Resonant Radiation Pressure on Neutral Particles in a Waveguide

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A theoretical analysis of electromagnetic forces on neutral particles in a hollow waveguide is presented. We show that the effective scattering cross section of a very small (Rayleigh) particle can be strongly modified inside a waveguide. The coupling of the scattered dipolar field with the waveguide modes induces a resonant enhanced backscattering state of the scatterer-guide system close to the onset of new modes. The particle effective cross section can then be as large as the wavelength even far from any transition resonance. As we will show, a small particle can be strongly accelerated along the guide axis while being highly confined in a narrow zone of the cross section of the guide.

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Demonstration of levitation and trapping of micronsized particles by radiation pressure dates back to 1970 and the experiments reported by Ashkin and co-workers [1]. Since then, manipulation and trapping of neutral particles by optical forces has had a revolutionary impact on a variety of fundamental and applied studies in physics, chemistry, and biology [2]. These ideas were extended to atoms and molecules where radiation pressure can be very large due to the large effective cross section (of the order of the optical wavelength) at specific resonances [1,3]. When light is tuned close to a particular transition, optical forces involve (quantum) absorption and reradiation by spontaneous emission as well as coherent (classical) scattering of the incoming field with the induced dipole [4]. Selective control of the strong interplay between these two phenomena is the basis of laser cooling and trapping of neutral atoms [5].

However, far from resonance, light forces on atoms, molecules, and nanometer-sized particles are, in general, very small. Here we show that the scattering cross section of a very small (Rayleigh) particle can be strongly modified inside a waveguide. The coupling of the scattered dipolar field with the waveguide modes induces a resonant enhanced backscattering state of the scatterer-guide system close to the onset of new modes. Just at the resonance, the effective cross section becomes of the order of the wavelength leading to an enhanced resonant radiation pressure which does not involve any photon absorption phenomena. As we will show, a small particle not only can be strongly accelerated along the guide axis but it can also be highly confined in a narrow zone of the cross section of the guide.

For the sake of simplicity we consider a two-dimensional xz waveguide with perfectly conducting walls and cross section D. The particle is then represented by a cylinder located at $\vec{r}_0 = (x_0, z_0)$ with its axis along oy and radius much smaller than the wavelength (see top of Fig. 1). However, apart from some depolarization effects, the analysis contains the same phenomena as the full three-dimensional problem [6] and hence it permits

an understanding of the basic physical processes involved in the optical forces without loss of generality.

An s-polarized electromagnetic wave is assumed (the electric field parallel to the cylinder axis), $\vec{E}(\vec{r}) = \exp(-i\omega t)E^0(\vec{r})\vec{y}$ with wave vector $k = \omega/c = 2\pi/\lambda$. For a single mode waveguide ($D/2 < \lambda \leq D$), the incoming electric field can be written as the sum of two interfering plane waves: $E^0(\vec{r}) = E_0[\exp(ik_z z + ik_x x) - \exp(ik_z z - ik_x x)]$ where $k_z = k\cos(\theta)$, $k_x = k\sin(\theta) = \pi/D$. The scatterer can be characterized by the scattering phase shift δ_0 or by its polarizability α and Rayleigh scattering cross section σ [7,8]. The time average force $\vec{F}$ can be written as the sum of an optical gradient force and a scattering force [9]:

$$\vec{F} = \left\{ \frac{1}{4}\, \alpha \vec{\nabla} |E^{\text{inc}}|^2 + \frac{\langle \vec{S} \rangle}{c}\, \sigma \right\}_{\vec{r}=\vec{r}_0}, \qquad (1)$$

where E^{inc} is the total incident field on the particle and $\langle \vec{S} \rangle$ is the time average Poynting vector.
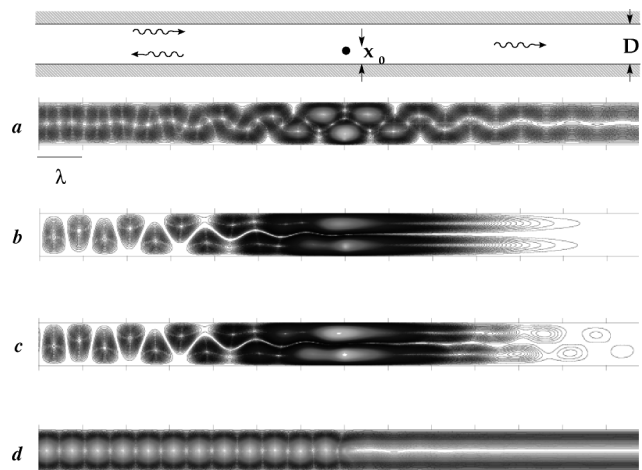

FIG. 1. Top: Sketch of the particle-waveguide system. Field intensity plots for different particle positions x_0 across the waveguide (a) $x_0/D = 0.15$, (b) $x_0/D = 0.22$, (c) $x_0/D = 0.25$, (d) $x_0/D = 0.5$. The field incides from the left side.

If we neglect the multiple scattering effects between the scatterer and the waveguide walls, the interaction would be equivalent to that of a particle placed in the interference pattern of two crossed plane wave beams. In this case $E^{\mathrm{inc}} = E^0$ and the theory of radiation pressure in a waveguide is straightforward. The longitudinal force F_z^0 (per unit length) can be written in terms of the average power density of the incident beams, $\langle S \rangle = \epsilon_0 c |E_0|^2$, as

$$F_z^0 = 2\sigma \epsilon_0 |E_0|^2 \cos(\theta) \sin^2\left(\frac{\pi x_0}{D}\right) \qquad (2)$$

which is maximum $[F_{z_{\max}}^0 = 2\sigma \epsilon_0 |E_0|^2 \cos(\theta)]$ just in the middle of the waveguide. The transversal force induces an optical potential along x given by

$$U_x^0 = -\alpha |E_0|^2 \sin^2(\pi x_0/D), \qquad (3)$$

which, for $\alpha > 0$, confines the particle near the center of the waveguide. This is the two-dimensional analog of previous approaches on laser guiding of atoms and particles in hollow-core optical fibers [10–12] where the interaction of the dipole field with the guide walls was neglected.

The scattering with the waveguide walls may induce however a dramatic effect on the optical forces on the particle. The scatterer radiates first a dipole field generated by the field of the incoming mode E^0. Then, the scattered field, perfectly reflected by the waveguide walls, goes back to the scatterer changing the field inciding on the scatterer and so on. This multiple scattering process can be regarded as produced by a set of infinite image dipoles [13,14]. From the exact solution for the total field together with Eq. (1), we found that, for a single mode waveguide, the forward component of the force F_z can be written in terms of the waveguide transmittance T (defined as the ratio between the outgoing and incoming energy flux; $0 \leq T \leq 1$) as

$$F_z = 2D \epsilon_0 |E_0|^2 \cos^2(\theta)[1 - T(x_0)], \qquad (4)$$

where T depends on the transversal position of the particle x_0.

The transmission coefficient T had been discussed before in the context of electronic conductance of quasi-one-dimensional conductors with pointlike attractive impurities [13–16]. T presents two peculiar properties: (i) when D/λ is just at the onset of a new propagating mode, the scatterer becomes transparent; (ii) interestingly, when D/λ is close but still below a mode threshold, the transmittance of a single mode waveguide presents a dip down to exactly $T = 0$. This backscattering resonance, that was associated with the existence of a quasibound state induced by the attractive impurity [14,15], can be achieved for any attractive scattering potential of arbitrarily small strength [14], i.e., *for arbitrarily small polarizability α and cross section σ* [8]. This resonance has a pronounced effect on the radiation forces.

In Fig. 2 we plot the transmission coefficient as a function of both the waveguide width, D/λ, and the scatterer
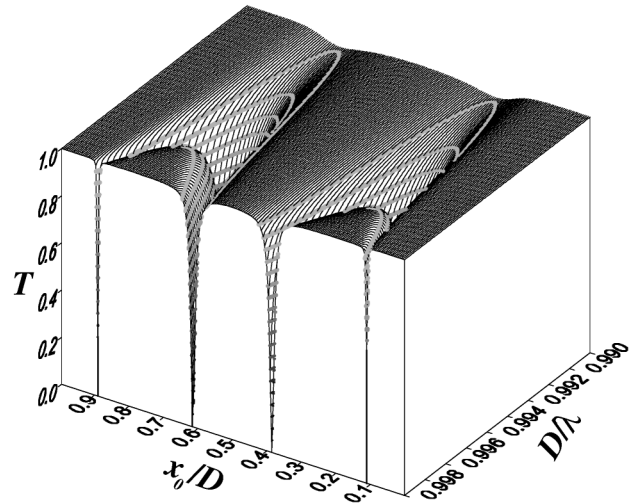

FIG. 2.   Transmittance of a single mode waveguide versus width D/λ and scatterer position x_0/D for a fixed phase-shift value $\delta_0 = 10°$.

position x_0 for $\delta_0 = 10°$. Near the threshold of the second mode ($D/\lambda \lesssim 1$), T presents sharp dips (down to $T = 0$) at some particular positions of the scatterer. Just at the enhanced backscattering resonances, the longitudinal force presents strong maxima $F_{z_{\max}}$ which can be compared with $F_{z_{\max}}^0$ (the maximum force obtained neglecting the interactions with the walls), $F_{z_{\max}}/F_{z_{\max}}^0 = \cos(\theta)D/\sigma$ (see Fig. 3), i.e., at resonance the interaction cross section of the particle in the waveguide can be as large as the total
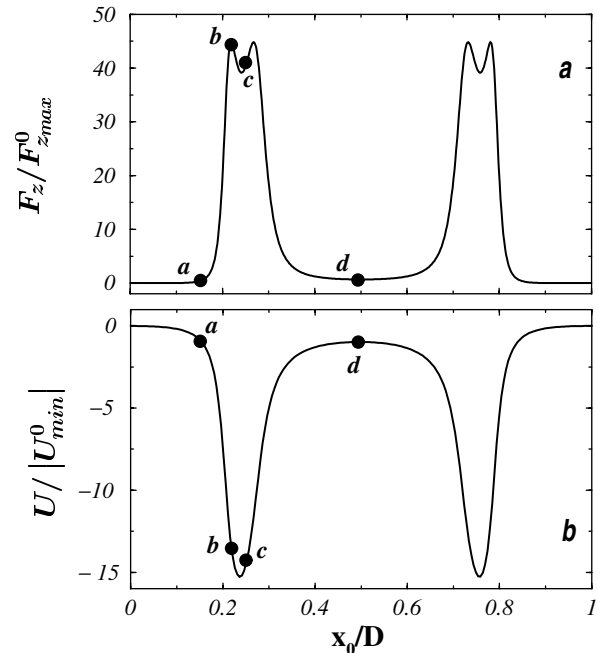

FIG. 3.   Longitudinal force F_z (a) and lateral confining potential U (b) versus particle position (normalized to the maximum force $F_{z_{\max}}^0$ and the minimum potential $U_{\min}^0$ in the unbounded system). The values of x_0/D corresponding to the dots are those of (a)–(d) in Fig. 1.

cross section of the waveguide independently of its value σ in the unbounded space. The force enhancement factor can be huge: for $\delta_0 = 10°$ ($\sigma \approx 20$ nm) and at micron wavelengths, it is of the order of 50, while for a nanometer scale particle ($\delta_0 \approx 2°$) this enhancement would be $\approx 10^3$ for two dimensions, but $\approx 10^6$ in true three-dimensional systems. This result is rigorously true for perfect walls. In actual waveguides however, the radiation losses through the walls and the scattering with surface defects [17] may modify the resonace behavior for σ values comparable to the surface roughness.

The resonance is related to the strong coupling between the incoming mode and the first evanescent mode in the waveguide. This can be seen by plotting the field intensity inside the waveguide for different particle positions (Fig. 1). At the resonance, the field around the particle corresponds to that of the second mode in the waveguide (with a node in the waveguide axis) which decays far from the defect. When the particle is located at the middle of the waveguide, there is no coupling of the scattered field with the first evanescent mode and F_z presents a minimum.

Transversal forces are also strongly affected by the resonances. Although the main contribution to these forces comes from polarization effects (i.e., proportional to $\nabla|E^{\mathrm{inc}}|^2$), in contrast with the free space case, the lateral forces have also a contribution of pure scattering origin due to the reflections of the flux from the walls. The induced transversal confining potential far from the mode threshold presents a single well similar to that discussed for the unbound system. However, near the resonance condition it presents two strong minima reflecting the excitation of the evanescent mode. In Fig. 3 we plot the normalized longitudinal force F_z and transverse confining potential U_x. The particle will be strongly confined in a small region inside the waveguide where the forward longitudinal force is maximum. For example, for $\delta_0 = 10°$ and at micron wavelengths, the potential well is more than 1 order of magnitude deeper than that obtained for the unbound system.

In summary, we have discussed the electromagnetic forces on small neutral particles in a hollow waveguide. In contrast with standard resonance radiation forces, the waveguide-particle backscattering resonances discussed here do not involve photon absorption processes and, we believe, open intriguing possibilities of atom and molecule manipulation. Specifically, the depth of the potential wells for the particle in resonant conditions and its remarkably large cross section suggest stable guiding of the particle along the waveguide with extremely large accelerations.

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