Is the Quantum Dot at Large Bias a Weak-Coupling Problem?

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We examine the two-lead Kondo model for a dc-biased quantum dot in the Coulomb blockade regime. From perturbative calculations of the magnetic susceptibility, we show that the problem retains its strong-coupling nature, even at bias voltages larger than the equilibrium Kondo temperature. We give a speculative discussion of the nature of the renormalization group flows and the strong-coupling state that emerges at large voltage bias.

For over a decade, transport measurements on quantum dot systems have stimulated theoretical interest in the properties of the Anderson model out of equilibrium [1–3]. Early predictions [4–9] of the Kondo effect in such systems have recently been experimentally verified [10–13]. In the small voltage regime, such experiments have produced impressive agreement with theoretical predictions. Recent interest [14–16] has turned to the question of how the Kondo effect behaves far from equilibrium, i.e., at large voltage bias.

In the Coulomb blockade regime, one may neglect charge fluctuations of a quantum dot and concentrate on transport processes that operate by flipping the dot's spin. In this limit, the physics is described by a two-lead Kondo model [14,17] [Eq. (1)]. A key property of the Kondo model in equilibrium is the presence of a "running coupling constant," whereby the antiferromagnetic coupling between the dot spin and the leads grows as the energy scale is reduced. This leads to a "strong coupling" low-energy regime where the spin of the dot is quenched by the lead electrons and the residual properties can no longer be obtained from a perturbative expansion. A single scale, the "Kondo temperature" T_K, governs the low temperature properties; $T_K = D\sqrt{g}\, e^{-1/2g}$, where g is the "bare coupling" between the spin of the dot and the leads and $D \gg T_K$ is the electron bandwidth. Thus the magnetization at temperature T and magnetic field B is a universal function $M = m(T/T_K, B/T_K)$, where M has a perturbative "weak coupling" expansion in g only when $T, B \gg T_K$. [See Fig. 1(a).]

In a quantum dot, the formation of an Abrikosov-Suhl resonance in the quasiparticle density of states which accompanies the Kondo effect substantially enhances the linear conductivity at low temperature, resulting in unitary transmission at absolute zero [4,5].

But what happens to the physics of the quantum dot at voltages V comparable to or larger than the Kondo temperature? In particular, does a large voltage act as a large magnetic field or temperature, and return the dot to a weak coupling regime where its spin is unquenched? Theoretical studies of the differential conductivity $G[V]$ do suggest that the large voltage physics is governed by weak-coupling perturbation theory, for [14] $G = (2e^2/h)F[eV/T_K]$ has a perturbative expansion in g for $V \gg T_K$. However, the large bias conductivity probes electrons that are far from the Fermi surface and does not in itself shed light on the coupling between the local moment and the electrons near the Fermi surface.

In this Letter, we argue that the quantum dot retains its strong-coupling character even at voltages $V \gg T_K$. We use explicit perturbative calculations of the magnetic susceptibility to show that the large bias quantum dot is characterized by a strong-coupling regime with a voltage-dependent Kondo temperature $T_K^* \propto T_K^2/eV$.

The Hamiltonian of the two-lead Kondo model is

$$H = \sum_{\alpha \mathbf{k}\sigma} \varepsilon_{\alpha \mathbf{k}} c_{\alpha \mathbf{k}\sigma}^\dagger c_{\alpha \mathbf{k}\sigma} + H_{\mathrm{refl}} + H_{\mathrm{trans}}, \qquad (1)$$

$$H_{\mathrm{refl}} = J_R \sum_{\mathbf{k},\mathbf{k}',\sigma,\sigma'} (c_{R\mathbf{k}\sigma}^\dagger \vec{\sigma}_{\sigma\sigma'} c_{R\mathbf{k}'\sigma'}) \cdot \vec{S} + (R \to L),$$

$$H_{\mathrm{trans}} = J_{LR} \sum_{\mathbf{k},\mathbf{k}',\sigma,\sigma'} (c_{R\mathbf{k}\sigma}^\dagger \vec{\sigma}_{\sigma\sigma'} c_{L\mathbf{k}\sigma'}') \cdot \vec{S} + (R \leftrightarrow L).$$

Here, $c_{\alpha \mathbf{k}\sigma}^\dagger$ creates an electron in lead $\alpha \in \{L, R\}$ with momentum $\mathbf{k}$ and spin σ, and J_L, J_R, and $J_{LR} = J_{RL}$ are positive (antiferromagnetic) Kondo coupling constants between the electrons and the dot ($\vec{S}$).
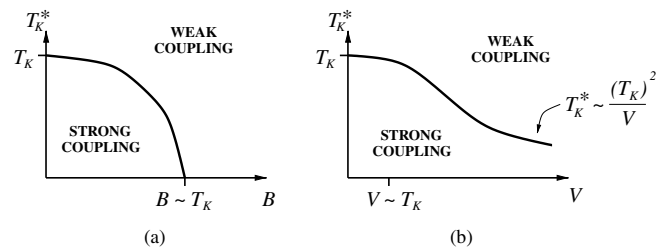

FIG. 1.   (a) Field dependence of the crossover temperature $T_K^*(B)$ separating weak- and strong-coupling regimes of the equilibrium Kondo model. At absolute zero, for fields larger than the Kondo temperature the quantum dot reenters weak coupling. (b) Voltage dependence of the crossover temperature $T_K^*(V)$, from our results. Note that T_K^* goes to zero only in the limit $V \to \infty$, i.e., for low enough temperatures, the model reaches a strong coupling state for all V.

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The first part of H describes the electrons in the leads, with energies $\varepsilon_{\alpha\mathbf{k}} = \varepsilon_{\mathbf{k}} - eV_\alpha$, where $V_\alpha = \pm V/2$ are the potentials of the left- and right-hand leads. H_{refl} describes regular Kondo processes, where an electron from a given lead is spin-flip scattered back into the *same* lead; H_{trans} describes "spin-flip cotunnelling," where an electron from one lead is spin-flip scattered into the *other* lead. We introduce the dimensionless couplings $\bar{J}_L, \bar{J}_R, \bar{J}_{LR}$, defined by $\bar{J}_i \equiv \rho J_i$, where ρ is the density of states at the Fermi levels. When the model is derived from an Anderson model via a Schrieffer-Wolff transformation [18,19], the coupling constants obey $|J_{LR}|^2 = J_L J_R$, restricting the scattering to a single channel. Symmetric dots ($J_L = J_R = J_{LR}$), are of particular interest, for in this case the differential conductivity G approaches the unitary limit ($2e^2/h$) at low temperatures.

In a magnetic field we replace $H \rightarrow H - BM_z$, where $M_z = 2S_z + \sum_{\alpha\mathbf{k}\sigma} \sigma n_{\alpha\mathbf{k}\sigma}$ is the magnetization. The impurity susceptibility is $\chi_i = \frac{\partial}{\partial B} M_z|_{B=0} - \chi_o$, where $\chi_o = 4\rho$ is the Pauli susceptibility of the leads. We expand χ_i perturbatively to order J^2 using the Keldysh method [20], and representing the spin as $\vec{S} = -\frac{i}{2}\vec{\eta} \times \vec{\eta}$, where the elements of $\vec{\eta}$ are Majorana fermions. In the large-bandwidth limit we find that

$$\chi_i = \frac{1}{T}\left\{1 - \frac{1}{2}(\bar{J}_R + \bar{J}_L) - (\bar{J}_L^2 + \bar{J}_R^2)\ln\left(\frac{D}{2\pi T e^{1+\gamma}}\right)\right.$$
$$\left. - 2|\bar{J}_{LR}|^2\left[\ln\left(\frac{D}{2\pi T e^{1+\gamma}}\right) - \phi\left(\frac{V}{T}\right)\right]\right\}, \quad (2)$$

where $\gamma = 0.577\ldots$ is the Euler constant. The crossover function $\phi(x)$, in terms of digamma functions $\psi(z)$, is

$$\phi(x) = \text{Re}\int_{-\infty}^{\infty} \frac{dy}{4\cosh^2(\frac{y}{2})}[\tilde{\psi}(y + x) - \tilde{\psi}(y)], \quad (3)$$

where $\tilde{\psi}(x) = \psi(\frac{1}{2} + i\frac{x}{2\pi})$.

The second-order terms in (2) describe the leading logarithmic enhancement of the Kondo coupling. Terms of order J_{LR}^2 involve interlead processes and, as expected, the logarithmic divergence in these terms is cut by the voltage. [To see this, note that $-\phi(V/T) \sim -\ln(V/T)$ for $V \gg T$ which cancels the logarithmic temperature divergence.] By contrast, the intralead terms of order J_R^2 and J_L^2 are completely unaffected by the voltage V, which guarantees that the leading logarithmic divergence survives at *arbitrarily high voltage*. This is easily seen in the large-V form of the susceptibility,

$$\chi_i = \frac{1}{T}\left\{1 - \frac{1}{2}(\bar{J}_R + \bar{J}_L) - (\bar{J}_L^2 + \bar{J}_R^2)\ln\left(\frac{D}{2\pi T e^{1+\gamma}}\right)\right.$$
$$\left. - 2|\bar{J}_{LR}|^2\ln\left(\frac{D}{V}\right)\right\}, \quad V \gg T. \quad (4)$$

The survival of these leading logarithms is a signature that the intralead Kondo effect continues unabated at temperatures smaller than the voltage.

Let us now be more precise by defining a *generalized crossover temperature* T_K^* as the temperature below which the perturbation expansion (2) breaks down, i.e., the temperature at which the $O(J)$ and $O(J^2)$ terms become equal in magnitude. This procedure yields an implicit formula for $T_K^*(V)$:

$$T_K^*(V) = T_K e^{-(1/2)\phi[V/T_K^*(V)]}, \quad (5)$$

where $T_K = De^{-1/2g}$ is the leading approximation to the Kondo temperature [21]. We plot $T_K^*(V)$ in Fig. 1(b).

Note that T_K^* goes to zero only when V goes to infinity. From the behavior of ϕ at large argument, we can extract the large voltage behavior

$$T_K^* = \frac{(T_K)^2}{V}. \quad (6)$$

This means that the model always enters a strong coupling regime at low temperature, even for $V \gg T_K$.

These perturbative results for χ_i may be interpreted within a simple renormalization picture. Let us stress, however, that there is not yet a general theory of renormalization out of equilibrium (though see [22]). In particular, it has not been established that there exists an appropriate family of effective Hamiltonians through which one passes as the high-energy cutoff scale is varied. Furthermore, in the presence of a voltage, long-time processes cannot be unambiguously connected with low-energy ones. Hence the following discussion is somewhat speculative, and steps beyond the perturbative calculation that led to (5).

In the spirit of a "poor man's scaling" approach [23], we consider varying a cutoff energy scale Λ in the problem, and studying the variation of the effective coupling constants J_L, J_R, J_{LR} as functions of this variable. The essential scaling behavior, as usual, can be read off from the perturbative expression (2). We may identify two scaling regimes:

(1) One-channel scaling, $\Lambda \gg V$. In this regime the effect of the voltage is not felt in any channel, so the dot behaves as an equilibrium single-channel Kondo model. The single coupling constant $g \equiv \bar{J}_L + \bar{J}_R$ rescales according to

$$\frac{dg}{d\ln\Lambda} = -2g^2 + O(g^3), \quad (7)$$

so to leading order in the scaling, the coupling constant at scale $\Lambda = V$ is given by $\frac{1}{g^*} = \frac{1}{g} - 2\ln(D/V)$. In this regime, the ratio J_{LR}/g is preserved.

(2) Two-channel regime $V \gg \Lambda \gg T_K^*(V)$. Around $\Lambda \sim V$ the logarithmic renormalization of interlead processes comes to a halt, freezing J_{LR} at its value $J_{LR}(V)$ and prompting a crossover to a two-channel scaling regime in which intralead processes independently renormalize J_R and J_L. This scenario is reminiscent of one proposed by Wen [24]: that the effect of the voltage is to destroy coherence *between* the two leads, suppressing the J_{LR} terms, and

leaving a two-channel Kondo Hamiltonian. We shall restrict our attention to symmetric dots, so that $J_{L,R} = \bar{J}(\Lambda)$ scale together according to

$$\frac{d\bar{J}}{d\ln\Lambda} = -2\bar{J}^2 + O(\bar{J}^3). \qquad (8)$$

At voltage V, $\bar{J}_{R,L} = g^*/2$, so to leading order, the coupling constants are given by

$$\frac{1}{\bar{J}(\Lambda)} = \frac{2}{g^*} - 2\ln(V/\Lambda). \qquad (9)$$

Scaling continues until these coupling constants become of order unity. Setting the left-hand side of (9) to one, we recover the same crossover scale obtained by comparing terms in the susceptibility: $T_K^* \sim Ve^{-1/g^*} = (T_K)^2/V$.

These flows are depicted in Fig. 2.

Thus we conclude that, for $V > T_K$, the scaling is of a two-channel character by the time strong coupling is reached. This suggests that an appropriate low-temperature description may be in terms of the infrared fixed point of the two-channel Kondo model. However, it is known that this point is unstable to perturbations of the form J_{LR}, so how can this be? We shall show that the presence of a finite voltage renders the J_{LR} perturbations irrelevant, making the two-channel infrared fixed point a likely candidate for the $V > T_K$ state of the model.

To demonstrate this, let us suppose that H_0 is the Hamiltonian where $J_{LR} = 0$, and consider the effects of introducing a small left-right coupling λ. For convenience, let us perform a gauge transformation on the lead-electron operators, $c_{\alpha\mathbf{k}\sigma} \rightarrow e^{iV_\alpha t}c_{\alpha\mathbf{k}\sigma}$. This removes the chemical potentials from the bare energies of the lead electrons, with the compensating effect of introducing a factor $e^{\pm iVt}$ into the J_{LR} terms. Therefore, the perturbation we wish to consider is given by $H_I(t) = \lambda[O^\dagger e^{-iVt} + Oe^{iVt}]$, where $O = c_L^\dagger \vec{\sigma} c_R \cdot \vec{S}$.
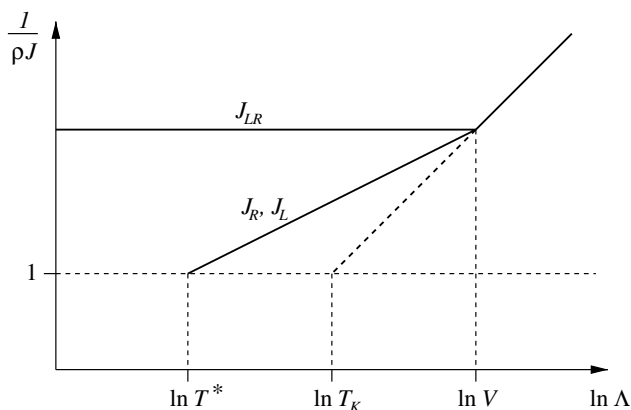
From the conformal field theory of the two-channel Kondo model [25], we know the scaling dimension of the interchannel operator, which scales according to $\langle O(t)O^\dagger(0)\rangle \sim \frac{1}{t}$ at long times. Any perturbation coupling statically to O is relevant, but we are interested in a coupling to the oscillatory operator Oe^{iVt}. If we calculate the expectation value of H_I, we obtain

$$\langle H_I\rangle = 2\lambda^2 \operatorname{Im}\int_0^\infty dt \, \langle[O(t), O^\dagger(0)]\rangle e^{-iVt} \sim \ln\left(\frac{\tilde{\Lambda}}{V}\right), \qquad (10)$$

showing that the perturbation is finite in a nonzero voltage, but becomes relevant, as expected, in the $V = 0$ case. This suggests that the finite voltage protects the two-channel fixed point against the damaging effects of interlead coupling. We expect that these arguments can be examined in more detail using a bosonized formulation of the two-channel Kondo model [16].

We may go one small step beyond this point, and examine briefly the nature of the current fluctuations in the leads at this fixed point. The current operator is given by $I(t) = -i(e/\hbar)\lambda[O^\dagger e^{-iVt} - \text{H.c.}]$. From the scaling dimension of the operators O and $O^\dagger$ we can see that the current-current correlator decays as e^{-iVt}/t, so that we expect that for small λ

$$C(\omega) \equiv \langle\{I(\omega), I(-\omega)\}\rangle \sim \ln(|\omega| - V), \qquad (11)$$

i.e., $C(\omega)$ will develop a weak logarithmic divergence at frequencies $\omega \sim eV/\hbar$. A similar divergence (resulting from the corresponding $1/t$ decay of the spin correlators) is responsible for the logarithmic divergence of the spin susceptibility in a two-channel Kondo model.

This discussion leads us to propose the crossover diagram presented in Fig. 3.

FIG. 2. Schematic renormalization flows for this model, for the case $V > T_K > T$. All couplings flow together down to scales of order V, after which the flow of J_{LR} is halted at its value $J_{LR}(V)$. The couplings J_L and J_R continue to flow to strong coupling values, but more slowly, yielding a generalized crossover temperature $T_K^*(V) \sim T_K^2/V$.
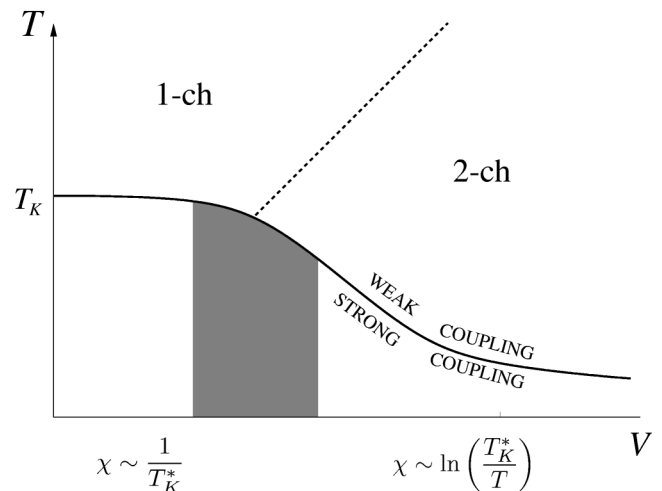

FIG. 3. Schematic crossover diagram for the two-lead Kondo model. The dashed line separates the regions $V < T$ and $V > T$ corresponding to one- and two-channel scaling behavior, respectively. The solid line represents the crossover from the weak- to the strong-coupling region, estimated from perturbative calculations.

If indeed T_K^* is the only scale in the problem, the susceptibility and frequency-dependent conductivity in this system are expected to display the following logarithmic forms at low temperature:

$$G(\omega) \sim \frac{e^2}{\hbar} \ln\left[\frac{T_K^*}{\max[|\omega - eV|, T]} \right], \qquad \omega \approx eV$$

(12)

$$\chi(T, V) \sim \frac{1}{T_K^*} \ln\left[\frac{T_K^*}{T} \right].$$

(13)

If these tentative forms hold true, then the spin susceptibility must go from a finite value at low voltages to a divergent form at high voltages, suggesting the possibility of a quantum critical point separating the low and high voltage regimes of the quantum dot.

We end by adding a brief remark on the issue of decoherence. A common view is that large voltages decohere the spin fluctuations of the quantum dot, introducing a decoherence time given by $\tau^{-1} \approx (\rho J)^2 V$. [See, for example, Eq. (43) of [8] and p. 387 of [19].] If this decoherence is to destroy the Kondo effect, it must cut the logarithms in expressions such as (2) before strong coupling is achieved. But in order for terms of the type $\ln[D/\max(\tau^{-1}, T)]$ to appear in the high-temperature expansion, the perturbation theory would have to break down at a temperature larger than T_K^*, for which there is no evidence in the low-order perturbation theory. Unless such a breakdown occurs at higher orders, we conclude that $\tau^{-1} < T_K^*$.

To conclude, we have presented perturbative arguments which suggest that the two-lead Kondo model, representing a quantum dot deep in the Coulomb blockade regime, retains its strong-coupling nature even for voltages $V > T_K$. To reach this conclusion, we have used a perturbative calculation of the magnetic susceptibility to read off the leading renormalization behavior. We concluded that J_{LR} ceases to renormalize at scales of order V, while J_L and J_R continue to renormalize, though more slowly, towards strong coupling. This suggests a picture in which the model scales to a two-channel infrared fixed point. To test this picture, we performed a partial analysis of the stability of this point with respect to small perturbations of the form J_{LR}. At finite voltages, the point was found to be stable to such perturbations, although the role of dangerous irrelevant operators may warrant further inspection. The voltage dependence of T_K^*, and the associated crossover to two-channel Kondo behavior, imply that the voltage, rather than simply providing a frequency shift, qualitatively changes the structure of the model's excitation spectrum.

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