Experimental Demonstration of Four-Photon Entanglement and High-Fidelity Teleportation

Jian-Wei Pan, Matthew Daniell, Sara Gasparoni, Gregor Weihs, and Anton Zeilinger
Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria

(Received 9 April 2001)

We experimentally demonstrate observation of highly pure four-photon GHZ entanglement produced by parametric down-conversion and a projective measurement. At the same time this also demonstrates teleportation of entanglement with very high purity. Not only does the achieved high visibility enable various novel tests of quantum nonlocality, it also opens the possibility to experimentally investigate various quantum computation and communication schemes with linear optics. Our technique can, in principle, be used to produce entanglement of arbitrarily high order or, equivalently, teleportation and entanglement swapping over multiple stages.

DOI: 10.1103/PhysRevLett.86.4435 PACS numbers: 03.65.Ud, 03.67.Hk, 42.50.Ar

Entanglement is not only the essence of quantum mechanics as suggested by Schrödinger [1], but is also at the basis of nearly all quantum information protocols such as quantum cryptography, quantum teleportation, and quantum computation [2]. While entanglement of two qubits is routine in the laboratory, entanglement of three photons [3] with high quality has only recently been experimentally realized [4] and used to experimentally demonstrate the extreme contradiction between local realism and quantum mechanics [5] in so-called Greenberger-Horne-Zeilinger (GHZ) states. In a parallel development, entanglement of the quantum states of three atoms [6] or four qubits in ions [7] has been demonstrated, yet in all these cases the quality of the entangled states still needs to be significantly improved in order to be useful for tests of quantum mechanics or in quantum information schemes.

A similar situation is found in the recent teleportation experiments [8–11]. To verify the nonlocal character of teleportation, two conditions must be satisfied in any experiment. On the one hand, one has to demonstrate that a genuinely unknown state (in the optimal case, a qubit which itself is still entangled to another one) is teleported [12]. On the other hand, a high experimental visibility is necessary in order to exclude local hidden variable (LHV) models [13–16]. The so-called entanglement swapping experiment [10] is the only one to date that demonstrates the teleportation of a genuinely unknown state. However, since its observed visibility was lower than 71%, one could, in principle, still doubt the nonlocal feature of teleportation [13].

In this Letter we report on an experiment that not only demonstrates the observation of four-photon entanglement but also shows high-fidelity entanglement swapping, thus proving the nonlocal character of quantum teleportation. Both features are not only important for performing novel fundamental experiments to test quantum mechanics or to demonstrate its counterintuitive features but also to expand our toolbox for quantum computation and quantum communication.

Our technique of observing four-photon GHZ entanglement uses two independently created photon pairs (Fig. 1). Suppose that the two pairs are in the state

$$|\Psi^i\rangle_{1234} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2) \otimes \frac{1}{\sqrt{2}}(|H\rangle_3|V\rangle_4 - |V\rangle_3|H\rangle_4), \qquad (1)$$

which is a tensor product of two polarization entangled photon pairs. Here $|H\rangle$ ($|V\rangle$) indicates the state of a horizontally (vertically) polarized photon.

One photon out of each pair is directed to the two inputs of a polarizing beam splitter (PBS). Since the PBS transmits horizontal and reflects vertical polarization, coincidence detection between the two PBS outputs implies that either both photons 2 and 3 are horizontally polarized or both vertically polarized, and thus projects the state (1) onto a two-dimensional subspace spanned by $|V\rangle_1|H\rangle_2|H\rangle_3|V\rangle_4$ and $|H\rangle_1|V\rangle_2|V\rangle_3|H\rangle_4$.

After the PBS, the renormalized state corresponding to a fourfold coincidence is

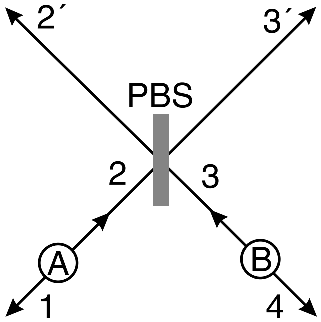

FIG. 1. Principle for observing four-photon GHZ correlations. Sources A and B each deliver one entangled particle pair. A polarizing beam splitter (PBS) combines modes 2 and 3. The two photons detected one each in its output port are either both H (horizontally) or both V (vertically) polarized projecting the complete four-photon state into a GHZ state.

$$|\Psi^f\rangle_{12'3'4} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_{2'}|V\rangle_{3'}|H\rangle_4 + |V\rangle_1|H\rangle_{2'}|H\rangle_{3'}|V\rangle_4). \qquad (2)$$

This is a GHZ state of four particles, which can exhibit nonlocal behavior according to the GHZ theorem.

The scheme described above has several notable features. First, it yields a fourfold coincidence with a success probability of 50%, which is much (4 times) more efficient than the one reported for observation of three-photon entanglement [4]. Second, the scheme does not only yield four-particle entanglement but—assuming perfect pair sources and detectors—could also produce freely propagating three-particle entangled states of modes 1, $3'$, and 4, if one puts a 45° polarizer into output $2'$. Detecting one photon in one of the outputs of this polarizer makes sure that there will be exactly one photon in each of the outputs 1, $3'$, and 4. Finally, by using this technique, one can also implement teleportation of entanglement, and hence a realization of entangled pair production with event-ready detectors [17]. To do this, two 45° polarizers are inserted into outputs $2'$ and $3'$. Conditioned on a coincidence detection of one photon in each of these outputs, we obtain an entangled pair in outputs 1 and 4 (for more details, see our further discussion below). Note that we can notify the observers at 1 and 4 before their measurements.

Obviously, an optimal realization of the above scheme would require perfect photon pair sources and ultimately perfect single-photon detectors. However, it is important to note that the absence of perfect sources and detectors does not prevent us from performing an experimental demonstration because, on the one hand, any practical application of our scheme would always need a final verification step by detecting a fourfold coincidence. On the other hand, any method to ensure that sources A and B each emit only one entangled pair is in essence equivalent to a fourfold coincidence detection. In the following we will describe our experimental verification of four-photon GHZ correlations.

In our experiment (see Fig. 2) we create polarization-entangled photon pairs by spontaneous parametric down-conversion from an ultraviolet femtosecond pulsed laser ($\sim$200 fs, $\lambda \simeq 394.5$ nm) in a $\beta - \mathrm{BaB_3O_6}$ (BBO) crystal [8,18]. The laser passes the crystal a second time having been reflected off a translatable mirror. In the reverse pass another conversion process may happen producing a second entangled pair. One particle of each pair is steered to a polarizing beam splitter where the path lengths of each particle have been adjusted such that they arrive simultaneously. On the polarizing beam splitter a horizontally polarized photon will always be transmitted, whereas a vertically polarized one will always be reflected, both with less than 10^{-3} error rate. The two outputs of the polarizing beam splitter are spectrally filtered (3.5 nm bandwidth) and monitored by fiber-coupled single-photon counters (D2 and D3). The filtering process stretches the coherence time to about 550 fs, substantially larger than the pump pulse duration [19]. This effectively erases any possibility to distinguish the two photons according to their arrival time and therefore leads to interference.

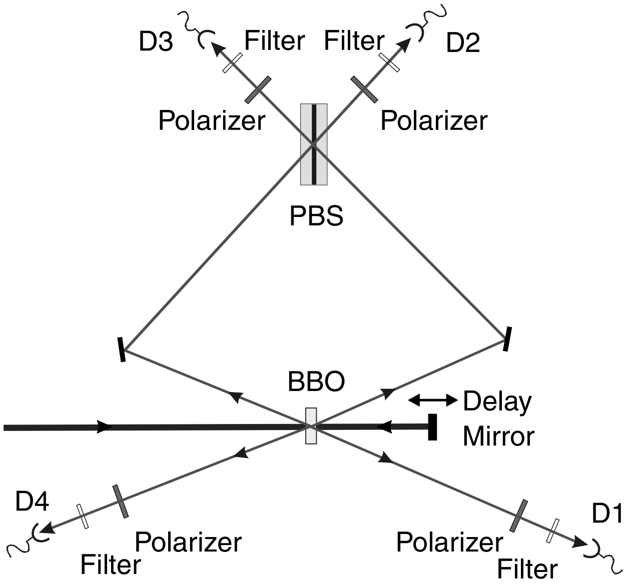

FIG. 2. Schematic of the experimental setup for the measurement of four-photon GHZ correlations. A pulse of UV light passes a BBO crystal twice to produce two entangled photon pairs. Coincidences between all four detectors 1–4 exhibit GHZ entanglement.

The remaining two photons—one from each pair—pass identical filters in front of detectors D1 and D4 and are detected directly afterwards. In front of each of the four detectors we may insert a polarizer to assess the correlations with respect to various combinations of polarizer orientations. A correlation circuit extracts only those events where all four detectors registered a photon within a small time window of a few ns. This is necessary in order to exclude cases in which only one pair is created or two pairs in one pass of the pump pulse and none in the other.

To experimentally demonstrate that the state $|\Psi^f\rangle$ of Eq. (2) has been obtained, we first verified that under the condition of having a fourfold coincidence only the $HVVH$ and $VHHV$ components can be observed, but no others. This was done by comparing the count rates of all sixteen possible polarization combinations, $HHHH, \ldots, VVVV$. The measurement results in the H/V basis (Fig. 3) show that the signal-to-noise ratio defined as the ratio of any of the desired fourfold events ($HVVH$ and $VHHV$) to any of the 14 other nondesired ones is about 200:1.

Showing the existence of $HVVH$ and $VHHV$ terms alone is just a necessary but not sufficient experimental criterion for the verification of the state $|\Psi^f\rangle$, since the above observation is, in principle, both compliant with $|\Psi^f\rangle$ and with a statistical mixture of $HVVH$ and $VHHV$. Thus, as a further test we have to demonstrate that the two terms $HVVH$ and $VHHV$ are indeed in a coherent superposition.

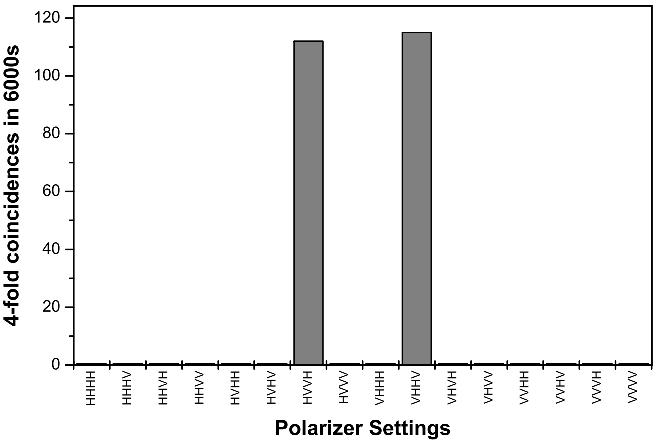

FIG. 3. Experimental data for horizontal and vertical polarizer settings. Only the two desired terms are present; all other terms which are not part of the state $|\Psi^f\rangle$ [Eq. (2)] are so strongly suppressed that they can hardly be discerned in the graph. The number of fourfold coincidences for any of the nondesired terms is 0.5 in 6000 s on the average, i.e., 7 events for all 14 possibilities.

This was done by further performing a polarization measurement in the 45° basis, where $|+45°\rangle = 1/\sqrt{2}\,(|H\rangle + |V\rangle)$ and $|-45°\rangle = 1/\sqrt{2}(|H\rangle - |V\rangle)$ are two corresponding eigenstates. Transforming $|\Psi^f\rangle$ to the $|+45°\rangle$, $|-45°\rangle$ linear polarization basis yields an expression containing 8 (out of 16 possible) terms, each with an even number of $|+45°\rangle$ components. Combinations with odd numbers of $|+45°\rangle$ components do not occur. As a test for coherence we can now check the presence or absence of various components. In Fig. 4 we compare the $(+45°/+45°/+45°/+45°)$ and $(+45°/+45°/+45°/-45°)$ count rates as a function of the pump delay mirror position. At zero delay—photons 2 and 3 arrive at the PBS simultaneously—the latter component is suppressed with a visibility of 0.79 ± 0.06. As explained in Ref. [19], many efforts have been made by us to obtain this high visibility reliably. In the experiment we observed that the most important ingredients for a high interference contrast were a high single pair entanglement quality, the use of narrow bandwidth filters, and the high quality of the polarizing beam splitter.

These measurements clearly show that we obtained four-particle GHZ correlations. The quality of the correlations can be judged by the density matrix of the state

$$\rho = 0.89|\Psi^f\rangle\langle\Psi^f|_{12'3'4} + 0.11|\Phi\rangle\langle\Phi|_{12'3'4}\,, \tag{3}$$

where $|\Phi\rangle = 1/\sqrt{2}\,(|HVVH\rangle - |VHHV\rangle)$. This density matrix describes our data under the experimentally well-justified assumption that only phase errors in the H/V basis are present, which appear as bit-flip errors in the 45° basis (see Fig. 4).

To show our experiment is also a realization of entanglement swapping, let us rewrite the state of Eq. (1) in the following way:

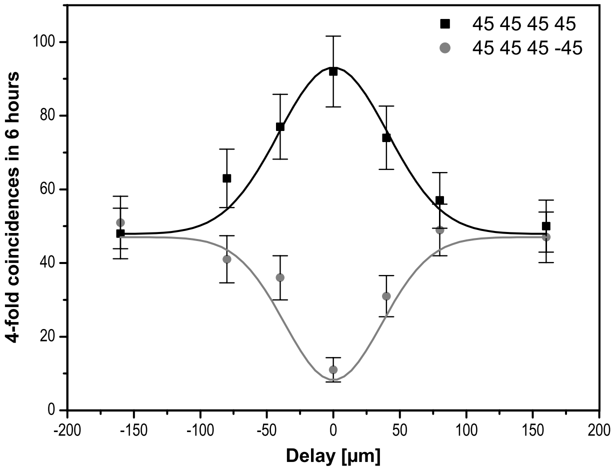

FIG. 4. Experimental data for 45° polarizer settings. The difference between the fourfold coincidence count rates for $(+45°/+45°/+45°/+45°)$ and $(+45°/+45°/+45°/-45°)$ shows that the amplitudes depicted in Fig. 3 are in a coherent superposition. Maximum interference occurs at zero delay between the photons 2 and 3 arriving at the polarizing beam splitter. The Gaussian curves that roughly connect the data points are only shown to guide the eye. Visibilities and errors are calculated only from the raw data.

$$|\Psi^i\rangle_{1234} = \frac{1}{2}\,(|\psi^+\rangle_{14}|\psi^+\rangle_{23} - |\psi^-\rangle_{14}|\psi^-\rangle_{23} - |\phi^+\rangle_{14}|\phi^+\rangle_{23} + |\phi^-\rangle_{14}|\phi^-\rangle_{23})\,, \tag{4}$$

where

$$\begin{aligned} |\psi^\pm\rangle &= \frac{1}{\sqrt{2}}\,(|H\rangle|V\rangle \pm |V\rangle|H\rangle)\,, \\ |\phi^\pm\rangle &= \frac{1}{\sqrt{2}}\,(|H\rangle|H\rangle \pm |V\rangle|V\rangle) \end{aligned} \tag{5}$$

are the four orthogonal Bell states.

Suppose that we now perform a joint Bell-state measurement on photons 2 and 3, i.e., project photons 2 and 3 onto one of the four Bell states. Equation (4) implies that this measurement also correspondingly projects photons 1 and 4 onto the same Bell state. After projection of photons 2 and 3, in all four cases photons 1 and 4 emerge entangled although they never interacted with one another in the past. This is the so-called entanglement swapping [17], which can also be seen as teleportation either of the state of photon 2 over to photon 4 or of the state of photon 3 over to photon 1 [12]. Apparently, in order to experimentally show the working principle of entanglement swapping it is sufficient to identify only one of the four Bell states [8,20].

In the experiment, we chose to analyze the projection onto $|\phi^+\rangle_{23}$. This projection is accomplished by performing a polarization decomposition in the 45° basis in outputs $2'$ and $3'$ and a subsequent coincidence detection [21]. More explicitly, detecting $+45°/+45°$ or $-45°/-45°$ coincidences between the outputs $2'$ and $3'$ acts as a projection onto $|\phi^+\rangle_{23}$, and thus leaves photons 1 and 4 in the identical state $|\phi^+\rangle_{14}$. This behavior is verified by the data shown in Figs. 3 and 4. Figure 3 proves that only *HH* and *VV* terms are present in the state of particles 1 and 4 conditioned on a fourfold coincidence. Figure 4 in turn can be viewed as the interference pattern showing the correlation in the conjugate basis. Specifically, the data of Fig. 4 indicate that the state of, say, photon 2 was teleported to photon 4 with a fidelity of 0.89. This clearly outperforms our earlier work [10] in this field, and for the first time fully demonstrates the nonlocal feature of quantum teleportation [13].

An experimental realization of four-particle GHZ entanglement and high-fidelity teleportation has rather profound implications. First, going to higher entangled systems the contradiction with local realism becomes ever stronger, because both the necessary visibility and the required number of statistical tests to reject the LHV models at a certain confidence level decrease with the number of particles that are entangled [22,23]. Second, based on the observed visibility of 0.79 ± 0.6, one could violate—with an appropriate set of polarization correlation measurements—Bell's inequality for photons 1 and 4, even though these two photons never interacted directly. As noted by Aspect, "This would certainly help us to further understand nonlocality" [24]. In our experiment, however, due to the low count rates and some instability in the pump laser it was not yet possible to carry out all the measurements needed. Note that, with the present experimental performance, a continuous measurement of more than six months would be necessary to collect statistically sufficient data.

Besides its significance in tests of quantum mechanics versus local realism, the methods developed in the experiment also have many useful applications in the field of quantum information. It was noticed very recently that, while our setup directly provides a simple way to perform entanglement concentration [25,26], a slight modification of the setup also provides a novel way to perform entanglement purification for general mixed entangled states [27]. Furthermore, following the recent proposal by Knill *et al.* [28], our four-photon experiment also opens the possibility to experimentally investigate the basic elements of quantum computation with linear optics.

In summary, we have demonstrated a method of creating higher order entangled states which can, in principle, be extended to any desired number of particles, provided one has efficient pair sources. Given that, more photon pair sources could be combined with polarizing beam splitters to yield entangled states of arbitrary numbers of particles. The latest developments in photon pair sources suggest that it should be possible in the near future to have sources with many orders of magnitude higher emission rates [29,30]. With these entanglement sources, one would be able to implement some quantum computation algorithms using only entanglement and linear optics [28]. Also, more elaborate entanglement purification protocols and high-fidelity teleportation over multiple stages as required for the construction of quantum repeaters [31] become possible.

We acknowledge the financial support of the Austrian Science Fund FWF, Project No. F1506, and the European Commission within the IST-FET project "QuComm" and TMR network "The Physics of Quantum Information."

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