# Creep of Superconducting Vortices in the Limit of Vanishing Temperature: A Fingerprint of Off-Equilibrium Dynamics

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We theoretically study the creep of vortex matter in superconductors. The low temperature experimental phenomenology, previously interpreted in terms of "quantum tunneling" of vortices, is reproduced by Monte Carlo simulations of a purely "classical" vortex model. We demonstrate that a nonzero creep rate in the limit of vanishing temperature is to be expected in systems with slow relaxations as a consequence of their off-equilibrium evolution in a complex free energy landscape.

There exists an abundance of experimental evidence that the relaxation rate of the magnetization in type-II superconductors (ranging from conventional to organic heavy fermions and a variety of $H$-$T_c$ superconductors [1–3]) does not vanish as the temperature $T$ is lowered towards zero (see, e.g., [1–3]). This may seem surprising if one assumes that the mechanism allowing the magnetization to relax is thermal activation over a characteristic energy barrier $\Delta E$. Namely, when $T \ll \Delta E$ the relaxation rate should vanish since the Arrhenius factor for thermal activation, $\exp(-\Delta E/T)$, goes to zero. The question then arises how relaxation can take place at a finite rate while the thermal activation factor exponentially approaches zero.

A number of researchers suggested that such a phenomenon is caused by quantum tunneling of vortices through the barriers in the random pinning potential (for a theoretical review, see [4]). The above "quantum" explanation is very intriguing and in good agreement with some experimental results in compounds such as YBCO [1,2] or BSCCO [5]. Also other even more exotic materials, such as organic superconductors [6], give good correspondences. However, in other nonconventional systems such as heavy fermion superconductors, the theory of quantum creep is totally unable to describe the observed low-$T$ relaxation [7]. In fact, strong discrepancies are found in many other systems ranging from PCeCO crystals to YBCO/PBCO multilayers or YBCO and BSCCO films and crystals [8]. One problem is [8] that the length of the tunneling vortex segment $L_c(0)$ needed to fit the creep rate data, $S$, can be orders of magnitude larger than the one theoretically predicted by quantum creep theory [4]. Also the experimental temperature dependence of the creep rate, $S$, is often very different from the one predicted by quantum theory (see [5]).

The above contradictory results suggest that one look for additional descriptions of the anomalous low-$T$ magnetic relaxation. It is worth stressing that the observation of a nonvanishing constant creep rate in the limit $T \rightarrow 0$ is found under very general circumstances: it does not crucially depend on the thickness of the sample [8] (i.e., on its dimensionality) or on whether the pinning is caused by columnar defects or random point pins [9]. Thus, the mechanism behind the low temperature creep seems to be of a fundamental and basic nature.

We demonstrate below that also in a "classical" system (i.e., not quantum) logarithmically slow glassy dynamics can naturally persist even at vanishing temperatures and can lead to the experimentally observed phenomenology. This is possible because the low-$T$ off-equilibrium dynamics consists of searching, among a very large number, for a few "downhill" or "flat" directions in the free energy landscape. The number of these directions decreases as relaxation proceeds though some always remain. They can be found only by collective cooperative rearrangements of the system, resulting in a slowing down of relaxation [10].

*The model.*— We study a statistical mechanics model for vortex matter called a restricted occupancy model (ROM) [11]. In the limit of zero temperature and infinite upper critical field, it reduces to a cellular automaton introduced in [12] to a study of vortex avalanches. We use Monte Carlo (MC) dynamics which enabled the ROM to depict a unified picture of creep and transport phenomena in vortex physics, ranging from magnetization loops with an "anomalous" second peak, logarithmic relaxation, and Bean profiles, to history-dependent behaviors in vortex flow and $I$-$V$ characteristics, and to the reentrant nature of the equilibrium phase diagram [11]. The model also predicts the existence of a "glassy region" at low temperature with strong "aging" effects [11].

Here we use the ROM to study the magnetic relaxation rate, $S$, in the very low temperature limit. Interestingly, the model reproduces the experimental anomalous relaxation and the observed behavior of $S$ [1–3,8].

The ROM model is described in full detail in Ref. [11]. A system of straight parallel vortex lines is coarse grained in the $xy$ plane by introducing a square grid of lattice spacing $l_0$ of the order of the London penetration length, $\lambda$ [11]. The number of vortices on the $i$th coarse grained plaquette is denoted by $n_i$. The occupancy of each plaquette is a number larger than zero and, importantly, smaller than $N_{c2} = B_{c2}l_0^2/\phi_0$, where $B_{c2}$ is the upper

critical magnetic field and $\phi_0 = hc/2e$ is the magnetic flux quantum. The ROM model is thus defined by the following coarse grained vortex interaction Hamiltonian [11]: $\mathcal{H} = \frac{1}{2}\sum_{ij} n_i A_{ij} n_j - \frac{1}{2}\sum_i A_{ii} n_i - \sum_i A_i^p n_i$. The first two terms describe the repulsion between the vortices and their self-energy. On-site and nearest neighbor interactions are included; i.e., the interaction between vortex lines with a separation greater than the London screening length (which by definition is close to $l_0$) is ignored. We choose $A_{ii} = A_0 = 1$; $A_{ij} = A_1$ if $i$ and $j$ are nearest neighbors and $A_{ij} = 0$ otherwise. The last term in $\mathcal{H}$ describes a delta-distributed random pinning $P(A^p) = (1 - p)\delta(A^p) + p\,\delta(A^p - A_0^p)$. Interestingly, the general scenario of creep phenomena we describe below does not depend on the details of pinning in the system (a fact in correspondence with experimental results [8]). In our model, $A_0$ sets the energy scale. Below we choose $A_0 = 1.0$, $A_0^p = 0.3$, $N_{c2} = 27$, $p = 1/2$, and $\kappa^* \equiv A_1/A_0 \in [0.26, 0.3]$.

The relaxation of the model is simulated by use of Monte Carlo dynamics on a square lattice in the presence of a thermal bath of temperature $T$. The system is periodic in the $y$ direction. The two edges parallel to the $y$-direction are in contact with a vortex reservoir. Particles can enter and exit the system only through the reservoir, which plays the role of the external magnetic field. Hence the reservoir density, $N_{ext}$, is used as the external control parameter. We perform the following zero field cooled experiment: at a low temperature $T$ we increase at constant rate $\gamma = \Delta N_0/\tau$ the reservoir density from zero to a working value $N_{ext}$. We keep $N_{ext}$ fixed while we monitor the time dependence of the magnetization $M = N_{in} - N_{ext}$. Here $N_{in} = \sum_i n_i/L^2$ is the vortex density inside the system (of size $L^2$ [13]). Time is measured in units of single attempted updates per degrees of freedom of the lattice (see Ref. [14]). The data presented below are averaged over 128 realizations of the pinning background.

In particular, we investigate the creep rate

$$S = \left| \frac{\partial \ln(M)}{\partial \ln(t)} \right| \qquad (1)$$

as a function of $T$, $N_{ext}$, and $\gamma$. In typical experiments the nature of the $t$ dependence of $M$ is such that $S$ decreases in time. So, usually, one deals with an average creep rate, $S_a$, in some given temporal window [1–7]. As shown in the upper inset of Fig. 2 below, in our model dynamics $M(t)$ at low temperatures behaves according to the known logarithmic interpolation formula (see Ref. [11]) found in experiments [4], namely, $M(t) - M(0) \simeq \Delta M_\infty\{1 - [1 + \frac{\mu T}{U_c}\ln(\frac{t+t_0}{t_0})]^{-1/\mu}\}$. Here, $M(0)$ is the magnetization at the time of preparation of the sample ($t = 0$), $\Delta M_\infty$ is its overall variation, the exponent $\mu$ is consistent with 1, and $\frac{\mu T}{U_c}$ and $t_0$ are fit parameters [4].

Consistently, we define $S_a$ as the average value of $S$ in the last time decade of our measurements (i.e., for $t \in [10^5, 10^6]$). The present choice, analogous to those

made for experimental data, is the most natural one and the general results presented below do not depend on it.

For the reasons explained earlier, a very important physical quantity is the distribution, $P(\Delta E)$, of the energy barriers, $\Delta E$, that vortex segments meet during their motion. Since at low $T$ the system is typically off equilibrium, $P(\Delta E)$ is itself a (logarithmically slow) function of $t$ and we consider its average over the last time decade of our measurements.

*Results.*—When the temperature is very low, the model exhibits the same type of anomalous creep found in the experiments on superconductors. In Fig. 1, we plot the creep rate, $S_a$, as a function of $T$ in a broad temperature range. For comparison we present equivalent experimental measurements in a BSCCO single crystal (from Ref. [5]) as the inset. The numerical values found for $S_a$ at low $T$ in our model and in real samples are interestingly very similar. The temperature scales of the simulations and of real experiments can be compared by considering that the ratio $T/A_0$ in our model is of the same order of magnitude as $T/T_c$ in a real superconductor. This is seen from a comparison of the $(T, N_{ext})$ equilibrium phase diagram of our model with the equilibrium temperature-magnetic field, $(T, H)$, phase diagram of, say, a BSCCO superconductor (see Ref. [11]).

In both experiments and simulations, $S_a$ approaches a finite value, $S_a^0$, when $T \to 0$. In particular, we find that a linear fit of $S_a(T)$ in the low-$T$ regime is very satisfactory (see Fig. 1):

$$S_a(T) = S_a^0 + \sigma T, \qquad (2)$$

where both $S_a^0$ and $\sigma$ are a function of the applied field $N_{ext}$. We also note that in the present model $S_a(T)$ is nonmonotonous in $T$: in the higher $T$ region it starts decreasing. This is also a known experimental fact [6,15]; we
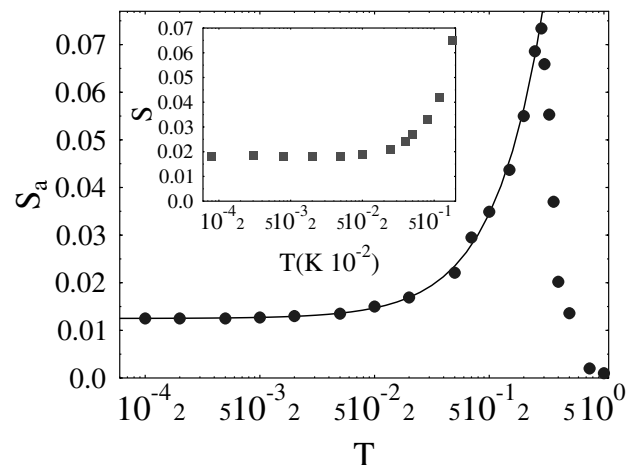


FIG. 1. Main frame: the creep rate, $S_a$, in the ROM model ($\kappa^* = 0.28$, $\gamma = 10^{-3}$) for $N_{ext} = 10$ as a function of the temperature, $T$, in units of $A_0$. The error bars are of the size of symbols. The superimposed line is a linear fit. Inset: the creep rate measured in BSCCO single crystal from Aupke *et al.* [5] for an external field of 880 Oe.

discuss it later. The maximum in $S_a(T)$ is just above a characteristic crossover "glassy" temperature, $T_g$, defined in [11].

In our model, by varying the applied field, we find a range of values for $S_a^0$ very similar to experimental ones [1–7] (see Fig. 2). In particular, $S_a^0$ seems to decrease on average by increasing the field $N_{ext}$. The overall behavior can be approximately interpolated with a power law: $S_a^0(N_{ext}) \simeq (N_{ext}/N_0)^{-x}$, where, for $\kappa^* = 0.28$, $N_0 \simeq 0.01$ and $x \simeq 0.6$. As shown in Fig. 2, the presence of a small exponent $x$ implies that sensible variations in $S_0^a$ can be seen only by changing $N_{ext}$ of orders of magnitude. Note that the dips in the $S_a(0)$ versus $N_{ext}$ data in Fig. 2 at certain values of $N_{ext}$ (namely, about 3, 13, and 18) are statistically significant. They are related to the low field order-disorder transition, the second peak transition, and the reentrant high field order-disorder transition, respectively [11].

In the lower inset of Fig. 2, we show that $S_a$ is essentially independent of the ramping rate, $\gamma$ (the values shown are for $N_{ext} = 10$ and $T = 10^{-4}$). This is another typical experimental observation [1]. However, a very small decrease of $S_a$ with increasing $\gamma$ cannot be excluded: we show a fit to the form $S_a^0 \simeq S_1 + s_2 \ln(\gamma)$, with $S_1 = 0.11$ and $s_2 = -2 \times 10^{-4}$. The fact that $S_a^0$ is practically independent of $\gamma$, far from being proof of the presence of equilibrium in the system, is due to the fact that at very low temperatures the characteristic equilibration time, $\tau_{eq}$, is enormous (see Ref. [11]). So whenever the driving rate, $\gamma$, is much larger than $\tau_{eq}^{-1}$ the off-equilibrium state and dynamics of the system are essentially inde-

pendent of $\gamma$. Stronger $\gamma$ effects have to be expected when $\gamma$ gets closer to $\tau_{eq}^{-1}$. In fact, it is experimentally well known that at higher temperatures the systems exhibit strong $\gamma$-dependent "memory" effects [11,15,16], the signature of off-equilibrium dynamics. Actually, in the present model at low $T$, it is possible to show that $\tau_{eq}(T)$ diverges exponentially [11]: $\tau_{eq}(T) \sim \exp(\frac{E_0}{T})$. In that region, the typical observation time windows, $t_{obs}$, are such that $t_{obs}/\tau_{eq} \ll 1$, and the system is in the early stage of its off-equilibrium relaxation from its initial state. This is schematically the origin of the flattening of $S_a$ at very low $T$. Notice that, if one could observe the system for an exponentially long time, i.e., if $t_{obs}/\tau_{eq} \gg 1$, then the creep rate, $S_a$, would indeed go to zero.

The above scenario is clarified by the analysis of the energy barrier distribution function, $P(\Delta E)$, recorded during the system evolution at very low $T$. Such a quantity also clearly shows the simple mechanical origin of the anomalous creep found at very low temperature in the present model. The function $P(\Delta E)$ (where $\Delta E$ is in units of $A_0$), recorded at $T = 10^{-4}$, is plotted in Fig. 3 for two values of the applied field, $N_{ext}$. We always find that $P(\Delta E)$ has support also on the negative axis. This is the mark of the off-equilibrium nature of the evolution on the observed time scales. The presence of a $P(\Delta E)$ which extends down to negative values also explains the presence of the recorded relaxation at low $T$: in the configuration space the system can still find directions where no positive barriers have to be crossed. The inset in Fig. 3 clarifies the mechanism behind the relaxation. Here, we plot the signal $A(t)$ defined, for each single MC step $t$, in the following way: $A(t) = 0$ if the MC trial is rejected; $A(t) = 1$ if
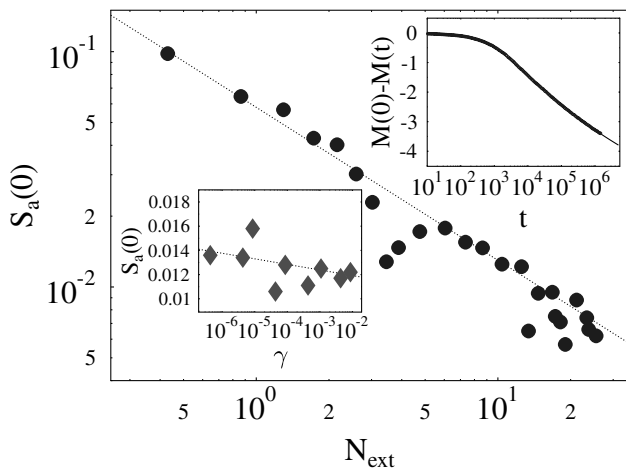


FIG. 2. Main frame: the zero temperature limit of the creep rate, $S_a$, in the ROM model as a function of the applied field $N_{ext}$ (for $T = 10^{-4}$ and $\gamma = 10^{-3}$). The superimposed curve is a power law to guide the eye. Inset top: the relaxation of the magnetization, $M(t)$, in the model ($\kappa^* = 0.28$, $\gamma = 10^{-3}$) for $N_{ext} = 10$ and $T = 0.25$ as a function of time. The continuous line is the logarithmic fit of the text. Inset bottom: $S_a$ is here plotted as a function of the sweep rate, $\gamma$, of the external field for $N_{ext} = 10$ and $T = 10^{-4}$. A very weak dependence is found.
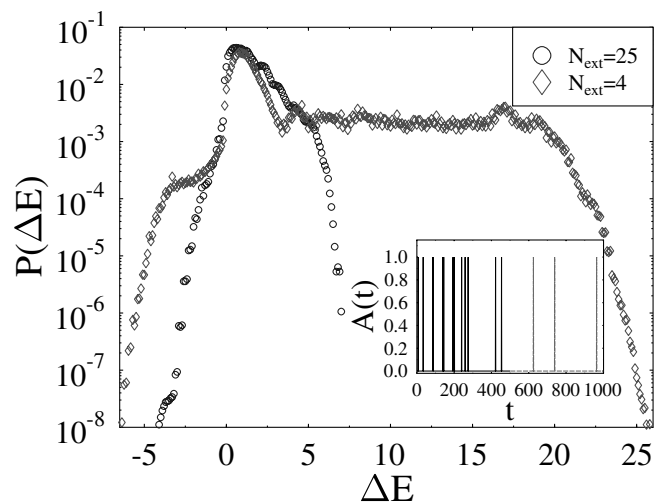


FIG. 3. The energy barrier distribution, $P(\Delta E)$, recorded at $T = 10^{-4}$ for $N_{ext} = 25, 4$ (circles, diamonds, respectively). Notice that $P(\Delta E)$ extends down to negative values. Inset: we plot the function $A(t)$ defined in the text to monitor the activity during relaxation. In the interval $[0, 500]$ we show $A(t)$ recorded at the beginning of the run and in $[500, 1000]$ $A(t)$ in the last steps of the same run.

the trial is accepted and the energy reduced, i.e., $\Delta E \leq 0$; finally, $A(t) = 2$ when a trial is accepted with $\Delta E > 0$. We plot two sequences of trials. One for $0 \leq t \leq 500$ was measured at the early stage of the relaxation, the second sequence, placed at the interval $500 \leq t \leq 1000$, was measured at the late stage of the relaxation. Most trials are rejected [$A(t) = 0$] and only once in a while does the system manage to find a route pointing downhill in the energy landscape. $A(t) = 2$ never occurs. As time proceeds, fewer and fewer "negative channels" are available to the relaxation and a decrease in the density of the spikes in $A(t)$ is observed.

As the temperature is increased, thermal activation over positive energy barriers becomes possible as the Arrhenius factor $\exp(-\Delta E/T)$ assumes a nonvanishing value for an appreciable range of barrier values $\Delta E < T$. When this happens the relaxation will occur sufficiently fast to allow one, within the experimental time window, to closely approach the equilibrium configurations where the vortex density profile is more or less flat and relaxation ceases, hence $S_a$ goes down, as seen in experiments [6,15] and in Fig. 1.

Finally, we stress that slow off-equilibrium relaxations at very low temperatures are also observed in glass forming liquids [10]. In those cases also, no activation over barriers occurs and the system simply wanders in its very high-dimensional phase space through the few channels where no energy increase is required.

*Discussion.*—We have demonstrated above that the phenomenological behavior of the creep rate at low temperatures can be understood in terms of the off-equilibrium nature of the inhomogeneous vortex density profile produced in magnetic creep experiments. In fact, cooperative mechanical rearrangements, possible even at very low $T$ (where thermal activation over positive barriers can be negligible), dominate the phenomenon [10]. In this perspective, it is very important to stress that the system's equilibration time at very low temperature is much larger than any experimentally accessible time window [11]. Accordingly, we can say that experimental findings do not enforce the interpretation in terms of macroscopic quantum tunneling of vortices.

Relaxation due to quantum tunneling might be present along with the mechanical relaxation discussed above. It is then important to ask how compelling the quantum tunneling interpretation is. The theory of quantum tunneling assumes a London picture and treats the position of the vortex core as the variable that is able to tunnel. It is not entirely clear if this is the right level at which to introduce quantum fluctuations, but, more importantly, the quantum tunneling description assumes the existence of a character-istic energy barrier [4], not a time-dependent distribution of barrier heights as typically found in many-body systems relaxing off-equilibrium. Finally, the quantum tunneling description also tacitly assumes the existence of a static equilibrium state in which barriers are always positive. As we have clearly shown above, this is typically not the case and a dynamical approach is more appropriate.

The present scenario, where off-equilibrium phenomena dominate the anomalous low-$T$ creep, could be experimentally verified by the discovery of aging phenomena [11] such as those recently observed in [17,18].

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