Inhibition of Decoherence due to Decay in a Continuum

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We propose a scheme for slowing down decay into a continuum. We make use of a sequence of ultra-short 2π pulses applied on an auxiliary transition of the system so that there is a destructive interference between the two transition amplitudes—one before the application of the pulse and the other after the application of the pulse. We give explicit results for a structured continuum. Our scheme can also inhibit unwanted transitions.

One of the fundamental causes of noise at the optical frequencies is the intrinsic spontaneous emission. It is well known that the performance of many systems is limited by spontaneous emission. For example, the noise figure of an amplifier is determined by spontaneous emission. One of the challenges therefore is to find ways in which spontaneous emission noise can be reduced if not totally inhibited. Several proposals exist in literature for reducing spontaneous emission noise. These include the placing of atoms in a photonic band-gap structure [1,2], as well as the use of external fields [3] and quantum interferences [4,5]. As is well known, spontaneous emission noise arises from considerations based on the interaction of an atom with the vacuum of the electromagnetic field. The vacuum acts like a zero temperature bath. It may be noted that the question of inhibiting the effects of a heat bath is being extensively studied and several new proposals exist for such an inhibition [6–10]. For example, for a spin system interacting with a bath the decoherence can be slowed [6] by applying a sequence of π pulses applied at intervals of a short period τ which is less than the bath correlation time.

In this Letter we consider the case of spontaneous emission from an excited atom. One has considerably more freedom with atoms than with spins, as in the former case we could use a different transition to control spontaneous emission. We propose a scheme to suppress spontaneous emission on (say) the emission from the state $|e\rangle$ to $|g\rangle$ by using an auxiliary transition $|e\rangle$ to $|f\rangle$. We apply a sequence of ultrashort 2π pulses separated by an interval τ. We demonstrate how a destructive interference between the evolution from t_o to $t_o + \tau$ and $t_o + \tau$ to $t_o + 2\tau$ can lead to suppression of decoherence. It should be borne in mind that the decoherence time scale is generally proportional to the decay time. The destructive interference is related to the very remarkable property that after the application of a 2π pulse the state $|e\rangle$ acquires a phase shift of π. An important by-product of our investigation is the possibility of suppressing undesirable weak transitions.

We first consider a simple case which is adequate to describe the main idea and which by itself is very relevant to the subject of quantum computation [11]. Consider a quantum system which can make an *unwanted* weak transition from the state $|g\rangle$ to $|e\rangle$ as a result of a perturbation v. Let δ be the detuning; i.e., the perturbation need not be resonant with the transition $|g\rangle \leftrightarrow |e\rangle$. The interaction Hamiltonian in the interaction picture is

$$H_1(t) = \hbar v |e\rangle\langle g| e^{-i\delta t} + \text{H.c.} \tag{1}$$

The perturbation theory leads to the probability of transition

$$p_{eg} = |v|^2 \frac{\sin^2(\delta t/2)}{(\delta/2)^2}. \tag{2}$$

We next demonstrate how this unwanted transition could be inhibited by applying a sequence of very short 2π pulses [12] on the transition $|g\rangle \leftrightarrow |l\rangle$ (Fig. 1). We thus divide the total time interval into a large number $2N$ of short intervals τ. The system evolves under v from t_o to $t_o + \tau$. At $t_o + \tau$ we apply an ultrashort 2π pulse on the transition $|g\rangle \leftrightarrow |l\rangle$. The system evolves from $t_o + \tau$ to $t_o + 2\tau$ under v. At the instant $t_o + 2\tau$ the 2π pulse is
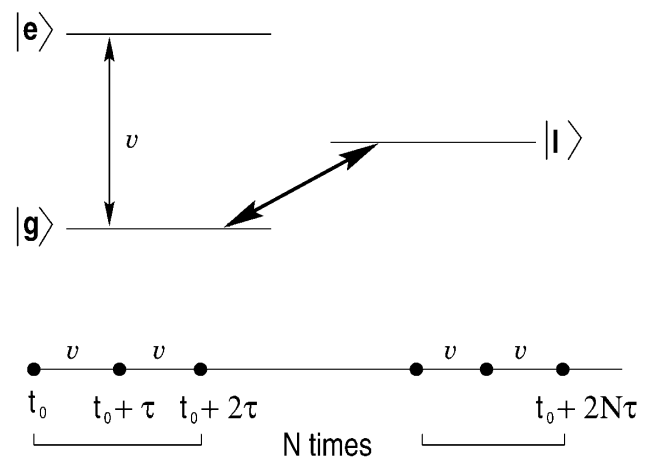

FIG. 1. Scheme for suppression of an unwanted transition $|g\rangle \rightarrow |e\rangle$ caused by a weak off-resonant perturbation v. The bold arrow represents the 2π pulse on the transition $|g\rangle \leftrightarrow |l\rangle$. The dots denote the times when an ultrashort 2π pulse is applied. In between the pulses the system evolves under v.

applied again. This process is repeated N number of times. The system then evolves as follows. For $t_o < t < t_o + \tau$, we have

$$|\psi(t)\rangle \sim |g\rangle - iv|e\rangle[X(t - t_o)/(-i\delta)]e^{-i\delta t_o};$$
$$X(t) = (e^{-i\delta t} - 1). \tag{3}$$

At time $t_o + \tau$ after the application of 2π pulse, the state of the system is

$$|\psi\rangle = -|g\rangle - iv|e\rangle X(\tau)e^{-i\delta t_o}/(-i\delta). \tag{4}$$

At a time $t_o + 2\tau$ just before the application of the second pulse the state would evolve into

$$|\psi\rangle = -|g\rangle - iv|e\rangle[X(\tau)/(-i\delta)]e^{-i\delta t_o} + iv|e\rangle[X(\tau)/(-i\delta)]e^{-i\delta(t_o+\tau)} + o(v^2), \tag{5}$$

which on application of the second 2π pulse changes to

$$|\psi\rangle \equiv |g\rangle + iv|e\rangle(X(\tau))^2 e^{-i\delta t_o}/(-i\delta) + o(v^2). \tag{6}$$

The transition amplitude χ_{eg} at the end of one cycle consisting of evolution of the system from t_o to $t_o + 2\tau$ but with 2π pulses applied at $t_o + \tau$ and $t_o + 2\tau$ will be

$$\chi_{eg} = iv(X(\tau))^2 e^{-i\delta t_o}/(-i\delta) + o(v^2). \tag{7}$$

The transition amplitude at the end of N such cycles will be

$$\chi_{eg} = iv\frac{(X(\tau))^2}{(-i\delta)}\sum_{p=0}^{N-1} e^{-2i\delta\tau p - i\delta t_o}, \tag{8}$$

which leads to the following result for the net transition probability

$$\tilde{p}_{eg} = |v|^2 \tan^2\left(\frac{\delta\tau}{2}\right)\frac{\sin^2(\frac{\delta}{2}(2\tau N))}{(\delta/2)^2}. \tag{9}$$

On using (2) we have one of our key results

$$\tilde{p}_{eg} = \tan^2\left(\frac{\delta\tau}{2}\right)p_{eg}. \tag{10}$$

We have thus proved that the application of a sequence of 2π pulses on an auxiliary transition leads to the suppression of an unwanted transition provided that the small interval and the detuning δ are chosen such that

$$\tan^2\left(\frac{\delta\tau}{2}\right) \ll 1. \tag{11}$$

The suppression arises from a *destructive interference* of the transition amplitudes [second and third terms in Eq. (5)]. This destructive interference is due to a phase change of the state $|g\rangle$ (and *not* of $|e\rangle$) by π due to the application of the 2π pulse. This also explains our choice of an auxiliary transition for the application of the 2π pulse as we selectively want to produce a phase change so that the interference can occur.

One might think that the procedure we describe is just the quantum Zeno effect [13]. We emphasize that it is *not* as we do not carry out repeated measurements and our results are in the framework of the standard unitary

evolution in quantum mechanics. The literature [14] on foundations of quantum mechanics considers the collapse of the state as an *essential ingredient* for quantum Zeno effect. In fact, the collapse of the wave function became such an issue that the authors of the important experiment [15] on quantum Zeno finally commented "the experiment neither verifies nor falsifies the notion of wave function collapse" [15(b)].

Our proposed scheme on the inhibition of unwanted transitions has similarities to the scheme of Itano *et al.* [15], in the sense that *pulses are applied on another* transition. Further, our scheme shares some features with the work of Viola and Lloyd [6] though there are differences in the nature of the interaction with the bath and the way the pulses are applied using the multilevel structure of the atom. Note further that for $N = 1$ and small $\delta\tau$, $\tilde{p}_{eg} \propto \tau^4$, whereas $p_{eg} \propto \tau^2$, and thus one might say that to leading order in τ^2 the unwanted transition gets decoupled (cf. Viola *et al.* in Ref. [6]).

We next demonstrate how the above procedure can be used to possibly inhibit spontaneous emission. We show the procedure schematically in Fig. 2. The atom makes a transition from the excited state $|e\rangle$ to the ground state $|g\rangle$ by emitting a photon. The photon can be emitted in any mode ω_k of the vacuum of the radiation field. The polarization of the emitted photon will be determined by the direction of the dipole matrix element. The interaction Hamiltonian in the interaction picture is [4].

$$H_1(t) = \hbar \sum_k |e\rangle\langle g|g_k a_k e^{-i\delta_k t} + \text{H.c.};$$
$$\delta_k = \omega_k - \omega_{eg}. \tag{12}$$

For brevity, we do not display the polarization "s" and vectorial indices of the mode. Thus k really stands for (k, s). In (12), a_k is the annihilation operator for the mode k of the radiation field. We now follow the procedure leading to Eq. (9). We quote the results of calculations for
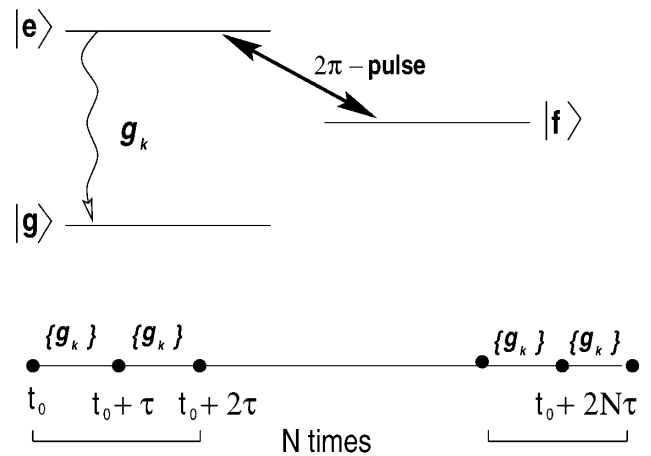

FIG. 2. Scheme for suppression of a decay into continuum with the atom going from $|e\rangle$ to $|g\rangle$ by the emission of a photon; the other specification is the same as in Fig. 1.

the transition probabilities with ($\tilde{p}_{ge}$) and without (p_{ge}) the application of 2π pulses

$$\tilde{p}_{ge} = \sum_k |g_k|^2 \tan^2\left(\frac{\delta_k \tau}{2}\right) \frac{\sin^2(\delta_k \tau N)}{(\delta_k/2)^2}, \quad (13)$$

$$p_{ge} = \sum_k |g_k|^2 \frac{\sin^2(\delta_k \tau N)}{(\delta_k/2)^2}. \quad (14)$$

We first note how (14) leads to the standard result and how the Einstein A coefficient emerges. If $t = 2N\tau$, then

$$\partial p_{ge}/\partial t = \sum_k 2|g_k|^2 \frac{\sin \delta_k t}{\delta_k} \quad (15)$$

which, under the assumption that the observation time t is large compared to the width of (δ_k) values (which is of the order of ω_{eg} for spontaneous emission in free space), reduces to the standard expression for the Einstein A coefficient

$$\partial p_{ge}/\partial t = 2\pi \sum |g_k|^2 \delta(\omega_k - \omega_{eg}) \equiv A. \quad (16)$$

We next examine the conditions under which the presence of the quantum interference term $\tan^2(\delta_k \tau/2)$ in Eq. (13) can lead to the suppression of spontaneous emission. Let us consider a kind of one-dimensional model in which we can replace (13) by

$$\tilde{p}_{ge} \equiv \int_{-\omega_{eg}}^{\infty} dx \, \tan^2\left(\frac{x\tau}{2}\right) \frac{\sin^2(\frac{x}{2} 2\tau N)}{(x/2)^2} \rho(x), \quad (17)$$

where $\rho(x)$ is the density of states for the one-dimensional vacuum. For $\rho(x)$, we can choose any of the functions like the Lorentzian or the exponential

$$\rho(x) = \frac{\rho_o \Gamma/\pi}{(x^2 + \Gamma^2)} \quad \text{or} \quad \rho(x) = \frac{\rho_o}{2\Gamma} e^{-|x|/\Gamma} \quad (18)$$

with $\Gamma \ll \omega_{eg}$ and where ρ_o is related to the square of the dipole matrix element. Note that these choices correspond to what are known as the "structured" vacua [16]. Thus we are essentially asking under what conditions the emission into structured vacua can be inhibited. We use the exponential model in (17) and show a typical result in Fig. 3. For comparison we also show the result p_{ge}. On comparing the two results we see that the use of a sequence of 2π pulses applied in the manner shown in Fig. 2 can suppress to a large extent the decay into a structured continuum. We next consider briefly the question of spontaneous emission in free space. We convert (13) into an integral using the standard expressions for g_k and letting the quantization volume go to infinity. The transition probability $\tilde{p}_{ge}$ depends on the following integral:

$$I = \int_{-1}^{\infty} dx \, (x + 1)^3 \tan^2\left(\frac{x\tau}{2}\right) \frac{\sin^2(\frac{x}{2} 2\tau N)}{(x/2)^2},$$
$$x = (\omega - \omega_{eg})/\omega_{eg}, \quad \tau \to \omega_{eg}\tau. \quad (19)$$

Note that the sine function is sharply peaked at $x = 0$ if $2N\tau$ is large. This is what enabled us to simplify the
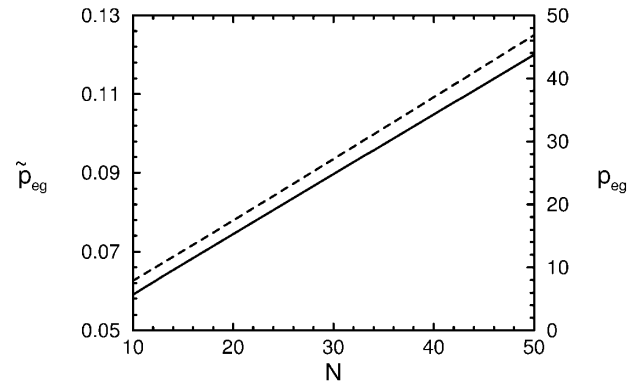

FIG. 3. A comparison of $\tilde{p}_{ge}$ (solid line) and p_{ge} (dashed line) (in units of A/Γ) as a function of N. The short interval τ is taken to be one-half of the bath correlation time.

expression for p_{ge} leading to (16). However, we now have the interference factor $\tan^2(x\tau/2)$ which starts growing as $x\tau/2 \to \pi/2$. Thus the idea of 2π-pulse induced slowing down of decoherence will obviously work if the bath has a cutoff such that $\tan^2(x\tau/2)$ remains much smaller than unity. In order to examine further the slowing down of the decoherence we analyze the integrand $\tilde{I}$ in (19). We can rewrite the integrand in the form

$$\tilde{I} \equiv (x + 1)^3 \frac{16 \sin^4(x\tau/2)}{x^2} \left[\frac{\sin^2(x\tau N)}{\sin^2(x\tau)}\right]. \quad (20)$$

Note that the integrand oscillates very rapidly due to the term $\sin^2(x\tau N)$. Though it has no singularities as the term in the square bracket is well behaved, it still grows as N^2 whenever $x\tau = n\pi$. Such a growth problem can be avoided if the vacuum mode frequencies are such that $x\tau \ll \pi$. If x is chosen to be of the order of unity, then the pulses have to be applied at *intervals much smaller than* ω_{eg}^{-1}, which is almost impossible in the optical domain though *not* difficult for *Rydberg transitions*. For example, for $\omega_{eg} \sim 10^{11} \, \text{sec}^{-1}$, $\tau \sim 10^{-11}$ sec, one should use subpicosecond pulses for changing the phase by π. Note that for applying 2π pulse one need not choose another Rydberg transition. Here it might even be advantageous to choose an optical transition. Note that with pulses applied at intervals of the order ω_{eg}^{-1}, the details of spontaneous emission become sensitive to the structure of the bath as shown in Fig. 4. Even the result I_0 in the absence of pulses exhibits a weak dependence on the cutoff frequency, though it can be shown to be approximately equal to $4\pi N\tau$ which is consistent with the dashed curves in Fig. 4. We note that Vitali and Tombesi [7] have examined the damping of a harmonic oscillator interacting with a bath with cutoff $\sim \omega_{eg}$ and have reached similar conclusions. The nature of the integrand in (19) also suggests that if the interval $\tau > \omega_{eg}^{-1}$, then we have the possibility of accelerating spontaneous emission (Fig. 4; curves for $\omega_{eg}\tau = \pi$) which is suggestive of an effect that is an analog of recently discovered quantum anti-Zeno effect [17]. In conclusion, we have shown how a sequence of
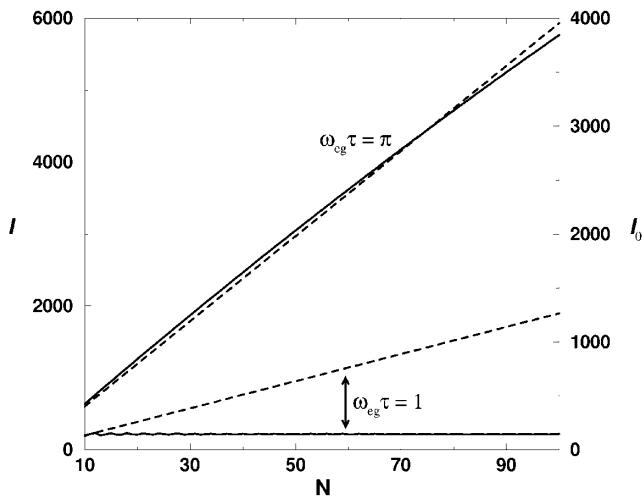
FIG. 4.   A comparison of I (solid lines) [Eq. (19)] and I_o (dashed lines) obtained by dropping tan function in the integrand of Eq. (19) when we introduce a cutoff at $x = 1$. We show results for $\omega_{eg}\tau = 1$ and π.

2π pulses applied on an auxiliary transition in a system can slow down considerably the decay into a continuum [18]. The same scheme also enables one to inhibit unwanted transitions.

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