Scale Dependent Dimension of Luminous Matter in the Universe

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We suggest a geometrical model for the distribution of luminous matter in the Universe, where the apparent dimension, $D(l)$, increases linearly with the logarithm of the scale l. Beyond the correlation length, ξ, the Universe is homogeneous, and $D = 3$. Comparison with data from the SARS redshift catalog, and the LEDA database provides a good fit with a correlation length $\xi \sim 300$ Mpc. This type of scaling structure was recently discovered in a simple reaction-diffusion "forest-fire" model, indicating a broad class of scaling phenomena.

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Uniformity of the background radiation requires that the Universe must be homogeneous at the largest scale; this is known as the cosmological principle. However, a decade ago, Coleman and Pietronero [1] suggested that the Universe, at length scales L up to a couple of Mpc, is fractal with fractal dimension, $D \sim 1.2$, based on a study of the CfA galaxy catalog. Subsequent studies seemed to confirm this picture: Guzzo *et al.* [2] found $D = 1.2$ for $L = 1-3$ Mpc, increasing to $D = 2.2$ for $L = 3-10$ Mpc, from the Perseus-Pisces catalog. Martinez and Coles [3] found that the dimension gradually increases from 2.25 to 2.77 at length scales increasing from 1–50 Mpc. These empirical studies have recently been reviewed by Wu *et al.* [4]. Even though there is general agreement about the existence of fractal galactic structures at moderate scales, there is still intense debate on whether or not the Universe is homogeneous at very large scales. If the Universe indeed becomes homogeneous, the question arises as to how the transition takes place [5,6]. The value of the homogeneity length scale and the matter distribution for smaller scales have important cosmological consequences.

We propose that the distribution of luminous matter in the Universe can be described by a new geometric scaling form that we discovered recently [7] in a different context. This description leads to a reconciliation of observational data at various scales. A sharp transition to homogeneity at 300 Mpc is predicted.

We studied a simple nonequilibrium reaction-diffusion "forest-fire" model [8,9], proposed to capture the essential features of turbulent systems, where energy is injected at the largest scale and dissipated at a small length scale. In a range of length scales between these two limits the dimension of the luminous field (fire distribution) varies gradually from zero to three. The distribution becomes homogeneous beyond a correlation length which depends on the energy injection rate. The model operates near a dynamical "critical point," with diverging correlation length. As we will show below, analysis of galaxy maps indicates that the geometrical structure of luminous matter in the Universe is very similar to that of the forest-fire model. We compare the actual distribution of luminous matter with the statistical properties of snapshots in the steady state of the forest-fire model.

While we hesitate to claim that the Universe should be viewed as one giant forest fire, we do suggest that the dimension dependent scaling picture may be a quite generic and robust geometrical form for dissipative turbulent systems. The underlying picture could be one where the galactic dynamics is turbulent, with stellar objects interacting with one another in reaction-diffusion-type processes through shock waves, supernovae explosions, galaxy mergers, etc. Moreover, it has been suggested that such reaction-diffusion processes may indeed be responsible for structure formation in the Universe, at least at smaller scales [10]. In any case, the similarity is appealing in that it suggests that the luminous matter in the Universe shares the basic characteristic features of other dynamical systems.

Usually, systems near criticality are self-similar, or fractal, for length scales below the correlation length; hence fractal behavior can often be viewed as a consequence of criticality [11]. However, the forest-fire model does not show self-similar scaling below the correlation length. Numerical studies show that the average amount of dissipation $n(l)$, within a cube box of size l that contains dissipation, obeys

$$n(l) = c\left(\frac{l}{l_0}\right)^{1.5\frac{\log(l/l_0)}{\log(\xi/l_0)}}, \tag{1}$$

where $l_0 \sim 1$ is the lattice spacing for the forest-fire model and c is a constant. The formula is valid for lengths smaller than the correlation length ξ, where there is a sharp crossover to a homogeneous 3D structure.

The equation can be interpreted in terms of an apparent fractal dimension that varies linearly with the logarithm of the length scale:

$$D(l) = \frac{d\log(n)}{d\log(l)} \sim 3\left(\frac{\log(l/l_0)}{\log(\xi/l_0)}\right). \tag{2}$$

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The length scale dependent behavior observed in this model may be sufficiently general that it is worthwhile to make a detailed comparison with astronomical data. Apart from an overall amplitude, there are only two fitting parameters in our proposed galaxy distribution, both have clear geometrical interpretation: beyond the upper length scale ξ, the distribution becomes uniform; at the lower cutoff, l_0, the distribution becomes pointlike.

In their seminal work, Sylos Labini *et al.* [12] analyzed several database catalogs of galaxy maps. From these databases, they created volume-limited samples containing all galaxies exceeding a certain absolute luminosity within a given volume. Then they calculated the conditional density $\Gamma^*(l)$, which is the average density of galaxies within a sphere of size l. This quantity corresponds to the density $n(l)$ defined above, divided by the volume l^3. Thus, the resulting prediction for $\Gamma^*(l)$ becomes

$$\log[\Gamma^*(l)] \sim \left[\frac{3}{2}\left(\frac{\log(l/l_0)}{\log(\xi/l_0)}\right) - 3\right]\log(l/l_0). \quad (3)$$

Namely, on a log-log plot there is a quadratic dependence on $\log(l/l_0)$ rather than the linear dependence found for self-similar fractal structures.

We have fitted Eq. (3) to the conditional densities extracted by Pietronero *et al.* from two widely different databases, with consistent results. The LEDA database is a heterogeneous compilation of data from the literature containing more than 200 000 galaxies. The Stromlo-APM redshift survey (SARS [13]) consists of 1797 galaxies. Figure 1 shows results from the fits, with two different cutoffs for the LEDA database. The labeling follows Sylos Labini *et al.,* with the numbers representing the lower luminosity cutoffs. Obviously, there are larger fluctuations for the sparser, but perhaps higher quality, Stromlo-APM data set than for the LEDA database.

The fits are very good in view of the fact that the only fitting parameters are the upper and lower length scales, ξ and l_0, respectively. In contrast to conventional self-similar critical phenomena, the correlation length enters the expression for length scales below the correlation length. We are therefore able to measure the correlation length from data taken from our local corner of the Universe, despite the fact that no data are available, as yet, at and beyond the projected correlation length.

The upper length scale is the one where the curves become flat, corresponding to the apparent dimension $D = 3$. The three fits yield very consistent values of this length scale, $\xi = 271 \pm 32$ Mpc from the LEDA16 data, $\xi = 255 \pm 80$ Mpc from the LEDA14 data, and $\xi = 389 \pm 220$ Mpc from the APM 18 data. The errors in the logarithm of the data points, $\sqrt{\chi^2}$, are 0.037, 0.046, and 0.145 for the three data sets, respectively. The correlation length is much smaller than the Hubble radius, so we can essentially view the observed density as recorded at an instant in time, making the equal time correlation function $n(l)$ for the forest-fire model the proper statistical quantity to compare with.

The logarithmic scale dependence of the dimension can be seen directly by replotting the data in Fig. 1. Figure 2 shows the scale dependence of the dimension $D(l) = 2[\log\Gamma^*(l)/\Gamma^*(l_0)]/\log(l/l_0) + 3$. All data sets yield linear behavior. The correlation length is found by linear extrapolation to the point where $D(l)$ assumes the value of 3. The dimensions derived from the intense galaxies, LEDA16 and APM 18, are essentially identical, but the LEDA14 data yield a somewhat steeper scale dependence. However, they all converge at almost the same

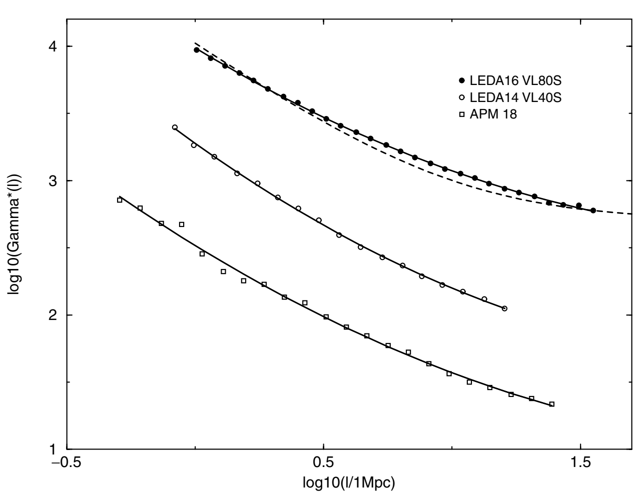

FIG. 1. Conditional average densities for various galaxy catalogs (arbitrary scale), as derived by Sylos Labini *et al.* [12], compared with fits to Eq. (1), yielding $\xi = 271$ Mpc from the LEDA16 data, $\xi = 255$ Mpc from the LEDA14 data, and $\xi = 389$ Mpc from the APM data. The broken line is a conventional fit to Eq. (4) with $\gamma = 1.3$, $r_0 = 10$ Mpc.

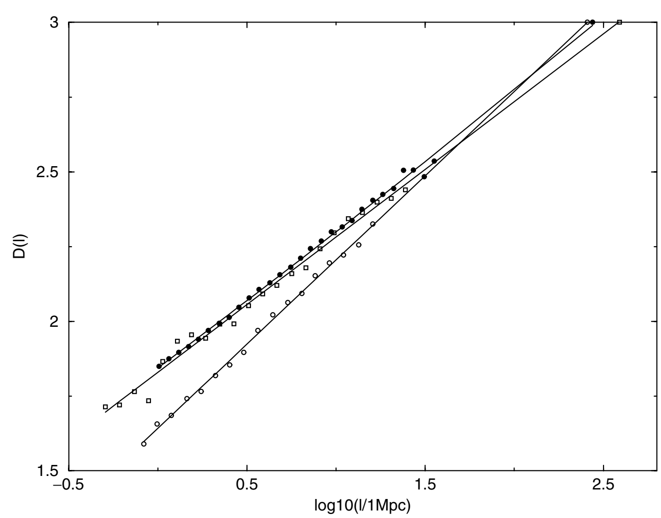

FIG. 2. Scale dependent dimension $D(l)$ derived from the data points in Fig. 1 as explained in text. We conjecture that future data points will follow the straight lines and saturate sharply to $D = 3$ at the correlation length.

homogeneity length. Figure 2 clearly shows that the distribution is not fractal, since this would imply a constant $D(l)$ over a range of length scales l.

We predict a sharp crossover to uniformity, i.e., a sharp kink in the curve, at the correlation length, which will be observable once data become available, presumably within the next decade. Actually, there is a recent analysis based on the ESO Slice Project galaxy redshift survey which indicates that the fractal dimension is close to 3 for the length scale greater than 300 Mpc [14]. Also, the intermediate data points are predicted to follow the straight lines in Fig. 2.

The lower cutoff, l_0, is the scale at which the slope of the extrapolated curves in Fig. 1 assumes the value of -3. We find $l_0 = 400$ light years, $l_0 = 4000$ light years, and $l_0 = 400$ light years for the three samples, respectively. This is comparable with the smallest intergalactic distances. This scale is determined with less precision than the correlation length ξ. It is not clear how well (if at all) our scaling form applies to the analysis of the galaxy distribution at small length scales.

The geometry of the luminous set is *not fractal* when viewed over the entire range of scales, since there is no self-similarity for different scales. It is not homogeneous either. The scale dependent dimension has a clear geometrical interpretation: At small distances, the Universe is zero dimensional and pointlike. Indeed, energy dissipation takes place on individual pointlike objects, such as stars and galaxies. At distances of the order of 1 Mpc the dimension is unity, indicating a filamentary, stringlike structure; when viewed at larger scales it gradually becomes 2-dimensional wall like, and finally at the correlation length, ξ, it becomes uniform.

It might be instructive to compare with more conventional interpretations of the large scale structure [15]. The conditional density can be related to a correlation function $g(r)$ for the overdensity through [12]

$$\Gamma^*(l) \sim \langle n \rangle [1 + g(l)], \tag{4}$$

where $\langle n \rangle$ is the mean density of galaxies. For instance, the field theory of de Vega *et al.* [16] yields an expression of this form. The overdensity $g(l)$ is often assumed to be of the form $g(l) = (r_0/l)^\gamma$.

Figure 1 also shows a fit to this expression, with $r_0 = 10$ Mpc and $\gamma = 1.3$. The fit is clearly inferior, flattening out at too small of length scales. The logarithmic error is 0.176, compared to 0.037 for our fit. This is in accordance with the observations by Sylos Labini *et al.* that the value of the fitted parameter r_0 depends heavily on the range of length scales used. At larger scales, the difference between the two fits is even more pronounced; when further data become available in the near future, one should be able to discriminate even better between the two pictures. In this traditional view, there is a smooth crossover to homogeneity when the *amplitude* of the overdensity, r_0/l, reaches unity. In contrast, within our picture the transition takes place when the length scale reaches the *correlation length*. One can trivially convert our result for correlation function, both for the Universe and the forest-fire model, to an expression for $g(l)$, by combining Eqs. (3) and (4). However, the resulting expression is complicated and unappetizing; it cannot be expressed by a simple power law.

In the conventional formulation, one usually visualizes that the amplitude r_0 of the power-law fluctuations increases with time, starting from the time of the decoupling of radiation from hadronic matter. In our phenomenology, it would be the correlation length ξ that increases with time, describing a universe with decreasing density. It would be interesting to compare our conjectured structure with numerical simulations, based for instance on cold dark matter models of the evolving Universe.

The novel geometrical scaling form has some important cosmological consequences. One can estimate the correlation length from data measured for distances shorter than the correlation length, in contrast to conventional critical phenomena. This allows us to estimate the average density of galaxies in the entire Universe, since this is equal to the density, Eq. (1), within the correlation length, i.e., $\langle n \rangle = \Gamma^*(\xi) = c/\xi^{3/2}$. From the fit to the APS 18 data we find that the density of galaxies with absolute luminosity greater than 18 is $\langle n \rangle = 3 \times 10^{-4}$ Mpc^{-3}. Assuming the size of the Universe to be 3000 Mpc this implies that, for instance, the total number of galaxies with luminosity greater than 18 is predicted to be approximately 10^7. Traditional fits to Eq. (4) give much larger values for the density of galaxies in the Universe, depending on the range of length scales used in the fit [12].

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