Measuring the P-Odd Pion-Nucleon Coupling $h_{\pi NN}^{(1)}$ in π^+-Photoproton Production near Threshold

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We show that $\vec{\gamma} p \to \pi^+ n$ in the threshold region is an excellent candidate for measuring the leading parity-violating pion-nucleon coupling $h_{\pi NN}^{(1)}$ to an uncertainty of 20% if it has a natural size from dimensional analysis. The conclusion is based on a large unpolarized cross section, a new low-energy theorem for the photon polarization asymmetry at the threshold $A_\gamma|_{\rm th} = \sqrt{2} f_\pi (\mu_p - \mu_n) h_{\pi NN}^{(1)}/g_A m_N \sim h_{\pi NN}^{(1)}/2$, and its strong dominance at forward and backward angles in the threshold region.

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Parity-violating, or P-odd, hadronic observables provide crucial information about the physics of nonleptonic weak interactions in hadronic structures and reactions. At low-energy, parity-violating hadronic interactions can be systematically classified in the framework of effective field theories [1–3]. At the leading order in chiral power counting, the most important is the isovector P-odd pion-nucleon coupling $h_{\pi NN}^{(1)}$ which is responsible for the longest range part of the parity-violating $\Delta I = 1$ NN forces [1,4,5]. In quantum chromodynamics (QCD), its value is dominated by the s-quark contribution through neutral current interaction [6]. A precise knowledge of $h_{\pi NN}^{(1)}$ not only is critical for understanding the P-odd NN force but will also shed important light on how parity violation takes place in nonleptonic systems.

For many years, serious attempts have been made to measure $h_{\pi NN}^{(1)}$ from parity-violating processes (see [5,7,8] for reviews). In many-body systems, parity-violating effects can be enhanced by strong correlations and have been detected experimentally. However, the theoretical analyses have not yet been fully reliable. The disagreement in the extraction of $h_{\pi NN}^{(1)}$ from ^{18}F [9] and ^{133}Cs [10] systems could be a reflection of poor understanding of many-body physics. In few-body systems, the theory is under better control; but the P-odd effects are generally small. While previous measurements could not reach the required precision [11], new experiments underway are expected to improve significantly. These include $\vec{n} p \to d\gamma$ at Los Alamos Neutron Science Center [12], $\vec{\gamma} d \to np$ at Jefferson Lab (JLab) [13], and the rotation of polarized neutrons in helium at National Institute of Standards and Technology [11]. Finally, in the single nucleon systems, new P-odd observables in Compton scattering on the proton were recently proposed to determine $h_{\pi NN}^{(1)}$ [14]. The process is theoretically "clean," however the experimental feasibility is marginal because of the small total cross section and P-odd asymmetries.

In this paper, we show that the polarized photon asymmetry in $\vec{\gamma} p \to n\pi^+$ at the threshold region is an excellent candidate to measure $h_{\pi NN}^{(1)}$. We derive a low-energy theorem for the asymmetry at the pion-production threshold in the chiral limit: $A_\gamma|_{\rm th} = \sqrt{2} f_\pi (\mu_p - \mu_n) h_{\pi NN}^{(1)}/g_A m_N \sim h_{\pi NN}^{(1)}/2$. A leading-order (LO) calculation in heavy-baryon chiral perturbation theory (HBχPT) shows that the result is modified only mildly by higher partial waves, particularly at forward and backward angles, and chiral corrections from the finite pion mass and momentum in the threshold region up to photon energy $E_\gamma \sim$ 200 MeV. With a total cross section $\sim$100 μb and the expected asymmetry $\sim 2 \times 10^{-7}$, the experiment is feasible at existing laboratories such as JLab. Theoretical studies of the same process have been carried out before by Woloshyn [15] and by Li, Henley, and Hwang [16] in the framework of meson exchange models. In particular, Ref. [16] has already noted the dominance of the $h_{\pi NN}^{(1)}$-type P-odd coupling in the asymmetry near the threshold. The present analysis sharpens the finding by deriving the low-energy theorem and defending its dominance in the threshold region using the modern theoretical tool—HBχPT [2,3].

We are interested in the following two-body process:

$$\vec{\gamma}(q^\mu; \epsilon^\mu) + p(P_i^\mu) \to \pi^+(k^\mu) + n(P_f^\mu), \qquad (1)$$

where $q^\mu = (\omega, \mathbf{q})$, P_i^μ, $k^\mu = (\omega_\pi, \mathbf{k})$, and P_f^μ are the center-of-mass four-momenta of photon, proton, pion, and neutron, respectively, and ϵ^μ is the photon polarization vector. In the threshold region, the pion and photon as well as the nucleon momenta are much smaller than the chiral symmetry breaking scale $\Lambda_\chi \sim 4\pi f_\pi \sim 1$ GeV; therefore chiral perturbation theory (χPT) is a useful tool in making theoretical analyses [3]. When the nucleon is explicitly involved, a natural scheme for systematic power counting is to treat its mass as a heavy scale as Λ_χ, and thus HBχPT [2]. In addition, since the delta resonance is only 300 MeV heavier than the nucleon (order $1/N_c$ in QCD with a large number of N_c colors) and is strongly coupled to the latter through electromagnetic excitations, it is sensible to extend HBχPT to include the resonance as dynamical degrees of freedom and to treat the mass difference $\Delta = m_\Delta - m_N$ as a small parameter [17]. The $SU(2)_L \times U(1)$ symmetry structure of electroweak interactions can be incorporated with the weak boson exchange described by contact interactions and the photon kept as dynamical degrees of freedom.

The unpolarized $\gamma p \to \pi^+ n$ reaction at the threshold represents a classical example of the successes of effective theory ideas. Simply relying on the symmetry properties of the strong interactions, Kroll and Ruderman made a prediction in 1954 on the s-wave scattering length in the chiral limit [18]. Away from this limit, the corrections have been successfully studied using effective field theories. A first analysis of the reaction in χPT was made by Bernard *et al.* [19], who found that the one-loop correction to the tree-order threshold s-wave amplitude (E_{0+}) is insignificant. A more detailed study of partial waves in the framework of HBχPT has recently been made by Fearing *et al.* [20], who found that the p-wave multiples at the threshold are well described by the leading [$\mathcal{O}(p)$] plus next-to-leading [$\mathcal{O}(p^2)$] order calculations. For example, M_{1+}, M_{1-}, and E_{1+} multiples are -4.7, 9.4, and 4.7 in unit $10^{-3}/m_\pi^{3+}$ at $\mathcal{O}(p)$. At order $\mathcal{O}(p^2)$, the results are -7.7, 5.6, and 5.1 which compare favorably with -9.6, 6.1, and 4.9 from a dispersion-theory analysis of experimental data [21].

For the process to be useful in studying nonleptonic parity-violating interactions, the cross section must be large enough to yield a sufficient number of events. Because of the severe phase space suppression at the threshold, we need to establish an *extended* threshold region in which the effective theory description remains effective and, at the same time, the cross section is appreciable. For this purpose, we consider the result of HBχPT at leading order. The parity-conserving T matrix depends on the four amplitudes,

$$T^{PC} = N^\dagger[i\mathcal{A}_1\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon} + i\mathcal{A}_2\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\boldsymbol{\epsilon}\cdot\hat{\mathbf{k}} + i\mathcal{A}_3\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\boldsymbol{\epsilon}\cdot\hat{\mathbf{k}} + \mathcal{A}_4\boldsymbol{\epsilon}\cdot\hat{\mathbf{q}}\times\hat{\mathbf{k}}]N\,, \tag{2}$$

where N is the proton Pauli spinor, $\boldsymbol{\sigma}$ is the Pauli matrix vector, and $\hat{\mathbf{q}}$ and $\hat{\mathbf{k}}$ are the unit vectors in the $\mathbf{q}$ and $\mathbf{k}$ directions. At leading order in HBχPT, $\mathcal{A}_1 = eg_A/\sqrt{2}f_\pi$, $\mathcal{A}_2 = \mathcal{A}_1\omega|\mathbf{k}|/q\cdot k$, $\mathcal{A}_3 = -\mathcal{A}_1\mathbf{k}^2/q\cdot k$, and $\mathcal{A}_4 = 0$ [20]. The resulting differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha_{\rm em}g_A^2}{8\pi f_\pi^2}\frac{m_N^2}{S}\frac{|\mathbf{k}|}{\omega}\mathcal{G}\,, \qquad \mathcal{G} = 1 - \frac{\sin^2\theta|\mathbf{k}|^2}{q\cdot k}\left[1 - \frac{(\mathbf{q}-k)^2}{2q\cdot k}\right], \tag{3}$$

where θ is the angle between $\hat{\mathbf{q}}$ and $\hat{\mathbf{k}}$, and $S = (q + P_i)^2$. A comparison between data [22] and the integrated cross section is shown in the upper graph of Fig. 1 as a function of the photon energy E_γ in the lab frame. The leading-order result describes the data (which have a considerable

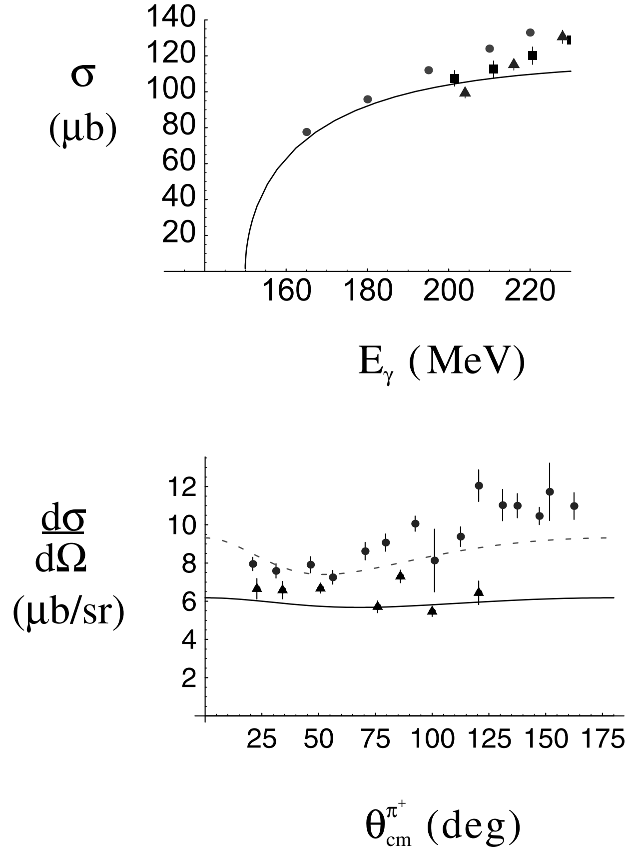

FIG. 1. Upper graph: $\gamma p \to \pi^+ n$ cross section shown as a function of the photon energy in the laboratory frame. The solid curve is the leading-order HBχPT prediction and the data shown are taken from Ref. [22]. Lower graph: the π^+ angular distribution in the center-of-mass frame. The solid and short-dashed curves, and the corresponding data [21,22], triangles, and solid circles are for $E_\gamma = 165$ and 200 MeV, respectively.

variation themselves) within 10% up to $E_\gamma = 200$ MeV. The difference indicates the size of the higher-order corrections expected of HBχPT and the level of convergence of the chiral expansion. According to the figure, we *define* the threshold region in terms of the laboratory photon energy from the threshold to 200 MeV. In the lower graph, we show the angular distributions of the pions in the center-of-mass frame and the data which show the largest deviation from the theory by about 20% at 200 MeV and backward angles.

Now we turn to parity-violating effects in the process. To calculate P-odd observables, we need to extend chiral perturbation theory to include nonleptonic weak interactions. A systematic construction of the P-odd effective chiral Lagrangian has been undertaken in Ref. [1]. To $\mathcal{O}(p^0)$ (we choose to ignore the weak coupling in power counting), it has one term,

$$\mathcal{L}^{PV} = -ih_{\pi NN}^{(1)}\pi^+ p^\dagger n + \text{H.c.} + \dots\,, \tag{4}$$

where the ellipses denote terms with more pion fields and derivatives, and the phase convention is taken from Refs. [23]. By matching onto four-quark interactions,

$h^{(1)}_{\pi NN}$ was found to be dominated by s-quark contributions, $|h^{(1)}_{\pi NN}| \sim G_F F_\pi \Lambda_\chi/\sqrt{2} \sim 5 \times 10^{-7}$ [1]. This estimation is consistent with the "best value" obtained in Ref. [4] and close to a result [24] from QCD sum rules. On the other hand, a recent calculation in the SU(3) Skyrme model yields $h^{(1)}_{\pi NN} \sim (0.8\text{–}1.3) \times 10^{-7}$ [25].

To the next-to-leading order (NLO) [$\mathcal{O}(p)$] in chiral expansion, the relevant Feynman diagrams for the P-odd $\gamma p \to \pi^+ n$ process are shown in Fig. 2. The resulting T matrix can be expressed in terms of two amplitudes,

$$T^{PV} = N^\dagger[i\mathcal{F}_1 \hat{\mathbf{k}} \cdot \boldsymbol{\epsilon} + \mathcal{F}_2 \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \times \hat{\mathbf{q}}]N\,, \quad (5)$$

where

$$\mathcal{F}_1 = -\frac{e h^{(1)}_{\pi NN}|\mathbf{k}|}{q\cdot k}\,, \qquad \mathcal{F}_2 = \frac{e h^{(1)}_{\pi NN}}{2m_N}\left[\mu_p - \left(\frac{\omega}{\omega_\pi}\right)\mu_n\right]. \quad (6)$$

P-odd observables can now be constructed from the interference between T^{PV} and T^{PC}. The leading single-spin asymmetry arises from the interference between $\mathcal{A}_{1-3}$ and $\mathcal{F}_1$, and is dependent on the proton polarization. Because of technical difficulties with a large volume, high-density polarized hydrogen target, an experimental measurement of this asymmetry is not within sight. Therefore, in the following we focus on the photon helicity-flip asymmetry which comes in at NLO from the interferences between $\mathcal{A}_{1-3}$ and $\mathcal{F}_2$ and between $\mathcal{A}_4$ and $\mathcal{F}_1$. $\mathcal{A}_4$ in HBχPT is found nonvanishing at NLO and is

$$\mathcal{A}_4 = \frac{e g_A|\mathbf{k}|}{2\sqrt{2} f_\pi m_N}\left[\mu_p - \left(\frac{\omega}{\omega_\pi}\right)\mu_n\right] - \frac{2 e g_{\pi N\Delta} G_1|\mathbf{k}|}{9\sqrt{2} f_\pi m_N}\left(\frac{\omega}{\omega - \Delta} + \frac{\omega}{\omega_\pi + \Delta}\right), \quad (7)$$

where the delta-resonance contribution has been included explicitly. G_1 is the M1 transition moment between the nucleon and delta, and $g_{\pi N\Delta}$ is the π-N-Δ coupling.

More explicitly, the photon helicity asymmetry $A_\gamma(\omega,\theta) = [d\sigma(\lambda_\gamma = +1) - d\sigma(\lambda_\gamma = -1)]/[d\sigma(\lambda_\gamma = +1) + d\sigma(\lambda_\gamma = -1)]$ at the leading order in HBχPT is

$$A_\gamma(\omega,\theta) = \frac{\sqrt{2}\, h^{(1)}_{\pi NN} f_\pi}{g_A m_N \mathcal{G}}\left\{\left[\mu_p - \left(\frac{\omega}{\omega_\pi}\right)\mu_n\right]\left(1 - \frac{\sin^2\theta \mathbf{k}^2}{q\cdot k}\right) + \frac{2}{9}\frac{g_{\pi N\Delta} G_1 \sin^2\theta \mathbf{k}^2}{g_A q\cdot k}\left(\frac{\omega}{\omega - \Delta} + \frac{\omega}{\omega_\pi + \Delta}\right)\right\}, \quad (8)$$

where $\mathcal{G}$ is given in Eq. (3). Although the result formally depends on the NLO amplitude $\mathcal{A}_4$, it is dominated in the threshold region by the "beat" between the parity-violating amplitude $\mathcal{F}_2$ and the leading-order parity-conserving amplitudes $\mathcal{A}_{1,2,3}$ which have already been tested in Fig. 1. Right at the threshold $|\mathbf{k}| = 0$, only the s-wave $\pi^+ n$ final-state contributes; we find the equivalent of the Kroll-Ruderman theorem for the P-odd photon-helicity asymmetry,

$$A_\gamma(\omega_{\rm th},\theta) = \frac{\sqrt{2} f_\pi(\mu_p - \mu_n)}{g_A m_N}\, h^{(1)}_{\pi NN}\,, \quad (9)$$

which depends only on the chiral symmetry. Plugging in the known physical quantities, the coefficient of $h^{(1)}_{\pi NN}$ is 0.52. So the asymmetry has the same size as $h^{(1)}_{\pi NN}$ and of order 10^{-7}. In Fig. 3, we show the angular dependence of the leading-order A_γ at $E_\gamma = 180, 200$ MeV (corresponding to the center-of-mass energy $\omega = 138, 168$ MeV), together with the low-energy theorem. At the forward and backward angles, we see hardly any deviation from the threshold result. Only near $\theta = 90°$ at $E_\gamma = 200$ MeV does the modification from high partial waves become significant (less than 40%).

Will the above result be changed significantly when going to higher orders in HBχPT? A complete answer to the question requires a systematic study of the contribution at the next order which we will communicate in a separate publication [28]. Here we just present a few qualitative arguments why it is unlikely that the higher-order corrections ruin the leading-order relation between A_γ and $h^{(1)}_{\pi NN}$ in the threshold region. Because the parity-conserving amplitudes are dominated by the leading order, we know at least one class of corrections—the interference between the next-to-next-to-leading order (NNLO) T^{PC} with LO T^{PV}—is small. The second class of corrections is an interference between NLO T^{PC} and NLO T^{PV}. No loop calculations are involved here and all couplings except $h^{(1)}_{\pi NN}$ are

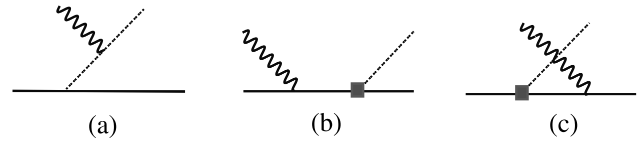

FIG. 2. Feynman diagrams contributing to the parity-violating amplitudes at LO [$\mathcal{O}(1)$] and NLO [$\mathcal{O}(p)$] in $\vec{\gamma} p \to \pi^+ n$.

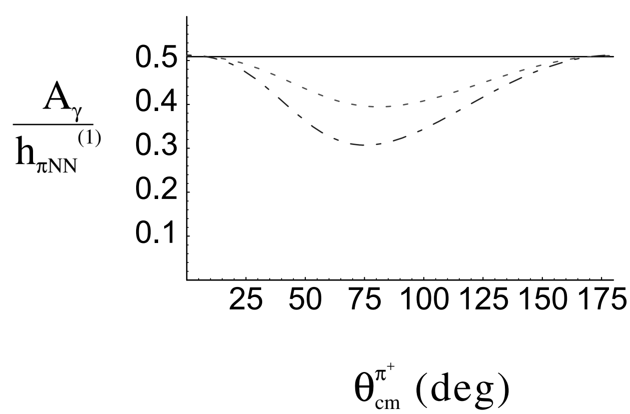

FIG. 3. The photon-helicity asymmetry A_γ in unit $h^{(1)}_{\pi NN}$. The solid line is the low-energy theorem in Eq. (9), and the short-dashed and dash-dotted lines are for $E\gamma = 180$ and 200 MeV, respectively.

known. The size of the correction will follow the canonical power counting, i.e., of order $\mathcal{O}(\epsilon/m_N)$, where ϵ stands for m_π, ω, ω_π, and Δ. The last class involves an interference between LO T^{PC} and NNLO T^{PV} amplitudes; the latter contains one-loop integrals as well as tree contributions from new P-odd effective couplings. The following is an example of P-odd interactions at NNLO,

$$\mathcal{L}^{PV} = \frac{eh_{\gamma\pi NN}}{m_N^2}\overline{p}[S^\mu, S^\nu]\pi^+ nF_{\mu\nu} + i\,\frac{e\tilde{G}}{m_N}\overline{\Delta^{+}}^{\mu} v^\nu F_{\nu\mu}p\,. \tag{10}$$

While the one-loop integrals are not expected to yield large corrections, the magnitude of the new couplings is unknown. Since an unnatural size of couplings in effective theory usually arises from new physics, we do not expect this to happen here from our experience with the corresponding parity-conserving amplitudes. This of course can be tested by the θ dependence of the asymmetry. In short, we expect the higher-order corrections to Eq. (8) is $\mathcal{O}(\epsilon/m_N)$, namely, about 20%. Recently, Zhu *et al.* have published a NLO calculation, questioning the validity of this [26]. Comments about their paper can be found in [27].

Finally, we briefly comment on the experimental feasibility for measuring the polarization asymmetry in $\vec{\gamma}p \to \pi^+ n$. To overcome statistics, a large number of events ($\sim 10^{14}$) are needed. This requires a luminosity of order $10^{37}/(\text{cm}^2\text{ sec})$ which is reasonable with the current technology and facilities such as JLab. With a total cross section $\sim 100\ \mu b = 10^{-28}\text{ cm}^2$, the π^+ production rate is $10^8/\text{sec}\cdot\text{rad}$. Thus $\sim 10^6$ sec of beam time will yield the required number of events. The challenge, however, could be $10^8\pi^+/\text{sec}$ detection.

In conclusion, we have shown that parity-violating $\vec{\gamma}p \to \pi^+ n$ is a theoretically clean and experimentally feasible process to measure $h^{(1)}_{\pi NN}$. Near the threshold region, the size of the photon helicity asymmetry is estimated to be $\sim 2\times 10^{-7}$ for an expected magnitude of $h^{(1)}_{\pi NN}$. Assuming a luminosity of $10^{37}/(\text{cm}^2\text{ sec})$, $h^{(1)}_{\pi NN}$ can be measured to an accuracy of 10^{-7} in a few months of running. Similar results for pion electroproduction will be published separately [28].

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