Long-Range Proximity Effects in Superconductor-Ferromagnet Structures

F. S. Bergeret,[1] A. F. Volkov,[1,2] and K. B. Efetov[1,3]

[1]*Theoretische Physik III, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

[2]*Institute of Radioengineering and Electronics of the Russian Academy of Sciences, 103907 Moscow, Russia*

[3]*L. D. Landau Institute for Theoretical Physics, 117940 Moscow, Russia*

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We analyze the proximity effect in a superconductor/ferromagnet (S/F) structure with a local inhomogeneity of the magnetization in the ferromagnet near the S/F interface. We demonstrate that not only the singlet but also the triplet component of the superconducting condensate is induced in the ferromagnet due to the proximity effect. The singlet component penetrates into the ferromagnet over a short length $\xi_h = \sqrt{D/h}$ (h is the exchange field and D the diffusion coefficient), whereas the triplet component penetrates over a long length $\sqrt{D/\epsilon}$ and leads to a significant increase of the ferromagnet conductance below the superconducting critical temperature T_c.

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In recent experiments on superconductor/ferromagnet (S/F) structures a considerable increase of the conductance below the superconducting critical temperature T_c was observed [1–3]. Although in a recent work [4] it was suggested that such an increase may be due to scattering at the S/F interface, a careful measurement of the conductance demonstrated that the entire change of the conductance was due to an increase of the conductivity of the ferromagnet [1,2].

Such an increase would not be a great surprise if instead of the ferromagnet one had a normal metal N. It is well known (see for review [5,6]) that in S/N structures proximity effects can lead to a considerable increase of the conductance of the N wire provided its length does not exceed the phase breaking length L_φ. However, in a S/F structure, if the superconducting pairing is singlet, the proximity effect is negligible at distances exceeding a much shorter length $\sim\xi_h$. This reduction of the proximity effect due to the exchange field h of the ferromagnet is clear from the picture of Cooper pairs consisting of electrons with opposite spins. The proximity effect is not considerably affected by the exchange energy only if the latter is small $h < T_c$. As concerns such strong ferromagnets as Fe or Co used in the experiments [1,2], whose exchange energy h is by several orders of magnitude larger than T_c, a singlet pairing is impossible due to the strong difference in the energy dispersions for the two spin bands. At the same time, an arbitrary exchange field cannot destroy a triplet superconducting pairing because the spins of the electrons forming Cooper pairs are already parallel. A possible role of the triplet component in transport properties of S/F structures has been noticed in Refs. [7,8], where the triplet component arose only as a result of mesoscopic fluctuations. However, in both cases the corrections to the conductance are much smaller than the observed ones.

In this paper, we suggest a much more robust mechanism of formation of the triplet pairing in S/F structures, which is due to a local inhomogeneity of the magnetization M in the vicinity of the S/F interface. We show that the inhomogeneity generates a triplet component of the superconducting order parameter with an amplitude comparable with that of the singlet pairing. The penetration length of the triplet component into the ferromagnet is equal to $\xi_\varepsilon = \sqrt{D/\varepsilon}$, where the energy ε is of the order of temperature T or the Thouless energy $E_T = D/L^2$, L is the sample size. The length ξ_ε is of the same order as that for the penetration of the superconducting pairs into a normal metal and therefore the increase of the conductance due to the proximity effect can be comparable with that in an S/N structure.

We consider a structure shown in Fig. 1 and assume that the magnetization orientation varies linearly from $\alpha = 0$ at $x = 0$ to $\alpha_w = Qw$ at $x = w$. Here α is the angle between M and the z axis. The case $Qw = \pi$ corresponds to a domain wall with thickness w located at the S/F interface, while the model with the homogeneous magnetization is recovered by putting $Q = 0$. This variation of M may also be brought about by an external magnetic field (see [9], and references therein). Of course, the variation

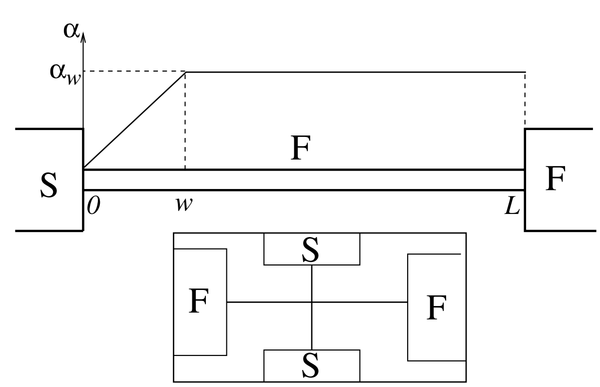

FIG. 1. Schematic view of the structure under consideration. In the inset is shown the structure, for which we calculate the conductance variation: two ferromagnetic wires connected to two ferromagnetic and two superconducting reservoirs.

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of the magnetization considered is the simplest model of what may happen at the interface in reality and we use it for simplicity. We consider the diffusive limit corresponding to a short mean free path. In this limit one may describe the S/F structure using the Usadel equation [10]. The proximity effect in a S/F structure with a uniform magnetization M was analyzed in [11]. For the system with a nonhomogeneous magnetization one should use a generalized form of this equation containing spin variables [12]. The equation is nonlinear and contains normal $\check{g}$ and anomalous $\check{f}$ quasiclassical Green's functions. These functions are 4×4 matrices in the Nambu $\otimes$ spin space.

Assuming that the anomalous condensate function f is small, one can linearize the Usadel equation. This can be done if the transmission coefficient through the S/F interface is small due to a strong mismatch of the Fermi surfaces. Moreover, the order parameter in the superconductor can be strongly suppressed near the S/F interface if the transparency is high. In both cases one may assume a weak proximity effect, which presumably corresponds to the experiments. We write the Usadel equation for the matrix element (12) of $\check{f}$ in the Nambu space. Then for the retarded matrix (in the spin space) Green's function $\hat{f}^R$ we obtain (the index R is dropped)

$$-iD\partial_{\mathbf{r}}^2\hat{f} + 2\epsilon\hat{f} - 2\Delta\hat{\sigma}_3 + (\hat{f}\hat{V}^* + \hat{V}\hat{f}) = 0. \quad (1)$$

Here ϵ is the energy, Δ is the superconducting order parameter, which vanishes in the ferromagnet; $\hat{\sigma}_i$ are the Pauli matrices in the spin space, and the matrix $\hat{V}$ is defined as $\hat{V} = h(\hat{\sigma}_3\cos\alpha + \hat{\sigma}_2\sin\alpha)$, where α varies with x as shown in Fig. 1. This matrix describes the interaction between the exchange field and spins of the conduction electrons and vanishes in the superconductor. Equation (1) could be written for temperature anomalous Green functions $\hat{f}^M$ at Matsubara frequencies ω, by replacing $\epsilon \to i|\omega|$ and multiplying the last term by $\mathrm{sgn}(\omega)$ [12]. Equation (1) is supplemented by the boundary conditions at the interface that can also be linearized [13]. Assuming that there are no spin-flip processes at the S/F interface, we have

$$\partial_x\hat{f}|_{x=0} = (\rho/R_b)\hat{f}_S\,, \quad (2)$$

where ρ is the resistivity of the ferromagnet, R_b is the S/F interface resistance per unit area in the normal state, and $f_S = \hat{\sigma}_3\Delta/\sqrt{\epsilon^2 - \Delta^2}$. The solution of Eq. (1) is trivial in the superconductor but needs some care in the ferromagnet. In the region $0 < x < w$ the solution $\hat{f}$ can be sought in the form $\hat{f} = \hat{U}(x)\hat{f}_n\hat{U}(x)$, where $\hat{U}(x) = \hat{\sigma}_0\cos(Qx/2) + i\hat{\sigma}_1\sin(Qx/2)$. Substituting this expression into Eq. (1) and assuming that the solution depends on the coordinate x only we obtain the following equation for $\hat{f}_n$:

$$-iD\partial_{xx}^2\hat{f}_n + i(DQ^2/2)(\hat{f}_n + \hat{\sigma}_1\hat{f}_n\hat{\sigma}_1) + DQ\{\partial_x\hat{f}_n, \hat{\sigma}_1\} + 2\epsilon\hat{f}_n + h\{\hat{\sigma}_3, \hat{f}_n\} = 0. \quad (3)$$

Here $\{\ldots\}$ is the anticommutator. In the region $x > w$, $\hat{f}_n$ satisfies Eq. (3) with $Q = 0$.

We see from Eq. (3) that the singlet component, commuting with $\hat{\sigma}_3$, and triplet component, anticommuting with $\hat{\sigma}_3$, are mixed by the rotating exchange field h. In the region $x > w$ the triplet and the singlet components decouple and their amplitudes should be found by matching the solutions at $x = w$. One should also use the boundary condition, Eq. (2), and match the solutions in the ferromagnet and superconductor. It is clear that the singlet and triplet components of the anomalous function $\hat{f}_n$ inevitably coexist in the ferromagnet. This fact is also known for the case of magnetic superconductors with $Q \neq 0$ [14]. In the region $x > w$ the singlet part decays sharply but the triplet one survives over long distances. We are able to confirm these statements solving Eq. (3) with the boundary condition Eq. (2). In the case of a homogeneous magnetization ($Q = 0$) the triplet pairing cannot be induced, which follows immediately from Eq. (2) connecting separately the singlet and triplet components at the opposite sides of the interface and Eq. (3).

Equation (3) can be solved exactly. The solution $\hat{f}_n$ can be written in the form

$$\hat{f}_n = \hat{\sigma}_0A(x) + \hat{\sigma}_3B(x) + i\hat{\sigma}_1C(x). \quad (4)$$

The function $C(x)$ in Eq. (4) is the amplitude of the triplet pairing, whereas the first and the second term describe the singlet one. Substituting Eq. (4) into Eq. (3) we obtain a system of three equations for the functions A, B, and C, which can be sought in the form

$$A(x) = \sum_{i=1}^{3}[A_i\exp(-\kappa_i x) + \bar{A}_i\exp(\kappa_i x)]. \quad (5)$$

The functions $B(x)$ and $C(x)$ can be written in a similar way. The eigenvalues κ_i obey the algebraic equations

$$(\kappa^2 - \kappa_\epsilon^2 - Q^2)C - 2(Q\kappa)A = 0,$$
$$(\kappa^2 - \kappa_\epsilon^2)B - \kappa_h^2A = 0, \quad (6)$$
$$(\kappa^2 - \kappa_\epsilon^2 - Q^2)A - \kappa_h^2B + 2(Q\kappa)C = 0,$$

where $\kappa_\epsilon^2 = -2i\epsilon/D$ and $\kappa_h^2 = -2ih/D$ (indices i were dropped). The eigenvalues κ are the values at which the determinant of Eqs. (6) turns to zero. From the first equation of Eqs. (6) we see that in the homogeneous case ($Q = 0$) the triplet component has a characteristic penetration length $\sim\kappa_\epsilon^{-1}$, but we see from Eq. (2) that its amplitude is zero. If $Q \neq 0$, the triplet component C is coupled to the singlet component (A, B) induced in the ferromagnet according to the boundary condition Eq. (2) (proximity effect). If the width w is small, the triplet component changes only a little in the region $(0, w)$ and spreads over a large distance of the order $|\kappa_\epsilon^{-1}|$ in the region $(0, L)$. In the case of a strong exchange field h, ξ_h is very short ($\xi_h \ll w, \xi_T$), the singlet component decays very fast over the length ξ_h, and its slowly varying part turns out to be small. In this case the first two eigenvalues $\kappa_{1,2} \approx (1 \pm i)/\xi_h$ can be used everywhere in the ferromagnet ($0 < x < L$), where L is the length of the ferromagnet. As concerns the third eigenvalues, we obtain $\kappa_3 = \sqrt{\kappa_\epsilon^2 + Q^2}$ in the interval $(0, w)$, and

$\kappa_3 = \kappa_\epsilon$ in the interval (w, L). The amplitude B_3 of the slowly varying part of the singlet component is equal to $B_3 = 2(Q\kappa_3/\kappa_h^2)C_3 \ll C_3$.

All the amplitudes should be chosen to satisfy the boundary conditions at $x = 0$ [Eq. (2)] and zero boundary condition at $x = L$. For the triplet component we obtain [we restore the indices R(A)]

$$C^{R(A)}(x) = \mp i\{QB(0)\sinh[\kappa_\epsilon(L - x)] \times [\kappa_\epsilon \cosh\Theta_\epsilon \cosh\Theta_3 + \kappa_3 \sinh\Theta_\epsilon \sinh\Theta_3]^{-1}\}^{R(A)}, \quad (7)$$

where $w < x < L$, $B^{R(A)}(0) = (\rho\xi_h/2R_b)f_S^{R(A)}$ is the amplitude of the singlet component at the S/F interface, $\Theta_\epsilon = \kappa_\epsilon L$, $\Theta_3 = \kappa_3 w$, and $\kappa_\epsilon^{R(A)} = \sqrt{\mp 2i\epsilon/D}$. One can see that the difference $C^R - C^A$ is an even function of ϵ. This is a direct consequence of the fact that $C^R - C^A$ is proportional to the Fourier transform of the correlator $K(t) = \langle\psi_\uparrow(t)\psi_\uparrow(0) + \psi_\uparrow(0)\psi_\uparrow(t)\rangle$, which is even in time. In the Matsubara representation, $C^{R(A)}$ in Eq. (7) should be replaced by C_ω with $\mathrm{sgn}\omega$ instead of $(\mp)$ and $\kappa_\omega = \sqrt{|\omega|/D}$, $f_S(\omega) = \Delta/\sqrt{\omega^2 + \Delta^2}$ instead of $\kappa_\epsilon^{R(A)}$, $f_S^{R(A)}$, respectively. Thus, C_ω corresponding to the temperature correlator $\mathcal{K}(\tau) = -\langle T_\tau\psi_\uparrow(0)\psi_\uparrow(\tau)\rangle$ is an odd function of ω and the sum over all ω is zero in accordance with $K(0) = \mathcal{K}(0) = 0$.

It is clear from Eq. (7), that the triplet component is of the same order of magnitude as the singlet one at the interface. Indeed, for the case $w \ll L$ we obtain from Eq. (7) $|C(0)| \sim B(0)/\sinh\alpha_w$, where $\alpha_w = Qw$ is the angle characterizing the rotation of the magnetization. Therefore if the angle $\alpha_w \leq 1$ and the S/F interface transparency is not too small, the singlet and triplet components are not small. They are of the same order in the vicinity of the S/F interface, but while the singlet component decays abruptly over a short distance ($\sim\xi_h$), the triplet one varies smoothly along the ferromagnet, turning to zero at the F reservoir. In Fig. 2 we plot the spatial dependence of the singlet $|B(x)|$ and the triplet $|C(x)|$ components for two different Q. One can see that the singlet component decays abruptly undergoing the well-known oscillations [15] while the triplet one decays to zero slowly. This decay in the region $(0, w)$ increases with increasing Q.

Thus, we come to a remarkable conclusion: the penetration of the superconducting condensate into a ferromagnet may be similar to the penetration into a normal metal. The only difference is that, instead of the singlet component in the case of the normal metal, the triplet one penetrates into the ferromagnet. Of course, in order to induce the triplet component one needs an inhomogeneity of the exchange field at the interface.

The presence of the condensate function (triplet component) in the ferromagnet can lead to interesting long-range effects. One of them is a change of the conductance of a ferromagnetic wire in a S/F structure (see inset in Fig. 1) when the temperature is lowered below T_c. This effect was observed first in S/N structures and later was successfully explained (see, e.g., reviews [5,6]). Now we consider the S/F structure shown in the inset of Fig. 1. The normalized conductance variation $\delta\tilde{G} = (G - G_n)/G_n$ is given by the expression [16]:

$$\delta\tilde{G} = -\frac{1}{32T}\,\mathrm{Tr}\int d\epsilon\, F_V'\langle[\hat{f}^R(x) - \hat{f}^A(x)]^2\rangle. \quad (8)$$

Here G_n is the conductance in the normal state, $F_V' = 1/2[\cosh^{-2}((\epsilon + eV)/2T) + \cosh^{-2}((\epsilon - eV)/2T)]$, and $\langle\ldots\rangle$ denotes the average over the length of the ferromagnetic wire between the F reservoirs. The function $\hat{f}$ is given by the third term of Eq. (4) with $C^R = -(C^A)^*$ (we neglect the small singlet component). Substituting Eqs. (4) and (7) into Eq. (8) one can determine the temperature dependence $\delta\tilde{G}(T)$. Figure 3 shows this dependence. We see that $\delta\tilde{G}$ increases with decreasing temperature and saturates at $T = 0$. This monotonic behavior of $\delta\tilde{G}$ contrasts with the so called reentrant behavior of $\delta\tilde{G}$ in S/N structures [17,18] and is a result

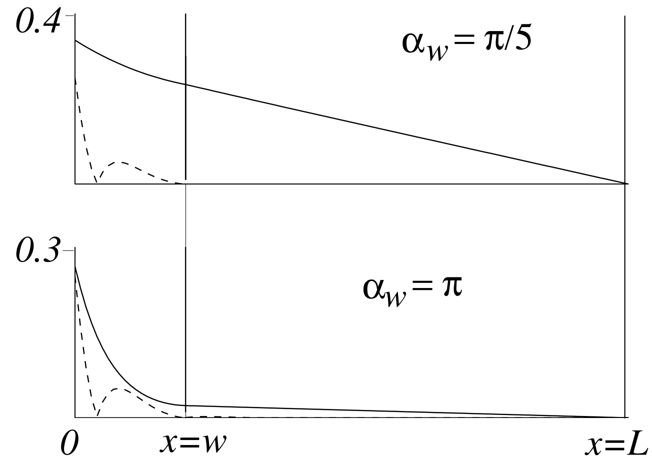

FIG. 2. Spatial dependence of the singlet (dashed line) and triplet (solid line) components of $|\hat{f}|$ in the F wire for different values of α_w. Here $w = L/5$, $\epsilon = E_T$, and $h/E_T = 400$. $E_T = D/L^2$ is the Thouless energy.

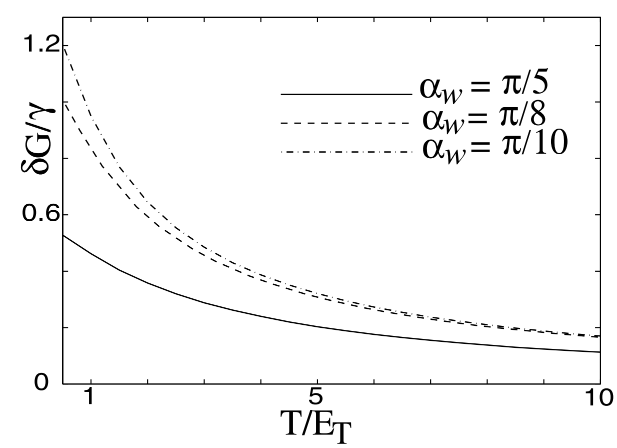

FIG. 3. The $\delta G(T)$ dependence. Here $\gamma = \rho\xi_h/R_b$. $\Delta/E_T \gg 1$ and $w/L = 0.05$.

of broken time-reversal symmetry of the system under consideration.

Available experimental data are still controversial. It has been established in a recent experiment [3] that the conductance of the ferromagnet does not change below T_c and all changes in δG are due to changes of the S/F interface resistance R_b. However, in other experiments R_b was negligibly small [1]. The mechanism suggested in our work may explain the long-range effects observed in the experiments [1,2]. At the same time, the result of the experiment [3] is not necessarily at odds with our findings. The inhomogeneity of the magnetic moment at the interface, which is the crucial ingredient of our theory, is not a phenomenon under control in these experiments. One can easily imagine that such inhomogeneity existed in the structures studied in Refs. [1,2] but was absent in those of Ref. [3]. The magnetic inhomogeneity near the interface may have different origins. Anyway, a more careful study of the possibility of a rotating magnetic moment should be performed to clarify this question.

In order to explain the reentrant behavior of $\delta G(T)$ observed in Refs. [1,2] one should take into account other mechanisms, as those analyzed in Refs. [4,7,19]. However, this question is beyond the scope of the present paper.

We note that at the energies ϵ of the order of Thouless energy $\epsilon \sim E_T$ the triplet component spreads over the full length L of the ferromagnetic wire (see Fig. 2). This long-range effect differs completely from the proximity effect in a ferromagnet with a uniform magnetization considered recently in Ref. [20]. In the latter case the characteristic wave vector is equal to $\kappa_{1,2} = \sqrt{-2i(\epsilon \pm h)/D}$ [cf. Eqs. (6)]. It was noted in Ref. [20] that if $\epsilon \to \pm h$, then $\kappa_{1,2} \to 0$ and the singlet component penetrates in the ferromagnet. If the characteristic energies $\epsilon_{ch} \sim E_T, T$ are much less than h, the penetration length $|\kappa_{1,2}|^{-1}$ is of the order ξ_h and is much shorter than ξ_T or L.

It is also interesting to note that a triplet component of the condensate function with the same symmetry (odd in frequency ω and even in momentum p) was suggested by Berenziskii [21] as a possible phase in superfluid ^{3}He (this so-called "odd" superconductivity was discussed in a subsequent paper [22]). Being symmetric in space, this component is not affected by potential impurities, in contrast to the case analyzed in Ref. [23], where the triplet component of the condensate was odd in space. While in ^{3}He this hypothetical condensate function is not realized (in ^{3}He it is odd in p but not in frequency), in our system this odd (in ω) triplet component does exist, although under special conditions described above.

In conclusion, we have shown that in the presence of a local inhomogeneity of magnetization near the S/F interface, both the singlet and triplet components of the condensate are created in the ferromagnet due to the proximity effect. The singlet component penetrates into the ferromagnet over a short length ξ_h, whereas the triplet component can spread over the full mesoscopic length of the ferromagnet. This long-range penetration of the triplet component should lead to a significant variation of the ferromagnet conductance below T_c.

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