TeV Strings and the Neutrino-Nucleon Cross Section at Ultrahigh Energies

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In scenarios with the fundamental unification scale at the TeV one expects string excitations of the standard model fields at accessible energies. We study the neutrino-nucleon cross section in these models. We show that duality of the scattering amplitude forces the existence of a tower of massive leptoquarks that mediate the process in the s channel. Using the narrow-width approximation we find a sum rule for the production rate of resonances with different spin at each mass level. We show that these contributions can increase substantially the standard model neutrino-nucleon cross section, although they seem insufficient to explain the cosmic ray events above the Greisen-Zatsepin-Kuz'min cutoff energy.

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Introduction.—Extensions of the standard model (SM) with extra dimensions offer new ways to accommodate the hierarchies observed in particle physics [1]. A very attractive possibility would be to bring the scale of unification with gravity from $M_{\text{Planck}} \approx 10^{19}$ GeV down to the electroweak scale $M_{\text{EW}} \approx 1$ TeV. This could result if gravity propagates along a $(4 + n)$-dimensional $[(4 + n)\text{D}]$ flat space with n compact submillimeter dimensions [2] or along a $(4 + 1)$D slice of anti–de Sitter space with a warp factor in the metric [3]. These higher dimensional field theories, however, must be considered effective low-energy limits valid only below the mass scale of a more fundamental theory. And nowadays, only string theory [4] provides a consistent framework for the unification of gravity with the standard model.

At energies where the effects of a higher dimensional graviton are unsuppressed one expects the presence of its string Regge (SR) excitations giving an effect of the same size. This will be necessary in order to avoid the pathologies of spin-2 field theories. In string theory the massless graviton comes as the zero mode of a closed string, whereas the gauge bosons are the lightest modes of an open string. Now, as emphasized in [5,6], the exchange amplitude of a closed string has an order g^2 suppression versus the exchange of an open string. In consequence, processes that receive sizable contributions from SR and Kaluza-Klein excitations of the graviton and also from SR excitations of the gauge bosons will be dominated by the second ones. This seems to be a generic feature in models of higher dimensional gravity embedded in a weakly coupled string theory.

In this Letter we explore some phenomenological consequences of the theories with unification at the TeV. In particular, we focus on the neutrino-nucleon cross section. Our interest is based on the possibility that new neutrino physics could explain the cosmic ray events above the GZK cutoff energy (see [7] and references therein). We discuss a genuine string effect, the presence of leptoquarks that mediate the process in the s and/or the u channel. This fact is a generic consequence required by the duality of the scattering amplitudes. The leptoquarks appear at the massive SR level even in string models where the only massless modes are the SM fields (string models with the SM gauge symmetry). The impact of leptoquarks on the neutrino-nucleon cross section at ultrahigh energies was first studied in [8], whereas in [9] they are proposed in a framework of strongly interacting neutrinos.

The $\nu_L u_L \to \nu_L u_L$ string amplitude.—Cullen, Perelstein, and Peskin build in Ref. [6] a TeV-string model for QED. It contains electrons and photons at low energies and massive SR excitations above the string scale. These excitations give corrections to QED processes that can be easily calculated. In this section we generalize their results in order to obtain the $\nu_L u_L \to \nu_L u_L$ string amplitude.

The model results from a simple embedding of the SM interactions into type IIB string theory. It is assumed that the 10D space of the theory has six dimensions compactified on a torus with common periodicity $2\pi R$ (the case with six extra dimensions tends to alleviate the graviphoton problem [10]) and that N coincident D3 branes (4D hypersurfaces where open strings may end) are stretched out in the four extended dimensions. We also assume that the extra symmetry of the massless string modes can be eliminated by an appropriate orbifold projection, resulting in an acceptable model with (at least) the SM fields. The parameters of this theory would be the string scale $M_S = \alpha'^{-1/2} \approx 1$ TeV and the dimensionless gauge coupling contant g, unified at M_S. Proposals for splitting these couplings can be found in [11]. For more general D-brane models, see [12] and references therein.

A tree-level amplitude of open string states on a D brane is given [6,13] as a sum of ordered amplitudes multiplied by Chan-Paton traces. For the process under study we have

$$\mathcal{A}(1,2,3,4) = g^2 \cdot S(s,t) \cdot F^{1243}(s,t,u) \cdot \text{tr}[t^1t^2t^4t^3 + t^3t^4t^2t^1] + g^2 \cdot S(s,u) \cdot F^{1234}(s,u,t) \cdot \text{tr}[t^1t^2t^3t^4 + t^4t^3t^2t^1] + g^2 \cdot S(t,u) \cdot F^{1324}(t,u,s) \cdot \text{tr}[t^1t^3t^2t^4 + t^4t^2t^3t^1]. \tag{1}$$

In this expression,

$$S(s,t) = \frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)} \quad (2)$$

is basically the Veneziano amplitude [14], $(1,2,3,4)$ label $(\nu_L^{\text{in}}, u_L^{\text{in}}, \nu_L^{\text{out}}, u_L^{\text{out}})$, the Chan-Paton factors t^a are representation matrices of $U(N)$, and $F^{abcd}(s,t,u)$ is a factor depending on the vertex operators for the external states and their ordering. In our case all the vertex operators will correspond to (massless) Weyl spinors of helicity (directed inward) $+$ or $-$, giving

$$F^{-++-}(s,t,u) = -4\frac{t}{s};$$

$$F^{-+-+}(s,t,u) = -4\frac{u^2}{st} = 4\left(\frac{u}{s} + \frac{u}{t}\right); \quad (3)$$

$$F^{--++}(s,t,u) = -4\,\frac{s}{t}.$$

We obtain

$$\mathcal{A}(\nu_L u_L \to \nu_L u_L) = -4g^2\left[\frac{s}{t}\,S(s,t)T_{1243} + \frac{s}{u}\,S(s,u)T_{1234} + \frac{s^2}{tu}\,S(t,u)T_{1324}\right], \quad (4)$$

with T_{abcd} the Chan-Paton traces.

To understand the phenomenological consequences of this amplitude let us start with the limit $s, t \to 0$. Since $\Gamma(1) = 1$, we have all the Veneziano factors $S(0,0) = 1$. The amplitude expresses then the exchange of massless vector modes in the t and the u channels. The former would correspond to the Z gauge boson, whereas the field exchanged in the u channel is in the $(\bar{\mathbf{3}},\mathbf{1})$ and/or the $(\bar{\mathbf{3}},\mathbf{3})$ representations of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L$ and has electric charge $Q = -2/3$. The $\mathrm{SU}(2)_L$ singlet can be found in the **10** of SU(5) or the adjoint **45** of SO(10), whereas the triplet is, for example, in the **35** of SU(5). We are interested, however, in models that reproduce the SM result at low energies, with no massless leptoquarks. We obtain this limit if the Chan-Paton factors assigned to u_L and ν_L are such that $T_{1243} - T_{1324} = -\frac{1}{10}$ and $T_{1234} = T_{1324}$, where we have used $\sin^2\theta_W = 3/8$. In terms of $T_{1234} = -a/10$ the amplitude becomes

$$\mathcal{A}(\nu_L u_L \to \nu_L u_L) = \frac{2}{5}\,g^2\left[\frac{s}{t}\,[(1+a)S(s,t) - aS(t,u)] + \frac{s}{u}\,[aS(s,u) - aS(t,u)]\right]. \quad (5)$$

At low s this amplitude is $\mathcal{A}_0 \approx (2/5)g^2 s/t$ and corresponds to the exchange of a Z boson in the t channel. The Z is then a massless SR mode that acquires its mass M_Z only through the Higgs mechanism. We neglect the corrections of order M_Z^2/M_S^2 that may affect the massive SR modes.

As the energy increases the Veneziano factor $S(s,t)$ gives a series of poles (at $1 - \alpha' s = 0, -1, -2, \ldots$) and zeros (at $1 - \alpha' s - \alpha' t = 0, -1, -2, \ldots$). It can be expressed as

$$S(s,t) = \sum_{n=1}^{\infty} \frac{\alpha' t + \alpha' s - 1}{\alpha' t + n - 1}\,\frac{\prod_{k=0}^{n-1}(\alpha' t + k)}{(\alpha' s - n)(n-1)!}. \quad (6)$$

At $s = nM_S^2$ the amplitude will describe the exchange of a collection of resonances with the same mass and different spin (see below). Away from the poles the interference of resonances at different mass levels will produce the usual soft (Regge) behavior of the string in the ultraviolet. Obviously, these resonances are not stable and at one loop will get an imaginary part in their propagator. When the total width of a resonance (which grows with its mass) is similar to the mass difference with the resonance in the next level one cannot see resonances and interference effects dominate also at $s = nM_S^2$.

Let us first analyze the case with $a = 0$ in Eq. (5). The amplitude is just $\mathcal{A}(\nu_L u_L \to \nu_L u_L) = (2/5)g^2(s/t) \cdot S(s,t)$. Near the pole at $s = nM_S^2$ it is

$$\mathcal{A}_n \approx \frac{2}{5}\,g^2\frac{nM_S^4}{t} \times \frac{(t/M_S^2)(t/M_S^2+1)\cdots(t/M_S^2+n-1)}{(n-1)!\,(s-nM_S^2)}. \quad (7)$$

This amplitude corresponds to the s-channel exchange of massive leptoquarks in the $(\mathbf{3},\mathbf{3})$ representation of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L$ with electric charge $Q = 2/3$. At each pole we have contributions of resonances with a common mass $\sqrt{n}\,M_S$ but different spin, going from zero to the order of the residue $P_n(t) = \mathcal{A}_n \cdot (s - nM_S^2)$. In this case the maximum spin at the n level is $J = n - 1$. To separate these contributions we first write the residue in terms of the scattering angle θ, with $t = -(nM_S^2/2)(1-\cos\theta)$. Then we express $P_n(\theta)$ as a linear combination of the d functions (rotation matrix elements):

$$P_n(\theta) = \frac{2}{5}\,g^2 nM_S^2 \sum_{J=0}^{n-1} \alpha_n^J d_{0,0}^J(\theta). \quad (8)$$

The coefficient α_n^J gives the contribution to our amplitude of a leptoquark X_n^J of mass squared nM_S^2 and spin J. For example, at the first SR level we find a scalar resonance with $\alpha_1^0 = 1$, at $s = 2M_S^2$ there is a single vector resonance with $\alpha_2^1 = 1$, whereas at $s = 3M_S^2$ there are modes of spin $J = 2$ ($\alpha_3^2 = 3/4$) and $J = 0$ ($\alpha_3^0 = 1/4$).

The general case with $a \neq 0$ is completely analogous, with resonant contributions from the terms proportional to $S(s,t)$ and $S(s,u)$. Taking $u = -(nM_S^2/2)(1+\cos\theta)$ and expressing again the residue in terms of d functions we find the same type of resonances but with different α_n^J coefficients: $\alpha_1^0 = 1 + 2a$, $\alpha_2^1 = 1$, $\alpha_3^2 = 3(1+2a)/4$, and $\alpha_3^0 = 1(1+2a)/4$.

The νN total cross section.—From the resonant amplitude $\nu_L u_L \to X_n^J \to \nu_L u_L$ we can now obtain the partial width $\Gamma_n^J \equiv \Gamma(X_n^J \to \nu_L u_L)$:

$$\Gamma_n^J = \frac{g^2}{40\pi} \frac{\sqrt{n}\, M_S |\alpha_n^J|}{2J + 1} \,. \tag{9}$$

Notice that for a given spin J, the variation with n of α_n^J gives the *running* of the coupling with the energy. We obtain numerically that the coupling of heavier resonances decreases like the power law $\alpha_n^J \approx 1/n$.

The partial width Γ_n^J can be used to obtain the cross section $\sigma_n^J(\nu_L u_L) \equiv \sigma(\nu_L u_L \to X_n^J)$ in the narrow-width approximation:

$$\sigma_n^J(\nu_L u_L) = \frac{4\pi^2 \Gamma_n^J}{\sqrt{n}\, M_S} (2J + 1) \delta(s - nM_S^2) \,. \tag{10}$$

At each mass level n there is a tower of resonances of integer spin J from 0 to $n - 1$. We find a sum rule for the production rate $\sigma_n(\nu_L u_L) \equiv \sum_J \sigma_n^J(\nu_L u_L)$ of any of these resonances:

$$\sigma_n(\nu_L u_L) = \begin{cases} \dfrac{2}{5} \dfrac{\pi g^2}{4} (1 + 2a) \delta(s - nM_S^2) & \text{for } n \text{ odd,} \\ \dfrac{2}{5} \dfrac{\pi g^2}{4} \delta(s - nM_S^2) & \text{for } n \text{ even.} \end{cases} \tag{11}$$

This is equivalent (for $a = 0$) to the production rate of a single resonance of mass $\sqrt{n}\, M_S$ and coupling $(2/5)g^2$ [8]. In our opinion this is a very interesting result. The coupling of heavier SR modes decreases quadratically with the energy, but the number of modes (and the highest spin) at each mass level n grows also quadratically making $\sum_J \alpha_n^J$ a constant independent of n.

In the narrow-width approximation $\sigma(\nu_L u_L) \equiv \sum_{n,J} \sigma(\nu_L u_L \to X_n^J \to \text{anything})$ is then $\sigma(\nu_L u_L) = \sum_n \sigma_n(\nu_L u_L)$. In this limit the cross section is proportional to a collection of delta functions and thus all interference effects are ignored. This is a good approximation as far as the total width of a resonance is smaller than the mass difference with the next resonance of same spin. Although the coupling (and any partial width) decreases with the mass, the total width of heavier resonances will grow due to the larger number of decay modes that are kinematically allowed. We estimate that contributions to $\sigma(\nu_L u_L)$ from modes beyond $n \cdot (g^2/4\pi) \approx 1$ are a continuum. In this regime any cross section goes to zero exponentially at fixed angle (s large, t/s fixed) and like a power law at small angles (s large, t fixed) [4]. We neglect these contributions. We keep only resonant contributions from levels $n < n_{\text{cut}} = 50$ and find that our result depends very mildly on the actual value of n_{cut}.

To evaluate the total νN cross section we also need the elastic amplitudes $\nu_L d_L$, $\nu_L u_R$, $\nu_L d_R$, and $\nu_L \overline{q}_{L(R)}$. $\mathcal{A}(\nu_L d_L \to \nu_L d_L)$ takes the same form as the amplitude in Eq. (5) with the changes $(2/5, a) \to (-3/5, a')$. The massive resonances exchanged in the s channel are now an admixture of an $SU(2)_L$ singlet and a triplet. The singlet contribution is required in n-even levels; otherwise an $SU(2)_L$ gauge transformation would relate the parameters α_n^J obtained here with the ones deduced from Eq. (5). In n-odd mass levels gauge invariance could be obtained with no singlets for $a' = -(2 + a)/3$. The cross section $\sigma_n(\nu_L d_L)$ can be read from Eq. (11) just by changing $(2/5, a) \to (3/5, a')$.

The calculation of amplitudes and cross sections for $\nu_L \overset{(-)}{q}_R$ are completely analogous. We obtain

$$\sigma_n(\nu_L u_R) = \begin{cases} \dfrac{2}{5} \dfrac{\pi g^2}{2} b\, \delta(s - nM_S^2) & \text{for } n \text{ odd,} \\ 0 & \text{for } n \text{ even.} \end{cases} \tag{12}$$

The cross sections $\sigma_n(\nu_L d_R)$, $\sigma_n(\nu_L \overline{u}_R)$, and $\sigma_n(\nu_L \overline{d}_R)$ coincide with the expression in Eq. (12) with the changes $(2/5, b) \to (1/5, b')$, $(2/5, b) \to (2/5, a)$, and $(2/5, b) \to (3/5, a')$, respectively. For the left-handed antiquarks, $\sigma_n(\nu_L \overline{u}_L)$ and $\sigma_n(\nu_L \overline{d}_L)$ can be read from Eq. (11) by changing $(2/5, a) \to (2/5, b)$ and $(2/5, a) \to (1/5, b')$, respectively.

Now the total neutrino-nucleon cross section due to the exchange of SR excitations can be very easily evaluated. In terms of parton distribution functions $q(x, Q)$ ($q = q_{L,R}, \overline{q}_{L,R}$) in a nucleon $[N \equiv (n + p)/2]$ and the fraction of longitudinal momentum x, it is

$$\sigma(\nu_L N) = \sum_{n=1}^{n_{\text{cut}}} \sum_q \frac{\tilde{\sigma}_n(\nu_L q)}{nM_S^2} xq(x, Q) \,, \tag{13}$$

where $x = nM_S^2/s$, $Q^2 = nM_S^2$, and $\tilde{\sigma}_n(\nu_L q)$ is the factor multiplying the delta function in the cross section $\sigma_n(\nu_L q)$.

In Fig. 1 we plot the neutrino-nucleon cross section at energies from 10^2 to 10^{13} GeV for $M_S = 0.5, 2$ TeV. We have used the CTEQ5 parton distributions in the DIS scheme [15] extended to $x < 10^{-5}$ with the methods in [16]. We include the SM cross section and plot the string corrections for $a = a' = b = b'$ equal 0 and 5 (notice that in the first case there are no s-channel resonances mediating the $\nu_L q_R$ amplitude). We do not include the modes beyond $n_{\text{cut}} = 50$, where we expect that the narrow width approximation is poor. We also find that around 80% of the effect comes from the ten first SR modes.

Conclusions.—Cosmic rays hit the nucleons in the atmosphere with energies of up to 10^{12} GeV. If the string scale is in the TeV range, these cosmic rays have the energy required to explore the fundamental theory and its interactions. In particular, ultrahigh energy neutrinos are

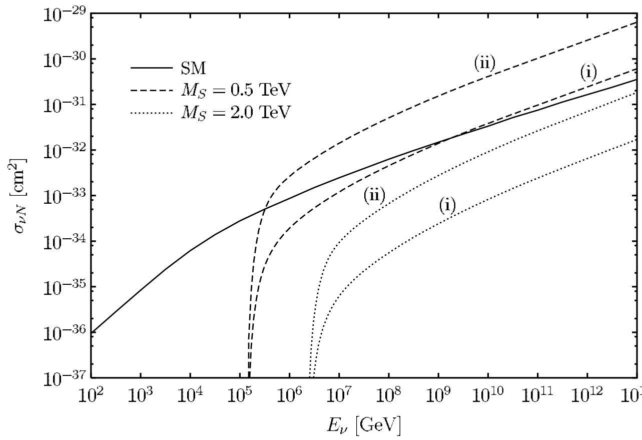

FIG. 1. Neutrino-nucleon cross section versus the incident neutrino energy E_ν. The SM contribution (solid) includes neutral and charged current interactions. We plot the SR contribution for $M_S = 0.5$ TeV (dashes) and $M_S = 2$ TeV (dots) for the cases (i) $a = a' = b = b' = 0$ and (ii) $a = a' = b = b' = 5$.

interesting since they can travel long distances without losing a significant fraction of energy. In addition, the SM interactions of a neutrino are much weaker than those of a quark or a charged lepton, which makes it easier to see deviations due to new physics.

With this motivation we have analyzed the string νN cross section at energies much larger than $M_S \approx 1$ TeV. We fix the arbitrary parameters of the model (four Chan-Paton traces) imposing phenomenological constraints, namely, the massless SR modes must account for the electroweak gauge bosons only. Then we find that the massive SR modes include leptoquarks that mediate the process in the s channel. The presence of massive leptoquarks is not a peculiarity of our toy model but a generic feature of any string model. The argument goes as follows. If an amplitude is mediated by a massless field in the t channel, there will be its higher-spin SR excitations mediating the process also in the t channel. However, the only known way to make sense of an amplitude mediated by (elementary) higher spin fields is *à la* Veneziano (open string) [14] or *à la* Virasoro (closed string) [17]. In the first case the amplitude has (s, t) and/or (t, u) duality [see, e.g., Eq. (4)], whereas in the second case the amplitude has (s, t, u) duality. Consequently, any amplitude with t-channel poles will also have s- and/or u-channel poles. Note that the u-channel poles of $\mathcal{A}(\nu_L u_L)$ become s-channel poles of $\mathcal{A}(\nu_L \overline{u}_R)$.

We have found a very simple sum rule for the production rate of all the leptoquarks, with spin from 0 to $n \pm 1$, in the same mass level n. This made possible the calculation of the total νN cross section in the narrow-width approximation. We obtain that the effect of these leptoquarks is not just a correction of order M_Z^2/M_S^2 to the SM cross section, as one would expect on dimensional grounds. SR excitations give a contribution that can dominate for $M_S \approx 1$ TeV. This deviation (see Fig. 1) could explain the observation of horizontal air showers in upcoming cosmic ray experiments [18]. However, for the expected flux of ultrahigh energy neutrinos it seems unlikely that the cosmic ray events observed above the GZK limit correspond to the decay of resonances produced in νN scattering. If the primordial particle is a neutrino, the most promising possibility would be $\nu\overline{\nu}$ scattering in the galactic halo with the resonant production of a Z boson [19] or other massive field [20] decaying into hadrons.

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