Surface Fluctuations of Normal and Superfluid ^{3}He Probed by Wigner Solid Dynamics

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Electron scattering from surface fluctuations on normal and superfluid ^{3}He has been measured by its effect on the linewidth of the low-wave-vector transverse magnetophonon mode of the electron crystal (the Wigner solid) floating on the helium surface. The relaxation rate becomes anomalously low below 70 mK, and reaches a plateau at about 3 times less than its expected value before dropping further at the superfluid transition. The absence of such anomalous behavior on ^{4}He suggests that the effect is specific to liquid ^{3}He.

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The strong contrast in low-temperature properties resulting from the different statistics of the two isotopes of helium has been extensively studied with regard to the bulk properties, but relatively little attention has been paid to the difference in *surface* properties. Surface electrons (SE) are an excellent probe for such a study. The fact that they crystallize into a Wigner solid (WS) at low temperature might be seen as an unwelcome complication, but, in fact, opens up the possibility of probing the low-frequency hydrodynamics of surface deformation by virtue of the formation of the dimple lattice which is formed on the helium surface under the lattice-localized electrons. This was the subject of previous work on the low-frequency mobility which is dominated by the scattering of bulk quasiparticles from the accompanying dimple lattice motion and little influenced by surface roughness [1,2].

In contrast, the present Letter reports measurements of electron scattering at frequencies where the motion of the dimple lattice no longer contributes and the effects of surface roughness are revealed most clearly. A comparative study of the effect of electron scattering from surface roughness of ^{3}He and ^{4}He substrates on the linewidth of a high-frequency vibration mode shows up striking differences between the low-temperature surface properties of the two isotopes. We find a marked decrease in scattering in the low-temperature Fermi liquid phase and a further decrease in the superfluid phase of ^{3}He while there is no anomaly in a similar WS on ^{4}He.

Grimes and Adams [3] studied the plasmon resonance of the liquid SE on ^{4}He and successfully explained the temperature dependence of the linewidth in terms of the electron scattering from He gas atoms and ripplons. The ripplons are quantized surface capillary waves, which are characteristic surface excitations of liquid ^{4}He. When the parameter $\Gamma = e^2\sqrt{\pi n_s}/k_B T$, where e is the elementary charge, n_s is the areal density of SE, k_B is the Boltzmann constant, and T is the temperature, exceeds 127, the SE solidify to form the WS. The formation of the WS causes a change in the phonon dispersion relation because of the appearance of the commensurate deformation on the liquid surface [4,5], which is called a dimple lattice (DL). Thus affected, the phonon dispersion is referred to as the coupled phonon-ripplon (CPR) modes [6]. The effect of the coupling is to split both longitudinal and transverse phonon modes into acoustic and optical branches at low wave vector. Because the optical modes are not accompanied by the bulk motion of the liquid, the origin of damping should be the same as that for the free plasmon of the liquid SE. The optical modes should provide information on the WS scattering from surface fluctuations. In particular, because the transverse-optical (TO)-CPR mode has very little dispersion, it has a narrower inhomogeneous linewidth resulting from the spread of the spatial Fourier spectrum of the excitation field and, therefore, is better suited to observing broadening from scattering [7].

The cell used in the experiment is shown schematically in Fig. 1. The surface of liquid helium was placed 0.6 mm above the lower of two plane parallel horizontal electrodes separated by 2.2 mm. The bottom electrode consisted of a pair of concentric Corbino electrodes and was used for the low-frequency conductivity measurement. The SE were confined laterally by a guard ring of 31 mm diameter. The resonance frequencies varied depending on temperature from 100 to 450 MHz, and hence a wideband spectrometer was necessary. The spectrometer we employed was similar to one used by Deville *et al.* [8]. The meander microstrip line was placed on the upper electrode, which was used to excite and detect the phonon oscillation in the SE sheet. The width of the microstrip line was 0.4 mm and the gap between neighboring lines was 0.5 mm. The 15 cycles of meander were placed in the entire area of the upper electrode. The impedance of the microstrip line was adjusted to 50 Ω so as to have a flat characteristic in a wide frequency range. A magnetic field perpendicular to the SE is necessary to couple the TO-CPR mode to the microstrip line. The electron source was a tungsten filament located 2.0 mm above the liquid surface. The cell contained a sintered silver heat exchanger in the bottom part in order to improve the heat contact between liquid ^{3}He and the cell body. The cell was mounted on a nuclear spin demagnetization cryostat and the temperature was measured using resistors, the ^{3}He melting curve, and Pt-NMR thermometers.

The spectrum of the optical modes of the WS in a magnetic field is obtained by solving

$$\begin{vmatrix} \omega_0^2 + \omega_l^2 - \omega^2 + i\omega/\tau & i\omega_c\omega \\ -i\omega_c\omega & \omega_0^2 + \omega_t^2 - \omega^2 + i\omega/\tau \end{vmatrix} = 0. \tag{1}$$

Here ω_l and ω_t are frequencies of the longitudinal and transverse WS phonons on a flat substrate without magnetic field, respectively. The cyclotron frequency $\omega_c = eH/mc$ is due to the Lorentz force where m is the mass of an electron, H is the magnetic field, and c is the speed of light. The relaxation time τ describes the damping of the modes. The appearance of the DL causes an additional restoring force and ω_0 is the frequency of electron oscillation in the bottom of the dimples, hereafter referred to as the dimple frequency. The resonance we measured here corresponds to the low-frequency mode of Eq. (1), ω_-. The ω_- mode inherits most properties from the TO phonon: weak dispersion and diminishing frequency at the melting temperature.

A theoretical expression for ω_0 is given by [6]

$$\omega_0^2 = \frac{1}{2}\sum_{\vec{g}} V_g^2, \qquad V_g = V_g^0 \exp(-nW_1), \tag{2}$$

where $\vec{g}$ is the reciprocal lattice vector of the WS, and n is the index of the reciprocal lattice vector defined as $|\vec{g}_n|^2 = n|\vec{g}_1|^2$. For the triangular lattice, $n = \{1, 3, 4, 7, 9, \ldots\}$. The coefficient V_g^0 is given by the following formula [9]:

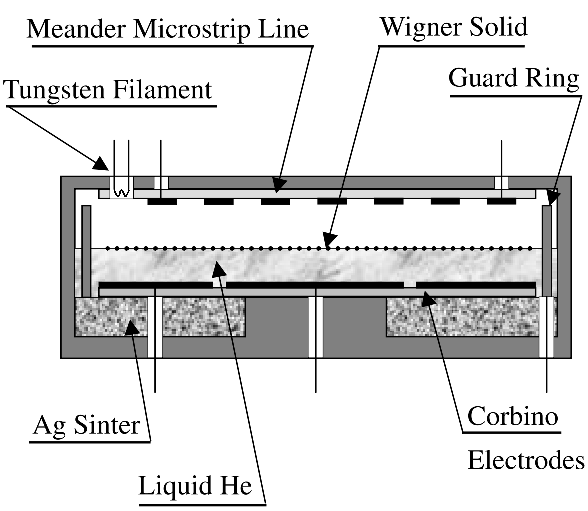

FIG. 1. Schematic drawing of the experimental cell. A 50-Ω impedance-matched meander-microstrip line is placed over the Wigner solid, through which an rf signal is fed and the absorption due to the WS-phonon resonance is analyzed. The electron distribution is tempered by applying electric potentials to the Corbino electrodes and guard ring.

$$\begin{aligned} \sqrt{\frac{m\alpha}{n_s}}\, V_g^0 &= eE_\perp + \frac{1}{2}\Lambda g^2 \varphi(gb/2), \\ \Lambda &= \frac{e^2}{4}\frac{\epsilon - 1}{\epsilon + 1}, \\ \varphi(x) &= -\frac{1}{1 - x^2} + \frac{1}{(1 - x^2)^{3/2}} \\ &\quad \times \ln\left[\frac{1 + \sqrt{1 - x^2}}{x}\right]. \end{aligned} \tag{3}$$

Here α and ϵ are the surface tension and dielectric constant of liquid He, respectively, b is the effective Bohr radius of the SE, with which the wave function in the direction perpendicular to the surface (z direction) may be expressed proportionally to $z\exp(-z/b)$, and $E_\perp$ is a pressing electric field. The pressing field under saturated electron density is equal to $2\pi e n_s$. We can deduce W_1 from the experimentally obtained resonance frequency by means of Eqs. (1)–(3).

In Fig. 2 we show the experimentally obtained W_1 on both ^{3}He and ^{4}He as a function of temperature. The experimental conditions are, for ^{3}He, $n_s = 3.7 \times 10^8$ cm^{-2}, $E_\perp = 388$ V cm^{-1}, and $H = 150$ Gs; for ^{4}He, $n_s = 3.9 \times 10^8$ cm^{-2}, $E_\perp = 355$ V cm^{-1}, and $H = 75$ Gs.

The Debye-Waller factor contains the term $W_1 = g_1^2\langle u^2\rangle/4$, where $\langle u^2\rangle$ is the mean square displacement of the electron about its lattice site. When one calculates $\langle u^2\rangle$, it is necessary to introduce a low-frequency cutoff in two dimensions. The dimple frequency ω_0 ought to be the cutoff, which does in turn depend on W_1, and hence the problem should be solved self-consistently [10]. One can find a general expression (a set of equations to solve) in Ref. [9], according to which the solid and dashed lines in Fig. 2 are calculated. The slight difference between ^{3}He and ^{4}He is due to a difference in surface tension. At temperatures below 10 mK the electron fluctuations are determined by zero point motion and W_1 does not change down to the lowest temperature. The value of W_1 in the low-temperature limit corresponds to the magnetophonon frequency of 435 MHz for ^{3}He or 408 MHz for ^{4}He. The agreement between the experiment and theory is satisfactory. It should be noted that it was important to take into account the contribution from large reciprocal lattice vectors, g_n's, in Eq. (2); otherwise the numerical

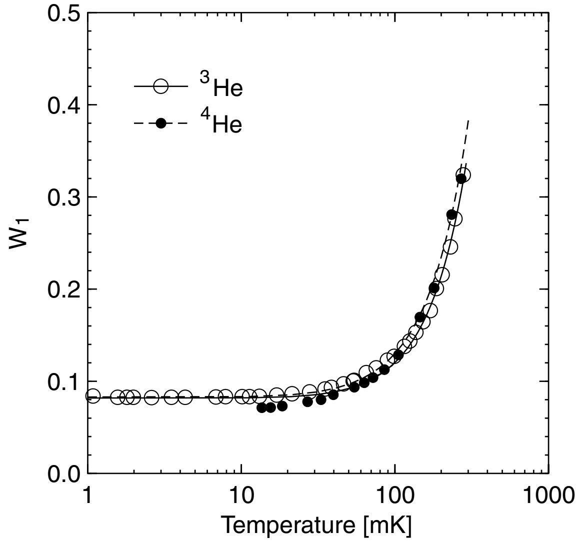

FIG. 2. Experimentally obtained W_1 as a function of temperature. Solid and dashed lines are theoretical (see text).

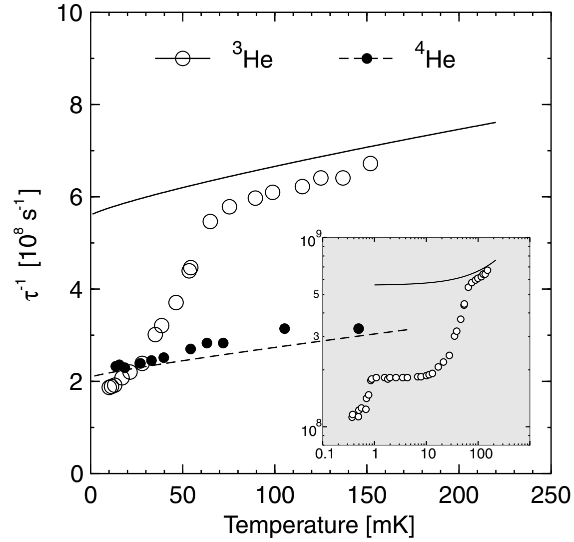

FIG. 3. Relaxation rate τ^{-1} of the high-frequency phonon mode of the Wigner solid as a function of temperature. Conditions are the same as for data presented in Fig. 2. Solid and dashed lines are from the SERS theory. Inset shows the logarithmic plot (including data below 10 mK) of the ^{3}He data to illustrate the low-temperature behavior.

agreement would be poor. Another important consequence is that the WS is in thermal equilibrium with liquid He at least down to 10 mK (below 10 mK, ω_0 becomes temperature independent in any case). Additionally, it was checked if the position and linewidth of the resonance remained unchanged upon varying the signal power. The measurements were performed at the rf power of -81 dBm, which corresponds to the power absorption of 0.1 pW by the SE. If the power is increased far above this level, the frequency of the resonance starts to decrease and the shape becomes sharper, consistent with the result obtained by Yucel *et al.* [11].

The influence of the magnetic field on the linewidth of the resonance was verified experimentally. The decrease of the linewidth was less than 8% at a magnetic field of 150 Gs. This is qualitatively consistent with what is expected from Eq. (1). Hereafter, we refer to τ^{-1} as the experimentally obtained linewidth of the magnetophonon resonance which is normalized so that it will coincide with τ^{-1} in Eq. (1) under zero magnetic field. Disregarding the influence of the magnetic field on τ^{-1} will not alter the present discussion.

Figure 3 shows the temperature dependence of τ^{-1}. Above 70 mK for ^{3}He and in the entire temperature range for ^{4}He, the temperature dependence of the relaxation rate is essentially the same for both ^{3}He and ^{4}He, and it is in good agreement with the single-electron ripplon scattering (SERS) theory [12,13], which is indicated by solid and dashed lines. Regarding ^{3}He, the relaxation rate of the phonon mode starts to drop from the SERS curve below 70 mK. This tendency was observed in the previous work [14], where the experimental conditions were not the same; in particular, the magnetic field was absent. Although there is no explicit theoretical explanation for why the momentum scattering rate of the SERS theory accounts so well for the relaxation rate of the vibrational modes of the WS phase for $T \ll T_D$, where T_D is Debye temperature, the excellent agreement for ^{4}He allows us to conclude that the anomalous behavior in ^{3}He results from the behavior of the ^{3}He surface.

We can observe the following tendency in both the present and previous works [14] that the inflection temperature at which the data start to deviate from the SERS curve decreases as the electron density n_s decreases. Because the decrease in n_s results in the decrease in the resonance frequency in the measurement, we cannot conclude decisively which quantity is the most relevant.

Makabe and Saitoh developed a theory of plasmon resonance of the WS, which predicted a relaxation rate linearly proportional to temperature assuming ripplon excitation on the ^{3}He surface [15]. According to Monarkha and Kono, a finite frequency theory predicts a T linear dependence of τ^{-1} assuming a Bose-type surface excitation [16]. One can speculate that such a surface excitation may be that predicted on the degenerate Fermi liquid in the collisionless limit by Fomin [17] and Ivanov [18]. Although the T linear dependence appears to be consistent with our ^{3}He data between 20 and 70 mK, this explanation encounters an essential difficulty in the explanation of why ^{4}He does not show the T linear behavior. Liquid ^{4}He clearly has a well-defined Bose-type surface excitation of ripplons down to very low temperatures. In addition, the explanation does not account for the plateau of τ^{-1} below 20 mK, which is clearly seen in the inset of Fig. 3. The damping introduced by the ^{3}He quasiparticles on the small dimple motion component of the optical mode, in the viscous or ballistic regimes, is estimated to contribute at most 10^{-2} of the observed relaxation rate.

The crossover temperature to the ballistic quasiparticle regime at the measurement frequency of ~400 MHz takes place at ~50 mK. This coincides with the inflection temperature of the τ^{-1} data. Since the quasiparticle collision frequency is $\propto T^2$, the crossover temperature decreases as the frequency decreases. This is qualitatively consistent with the observations in the present and previous works [14]. Possibly the crossover to the ballistic regime is reflected in the surface properties. We need, however, a sounder theoretical basis for the electron-surface-roughness scattering in this regime as well as more systematic experimentation.

Finally, we mention another striking result obtained in this experiment, as shown in the inset of Fig. 3. Below 10 mK, τ^{-1} remains temperature independent down to 1 mK and suddenly starts to decrease again at the superfluid ^{3}He transition temperature of 0.93 mK. It is as if surface fluctuation were reduced in the superfluid state.

In conclusion, we have shown that $W_1 = g_1^2 \langle u^2 \rangle / 4$, where g_1 is the shortest reciprocal lattice vector and $\langle u^2 \rangle$ is the mean square displacement of the electron in the WS, on ^{3}He and ^{4}He obtained from the frequency of TO mode magnetoplasmon resonances agrees well with self-consistent theory [9,10]. The relaxation rate τ^{-1} of the WS obtained from the linewidth is in accordance with the SERS theory in the entire temperature region of the experiment for ^{4}He and above 70 mK for ^{3}He. This is a strong indication that surface fluctuations are described by ripplons in these regimes, but we lack a sound theoretical explanation for why the SERS result works so well. Below 70 mK on ^{3}He, τ^{-1} shows significant deviation from the expected behavior extrapolated from high temperature, in marked contrast to the ^{4}He case. Below the ^{3}He superfluid transition at 0.93 mK, relaxation rate τ^{-1} shows another considerable decrease. All the experimental signatures indicate that something quite different is happening on the surface of liquid ^{3}He as compared to liquid ^{4}He. We hope that the present work prompts the further study of the dynamic properties of the ^{3}He surface at ultralow temperatures.

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