matrix $z^{(0)}_{\gamma\delta}$ can be chosen as follows:

$$z^{(0)}_{\gamma\delta} = \mathrm{diag}(1, \overset{n}{\ldots}, 1, 0, \overset{k}{\ldots}, 0) \,. \tag{7}$$

Thus, we see that Eq. (6) can be written in the form (5) provided that

$$\lambda^i_\alpha = G^i_{.\alpha}, \qquad i = 1, \ldots, n = 32 - k \,. \tag{8}$$

In the new basis $Q^{(0)}_\alpha$ defined by $Q^{(0)}_\alpha = (G^{-1})^{\beta}_{.\alpha} Q_\beta$, the M algebra (1) diagonalizes on BPS states so that

$$\begin{aligned} \{Q^{(0)}_i, Q^{(0)}_j\}|k\rangle &= \delta_{ij}|k\rangle, \\ \{Q^{(0)}_i, Q^{(0)}_r\}|k\rangle &= \{Q^{(0)}_r, Q^{(0)}_s\}|k\rangle = 0, \end{aligned} \tag{9}$$

$r, s = n + 1, \ldots, 32$. Hence, the set of 32 supercharges $Q^{(0)}_\alpha = (Q^{(0)}_i, Q^{(0)}_r)$ acting on the BPS state $|k\rangle$ splits into k generators $Q^{(0)}_r$ of supersymmetries preserving the BPS state (i.e., one can put consistently $Q^{(0)}_r|k\rangle = 0$), and $n = 32 - k$ generators $Q^{(0)}_i$ which describe the set of broken supersymmetries. To summarize, the eigenvalues $z_{\alpha\beta}$ of the tensorial charges that characterize a BPS state preserving $k < 32$ supersymmetries may be expressed by Eq. (5) in terms of $32 - k$ Majorana spin$(1,10)$ bosonic spinors λ^i_α. A BPS preon state with $z_{\alpha\beta} = \lambda_\alpha\lambda_\beta$ preserves $k = 31$ supersymmetries and $\nu = \frac{k}{32}$ BPS states are composed of $n = 32 - k$ preons.

Equation (5) implies that the tensorial charges are preserved under $O(n)$ transformations of the n spinors λ^i_α:

$$\lambda'^{i}_{\alpha} = O^i{}_j \lambda^j_\alpha, \qquad O^T O = \mathbf{1}_n, \qquad i = 1, \ldots, n \,. \tag{10}$$

The SO(n) = SO$(32 - k)$ rotations constitute an internal symmetry of the $\frac{k}{32}$-BPS states. For the $\frac{1}{2}$-BPS states associated with the fundamental $M2$ and $M5$ branes, in particular, SO(n) = SO(16). This group corresponds in $D = 11$ to the unitary internal symmetry $U(n)$ in $D = 4$ that was introduced in the framework of twistor theory for n-twistor composite systems [15,16]. In our scheme we see that the dimension n of SO(n) is equal to the number of broken supersymmetries.

(3) *Physical $\frac{1}{2}$ BPS states and their superpositions.*—The fundamental BPS preons have all the bosonic charges $p_\mu \propto \lambda C\Gamma_\mu\lambda$, $z_{\mu\nu} \propto \lambda C\Gamma_{\mu\nu}\lambda$, $z_{\mu_1\ldots\mu_5} \propto \lambda C\Gamma_{\mu_1\ldots\mu_5}\lambda$ nonvanishing. We show now in our framework that the $M2$ and $M5$ $\frac{1}{2}$-BPS states can be composed out of 16 BPS preons.

Let us consider the $M2$ brane, what implies $z_{\mu_1\ldots\mu_5} = 0$. Thus, Eq. (5) acquires the form

$$z_{\alpha\beta} = p_\mu\Gamma^\mu_{\alpha\beta} + z_{\mu\nu} i\Gamma^{\mu\nu}_{\alpha\beta} = \Sigma^n_{i=1}\lambda^i_\alpha\lambda^i_\beta \,, \tag{11}$$

where, at this stage, we do not fix n. In the rest frame of a BPS massive state, $p_\mu = m(1, 0, \ldots, 0)$. As we are dealing with the $M2$ brane, but not with the $M9$ brane, we shall assume that in this frame $z_{\mu\nu}$ has only spacelike components $z_{\mu\nu} = \delta^I_{[\mu}\delta^J_{\nu]} z_{IJ}$. In particular, we may choose the space slice of the $M2$ world volume in the $\{12\}$ plane, $z_{\mu\nu} = \delta^1_{[\mu}\delta^2_{\nu]} z$ where $z = 2z_{12}$. Using the spin$(1,2) \otimes$ spin(8) covariant splitting of $D = 11$ spinors and Γ matrices (see, e.g., [22]),

$$\lambda^i_\alpha = \begin{pmatrix} \lambda^i_{aq} \\ \lambda^{ia}_{\dot{q}} \end{pmatrix}, \qquad a = 1, 2, \qquad q, \dot{q} = 1, \ldots, 8 \,, \tag{12}$$

$i\Gamma_{12} = \begin{pmatrix} I_{16} & 0 \\ 0 & -I_{16} \end{pmatrix}$, $\gamma^0_{ab} = \delta_{ab}$, Eq. (11) can be written as

M2 : 1 2 − − − − − − − − − −

$$z_{\alpha\beta} = \begin{pmatrix} (m + z)\delta_{ab}\delta_{qp} & 0 \\ 0 & (m - z)\delta^{ab}\delta_{\dot{q}\dot{p}} \end{pmatrix}$$

$$= \Sigma^n_{i=1} \begin{pmatrix} \lambda^i_{aq}\lambda^i_{bp} & \lambda^i_{aq}\lambda^{ib}_{\dot{p}} \\ \lambda^{ia}_{\dot{q}}\lambda^i_{bp} & \lambda^{ia}_{\dot{q}}\lambda^{ib}_{\dot{p}} \end{pmatrix}. \tag{13}$$

The matrix $z_{\alpha\beta}$ has either rank 32 (when $m \neq \pm z$) or rank 16 (when $m = \pm z$). Assuming $z > 0$ we conclude that the $M2$ brane BPS state appears when $m = z$ and that preserves 1/2 of the target supersymmetries. In this case Eq. (13) implies

$$\Sigma^n_{i=1}(\lambda^i_{aq}\lambda^i_{bp}) = 2z\delta_{ab}\delta_{qp} \,, \tag{14}$$

$$\Sigma^n_{i=1}(\lambda^{ia}_{\dot{q}}\lambda^i_{bp}) = 0 = \Sigma^n_{i=1}(\lambda^{ia}_{\dot{q}}\lambda^{ib}_{\dot{p}}) \,. \tag{15}$$

Equation (14) has a solution only if $n \geq 16$. Moreover, as rank $z_{\alpha\beta} = 16$ for $m = z$, we need just 16 BPS preons, described by $\lambda^i_{aq} \in \mathbb{R}^+ \otimes O(16)$; from (15) $\lambda^{ia}_{\dot p} = 0$ follows. Using the $O(16)$ symmetry [Eq. (5)] we see that in a special frame the spinors λ^i_α ($i = 1, \ldots, 16$) satisfying Eqs. (14) and (15) and describing a $M2$-brane BPS state may be written as $\lambda^i_{(\alpha)}$,

$$\mathbf{M2}\ (m = z): \qquad \lambda^i_{(\alpha)} = \begin{pmatrix} \sqrt{2z}\, \delta^i_{aq} \\ 0 \end{pmatrix}. \tag{16}$$

To obtain the set of λ^i_α in an arbitrary frame we perform a Lorentz rotation of (16) by means of a spin$(1,10)$ matrix

$$\begin{gathered} v_\alpha{}^{(\beta)} \equiv (v_\alpha{}^{aq}, v_{\alpha a \dot q}) \in \mathrm{spin}(1,10)\,, \\ a = 1,2, \qquad q = 1, \ldots, 8, \qquad \dot q = 1, \ldots, 8\,, \end{gathered} \tag{17}$$

and we get

$$\mathbf{M2}: \qquad \lambda^i_\alpha = v_\alpha{}^{(\beta)} \lambda^i_{(\beta)} = \sqrt{2z}\, v_\alpha{}^{aq} \delta^i_{aq}\,. \tag{18}$$

Note that only one of the two [spin$(1,2) \otimes$ spin(8)]-covariant 32×16 blocks (cf. [22]) of the spinorial Lorentz frame matrix (17), $v_\alpha{}^{aq}$, enters in Eq. (18) [23].

For a $M5$-brane BPS state corresponding to the vanishing world volume gauge field the central charges matrix is

$$z_{\alpha\beta} = p_\mu \Gamma^\mu_{\alpha\beta} + z_{\mu_1 \ldots \mu_5} \Gamma^{\mu_1 \ldots \mu_5}_{\alpha\beta} = \Sigma^{16}_{i=1} \lambda^i_\alpha \lambda^i_\beta\,. \tag{19}$$

By considerations analogous to those of the $M2$-brane case one finds that the 16 BPS preons needed for such a BPS state of a $M5$ brane are associated with

$$\begin{gathered} \mathbf{M5}: \qquad \lambda^i_\alpha = v_\alpha{}^{(\beta)} \lambda^i_{(\beta)} = \sqrt{2z}\, v_\alpha{}^{aq} \delta^i_{aq}\,, \\ a = 1, \ldots, 4 \quad [\mathrm{spin}(1,5)] \qquad q = 1, \ldots, 4 \quad [\mathrm{spin}(5)]\,, \end{gathered} \tag{20}$$

where $z = m = 5!\, z_{1 \ldots 5} > 0$, and $v_\alpha{}^{aq}$ is the 16×32 [spin$(1,5) \otimes$ spin(5)]-covariant block of the Lorentz frame matrix $v_\alpha{}^{(\beta)}$. Thus, the $M2$ and $M5$ BPS states are described by a highly constrained set of $D = 11$ spinors λ^i_α [since $v_\alpha{}^{(\beta)}$ in Eq. (18) as well as in Eq. (20) belongs to spin$(1,10)$].

To describe a superposition of M branes preserving $\nu = \frac{k}{32} < \frac{1}{2}$ supersymmetry one needs $n > 16$ BPS preons. In particular, for the system of two orthogonal $M2$ branes

$$\begin{array}{ll} & 1\ 2\ -\ -\ -\ -\ -\ -\ -\ -\ - \\ \mathbf{M2} \otimes \mathbf{M2} & \\ & -\ -\ 3\ 4\ -\ -\ -\ -\ -\ -\ - \end{array}$$

with equal positive charges $z_{12} = z_{34} = m/4$ in the rest frame we get

$$z_{\alpha\beta} = m \begin{pmatrix} \gamma^0_{ab} \delta_{qp} + \mathcal{P}^{(-)}_{aq\, bp} & 0 \\ 0 & \mathcal{P}^{(-)ab}_{\dot q \dot p} \end{pmatrix} = \Sigma^{24}_{i=1} \lambda^i_\alpha \lambda^i_\beta\,, \tag{21}$$

where $\mathcal{P}^{(-)ab}_{\dot q \dot p} = \frac{1}{2}(\gamma^{0\,ab} \delta_{\dot q \dot p} - \varepsilon^{ab} \gamma^1_{q \dot q} \gamma^2_{q \dot p})$ is the orthogonal projector, $\epsilon^{12} = 1, \gamma^{0,1,2}_{ab}, \gamma^{0\,ab}, \ldots$, are SO$(1,2)$ gamma matrices, and $\gamma^1_{q\dot q}, \ldots, \gamma^8_{q \dot q}$ are the 8×8 SO(8) Pauli matrices. Thus, rank $z_{\alpha\beta} = 24$ and we need 24 BPS preons which can be characterized by ($\hat i = 1, \ldots, 16$; $\tilde i = 1, \ldots, 8$, $i = 1, \ldots, 24$)

$$\lambda^i_\alpha = \left(\begin{pmatrix} \hat\lambda^{\hat i}_{aq} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \tilde\lambda^{a \tilde i}_{\dot q} \end{pmatrix} \right), \tag{22}$$

where the $\hat\lambda^{\hat i}_{aq}$ and the $\tilde\lambda^{a\tilde i}_{\dot q}$ are constrained by

$$\begin{gathered} \Sigma^{16}_{\hat i=1} \hat\lambda^{\hat i}_{aq} \hat\lambda^{\hat i}_{bp} = m(\gamma^0_{ab} \delta_{qp} + \mathcal{P}^{(-)}{}_{aq\, bp})\,, \\ \Sigma^8_{\tilde i=1} \tilde\lambda^{a \tilde i}_{\dot q} \tilde\lambda^{b \tilde i}_{\dot p} = m \mathcal{P}^{(-)ab}_{\dot q \dot p}\,. \end{gathered} \tag{23}$$

In an arbitrary frame $\lambda^i_\alpha = (v_\alpha{}^{aq} \hat\lambda^{\hat i}_{aq}, v_{\alpha a \dot q} \tilde\lambda^{a \tilde i}_{\dot q})$, where $v_\alpha{}^{aq}, v_{\alpha a \dot q}$ are the spinor frame variables (17).

The $\frac{1}{4}$-BPS state (21) as well as many other $\nu = \frac{k}{32} < \frac{1}{2}$ BPS states are described by solitonic solutions of the $D = 11$ supergravity [10]. Our approach allows us to consider as well the "exotic" BPS states with $\nu > \frac{1}{2}$. They are described by $n = 32 - k < 16$ BPS preons. For instance, the massive $\frac{3}{4}$-BPS state in [13], with $z_{12} = -m/2$, $z_{2345} = -z_{26789\#} = \mp m/5!$ in the rest frame, is characterized by

$$z_{\alpha\beta} = 4m \begin{pmatrix} 0 & 0 \\ 0 & \mathcal{P}^{(\pm)ab}_{\dot q \dot p} \end{pmatrix} = \Sigma^8_{i=1} \lambda^i_\alpha \lambda^i_\beta\,, \tag{24}$$

where now $\mathcal{P}^{(\pm)ab}_{\dot q \dot p} = \frac{1}{2}(\gamma^{0\,ab} \delta_{\dot q \dot p} \pm \gamma^{2\,ab} \gamma^{1234}_{\dot q \dot p})$ are orthogonal projectors ($\mathcal{P}^{(+)} + \mathcal{P}^{(-)} = \gamma^0 \otimes I$, $\gamma^{1234}_{\dot q \dot p} \equiv \gamma^1_{q\dot q} \gamma^2_{q \dot r} \gamma^3_{p \dot r} \gamma^4_{p \dot p} = -\gamma^{5678}_{\dot q \dot p}$). Thus, rank $z_{\alpha\beta} = 8$ and one concludes that the BPS state preserves $\nu = 3/4$ of supersymmetry [13]. In an arbitrary frame $z_{\alpha\beta} = 4m v_{\alpha a \dot q} \times \mathcal{P}^{(\pm)ab}_{\dot q \dot p} v_{\beta b \dot q}$ and the $\frac{3}{4}$-BPS state can be described by 8 BPS preons

$$\lambda^{a \tilde i}_\alpha = v_{\alpha a \dot q} \tilde\lambda^{a \tilde i}_{\dot q}\,, \qquad \tilde i = 1, \ldots, 8\,, \tag{25}$$

where the 8 $\tilde\lambda^{a\tilde i}_{\dot q}$ are constrained by $\Sigma^8_{\tilde i=1} \tilde\lambda^{a \tilde i}_{\dot q} \tilde\lambda^{b \tilde i}_{\dot p} = 4m \mathcal{P}^{(\pm)ab}_{\dot q \dot p}$ and $v_\alpha{}^{aq}, v_{\alpha a \dot q}$ are the spinorial Lorentz frame variables of Eq. (17).

There are no solitonic solutions for the exotic BPS states known. Moreover, general κ-symmetry arguments [24] and the study of the simplest supersymmetric field theories [26] indicate that, probably, such solitonic solutions do not exist in the standard (i.e., unenlarged) $D = 11$ spacetime. This suggests that only composites of $n \geq 16$ BPS preons can be described in a $D = 11$ standard spacetime framework.

(4) *BPS preons, enlarged superspaces, and $OSp(1|64)$ supertwistors.*— It seems natural to assume that a dynamical realization of exotic states requires a new geometric framework, going beyond the standard $D = 11$ spacetime [27]. The most straightforward idea is to treat all tensorial charges as generalized momenta in a large conjugate space of 528 dimensions. The simplest supersymmetric dynamics in $D = 4$ superspace— Brink-Schwarz massless superparticle— can be extended to such a large space by two different ways of generalizing the mass-shell condition.

(i) The Sp(32)-invariant generalization $z_{\alpha\beta} C^{\beta\gamma} z_{\gamma\delta} = 0$ of $p^2 = 0$ (cf. [29]), where the Sp(32) metric C is the antisymmetric $D = 11$ Majorana charge-conjugation matrix.

(ii) The general, less restrictive, $GL(32,\mathbb{R})$-invariant condition (see, e.g., [26]) $\det z_{\alpha\beta} = 0$, characterizing all $\frac{k}{32}$-BPS states with $1 \leq k < 32$.

To introduce a dynamical scheme for the proposed BPS preons of M algebra one can develop a new spinorial geometry by doubling the $D = 11$ Lorenz spinors to introduce a $D = 11$ twistor $T_A = (\lambda_\alpha, \omega^\alpha)$ $(A = 1,\ldots,64)$ satisfying a generalized Penrose incidence equation

$$\omega^\alpha = x^{\alpha\beta}\lambda_\beta, \qquad x^{\alpha\beta} = x^{\beta\alpha}, \tag{26}$$

where

$$x^{\alpha\beta} = x^\mu \Gamma_\mu^{\alpha\beta} + y^{\mu\nu} i \Gamma_{\mu\nu}^{\alpha\beta} + y^{\mu_1\ldots\mu_5}\Gamma_{\mu_1\ldots\mu_5}^{\alpha\beta} \tag{27}$$

describes the $528 = 11 + 55 + 462$ coordinates dual to the P_μ, $Z_{\mu\nu}$, $Z_{\mu_1\ldots\mu_5}$ generalized momenta. In a supersymmetric theory, Eq. (26) has to be supplemented by (cf. [17,18,30])

$$\xi = \theta^\alpha \lambda_\alpha . \tag{28}$$

Then, $\mathcal{T}_{\mathcal{A}} = (T_A, \xi)$ defines a supertwistor, which is the fundamental representation of the generalized $D = 11$ conformal superalgebra $osp(1\,|\,64)$. In such a framework the basic geometry is described by the $D = 11$ supertwistors (T_A, ξ) which we propose to interpret as BPS preon phase space coordinates. Indeed, using Eqs. (26) and (5) one obtains a relation (modulo an exterior derivative) between the canonical Liouville one-forms describing the symplectic structure in the enlarged spacetime (27) and the $D = 11$ twistor space coordinates,

$$z_{\alpha\beta} dx^{\alpha\beta} = \Sigma_{i=1}^n \lambda_\alpha^i \lambda_\beta^i dx^{\alpha\beta} = 2\Sigma_{i=1}^n \omega^{\alpha i} d\lambda_\alpha^i , \tag{29}$$

a relation that can be supersymmetrized [17,18]. For non-BPS states, for which $\det z_{\alpha\beta} \neq 0$, one needs the maximal number, 32, of BPS preons described by 32 supertwistors $\mathcal{T}_{\mathcal{A}}^i = (\lambda_\alpha^i, \omega^{\alpha i}, \xi^i)$. Using 32 copies of the Eqs. (26) and (28) one can express all enlarged superspace coordinates $(x^{\alpha\beta}, \theta^\alpha)$ as composites of spinorial preonic coordinates as follows:

$$x^{\alpha\beta} = \Sigma_{i=1}^{32} \omega^{\alpha i} (\lambda^{-1})_i^\beta, \qquad \theta^\alpha = \Sigma_{i=1}^{32} \xi^i (\lambda^{-1})_i^\alpha . \tag{30}$$

If we diminish the number of BPS preons, the geometry becomes gradually more spinorial and detached from a spacetime framework. In particular, the most elementary constituent of M-theory matter in the present approach, a single BPS preon, is described by the purely spinorial geometry of a single supertwistor.

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