Domain Walls of High-Density QCD

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We show that in very dense quark matter there must exist metastable domain walls where the axial U(1) phase of the color-superconducting condensate changes by 2π. The decay rate of the domain walls is exponentially suppressed and we compute it semiclassically. We give an estimate of the critical chemical potential above which our analysis is under theoretical control.

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Introduction.—Domain walls are common in field theory [1]. They are configurations of fields interpolating between two vacua. If these two vacua are distinct the domain wall cannot decay. There are, however, theories with only a single vacuum, in which, nevertheless, domain-wall configurations exist. The most notable example is the theory of $N = 1$ axion [2]. In such theories the decay of the domain wall is possible, though, the decay rate is often suppressed. It is generally believed that there are no domain walls of either kind in the standard model. It was advocated only recently that long-lived domain walls may exist in QCD [3] at zero temperature and baryon density. The possibility of very unstable walls was noticed earlier in Ref. [4]. Unfortunately, no theoretical control is possible in this nonperturbative regime. Somewhat related but different domain-wall configurations were also discussed in Ref. [5] in the context of the decay of the metastable vacua possibly created in heavy ion collisions.

In this paper we show that, in the regime of high baryon densities, where relevant physics is under theoretical control, QCD must have domain walls. Across the wall, the $U(1)_A$ phase of the color-superconducting condensate varies from 0 to 2π. Thus, the same ground state is on both sides of the domain wall and, consequently, the domain wall is metastable. Our proof of the existence and the long lifetime of such domain walls relies on the following three facts: (i) the instanton density is small at large chemical potential, suppressing the effect of chiral anomaly, (ii) the $U(1)_A$ symmetry is spontaneously broken, and (iii) the decay constant of the pseudoscalar singlet boson is large. All these effects are known, from earlier studies, to occur in the color-superconducting phases of QCD. However, as far as we know, their implication for the domain walls has not been explored. These domain walls are similar to the walls of Ref. [3], which may exist, but no definite statement can be made in this case. Asymptotic freedom of QCD allows us to assert the existence of domain walls in the high baryon density regime reliably. The properties of the walls can be determined by controllable weak-coupling calculations.

We will also make a rough estimate of the critical chemical potential above which such domain walls must appear within weak-coupling instanton calculation. This chemical potential is relatively high (though not unreasonably high). In the model of QCD with two light flavors, the critical chemical potential is estimated to be about $6\Lambda_{\rm QCD} \sim 1$ GeV, close to the scale at which instanton interactions become relevant [6]. We also calculate the semiclassical lifetime of the domain wall and find it to be exponentially long.

Domain walls in two-flavor high-density QCD.—The simplest model with high-density domain walls is QCD with $N_f = 2$ massless quark flavors (u and d). This model is a rather good approximation to realistic quark matter at moderate densities, such as in the neutron star interiors. We recall that the ground state of this model at high baryon densities is the two-flavor color-superconducting state [7,8], characterized by the condensation of diquark Cooper pairs. These pairs are antisymmetric in spin (α, β), flavor (i, j), and color (a, b) indices:

$$\langle q_{L\alpha}^{ia} q_{L\beta}^{jb} \rangle^* = \epsilon_{\alpha\beta}\epsilon^{ij}\epsilon^{abc}X^c, \qquad \langle q_{R\alpha}^{ia} q_{R\beta}^{jb} \rangle^* = \epsilon_{\alpha\beta}\epsilon^{ij}\epsilon^{abc}Y^c. \tag{1}$$

The condensates X^c and Y^c are complex color three-vectors. In the ground state, X^c and Y^c are aligned along the same direction in the color space, and they break the color $SU(3)_c$ group down to $SU(2)_c$. The lengths of these vectors are equal, $|X| = |Y|$, and can be computed perturbatively (see below).

In perturbation theory, there is a degeneracy of the ground state with respect to the relative U(1) phase between X^a and Y^a. This is due to the $U(1)_A$ symmetry of the QCD Lagrangian at the classical level. This fact implies that the $U(1)_A$ symmetry is spontaneously broken by the color-superconducting condensate. Since this is a global symmetry, its breaking gives rise to a Goldstone boson, which we denote by η since it carries the same quantum numbers as the η boson in vacuum.

It is possible to construct the field corresponding to η boson explicitly. The following object,

$$\Sigma = XY^{\dagger} \equiv X^{a} Y^{a*}, \tag{2}$$

in contrast to X and Y, is a gauge-invariant order parameter. Furthermore Σ carries a nonzero $\mathrm{U}(1)_{\mathrm{A}}$ charge. Indeed, under the $\mathrm{U}(1)_{\mathrm{A}}$ rotations,

$$q \rightarrow e^{i\gamma_5 \alpha/2} q, \tag{3}$$

the fields (1) transform as $X \rightarrow e^{-i\alpha}X$, $Y \rightarrow e^{i\alpha}Y$, and therefore $\Sigma \rightarrow e^{-2i\alpha}\Sigma$. Thus, the color-superconducting ground state, in which $\langle\Sigma\rangle \neq 0$, breaks the $\mathrm{U}(1)_A$ symmetry. The Goldstone mode η of this symmetry breaking is described by the phase φ of Σ,

$$\Sigma = |\Sigma| e^{-i\varphi}. \tag{4}$$

Under the $\mathrm{U}(1)_{\mathrm{A}}$ rotation (3), φ transforms as

$$\varphi \rightarrow \varphi + 2\alpha. \tag{5}$$

At low energies, the dynamics of the Goldstone mode φ is described by an effective Lagrangian, which, to leading order in derivatives, must take the following form:

$$L = f^2[(\partial_0\varphi)^2 - u^2(\partial_i\varphi)^2]. \tag{6}$$

This Lagrangian (6) contains two free parameters: the decay constant f of the η boson and its velocity u. In general, the velocity u of the η boson may be different from the speed of light (i.e., unity) since the Lorentz invariance is violated by the dense medium. For large chemical potentials $\mu \gg \Lambda_{\mathrm{QCD}}$, the leading perturbative values for f and u have been determined by Beane, *et al.* [9]:

$$f^2 = \frac{\mu^2}{8\pi^2}, \qquad u^2 = \frac{1}{3}. \tag{7}$$

In particular, the velocity of the η bosons, to this order, is equal to the speed of sound. The fact that $f \sim \mu$ plays an important role in our further discussion.

It is well known that the $\mathrm{U}(1)_{\mathrm{A}}$ symmetry is not a true symmetry of the quantum theory, even when quarks are massless. The violation of the $\mathrm{U}(1)_{\mathrm{A}}$ symmetry is due to nonperturbative effects of instantons. Since at large chemical potentials the instanton density is suppressed (see below), the η boson still exists but acquires a finite mass. In other words, the anomaly adds a potential energy term $V_{\mathrm{inst}}(\varphi)$ to the Lagrangian (6),

$$L = f^2[(\partial_0\varphi)^2 - u^2(\partial_i\varphi)^2] - V_{\mathrm{inst}}(\varphi). \tag{8}$$

The curvature of V_{inst} around $\varphi = 0$ determines the mass of the η.

A standard symmetry argument determines periodicity of $V_{\mathrm{inst}}(\varphi)$. One can formally restore the $\mathrm{U}(1)_{\mathrm{A}}$ symmetry by accompanying (3) by a rotation of the θ parameter,

$$\theta \rightarrow \theta + N_f\alpha = \theta + 2\alpha. \tag{9}$$

This symmetry must be preserved in the effective Lagrangian, so the latter is invariant under (5) and (9). This means that the potential V_{inst} is a function of the variable $\varphi - \theta$, unchanged under $\mathrm{U}(1)_{\mathrm{A}}$. Since we know that the physics is periodic in θ with period 2π, we can conclude that, at the physical value of the theta angle $\theta = 0$, V_{inst} is a periodic function of φ with period 2π.

Moreover, at large μ, V_{inst} can be found from instanton calculations explicitly. The infrared problem that plagues these calculations in vacuum disappears at large μ: large instantons are suppressed due to Debye screening [10,11]. As a result, most instantons have small size $\rho \sim \mathcal{O}(\mu^{-1})$ and the dilute instanton gas approximation becomes reliable. One-instanton contribution, proportional to $\cos(\varphi - \theta)$, dominates in V_{inst}. Therefore,

$$V_{\mathrm{inst}}(\varphi) = -a\mu^2\Delta^2\cos\varphi, \tag{10}$$

where Δ is the Bardeen-Cooper-Schrieffer (BCS) gap, and a is a dimensionless function of μ which will be found later. Here we note only that a vanishes in the limit $\mu \rightarrow \infty$. This is an important fact, since it implies that the mass of the η boson,

$$m = \sqrt{\frac{a}{2}}\,\frac{\mu}{f}\,\Delta = 2\pi\sqrt{a}\,\Delta, \tag{11}$$

becomes much smaller than the gap Δ at large μ. In this case the effective theory (8) is reliable, since meson modes other than η have energy of order Δ, i.e., are much heavier than η and decouple from the dynamics of the latter.

The Lagrangian (8) with the potential (10) is just the sine-Gordon model, in which there exist domain-wall solutions to the classical equations of motion. The profile of the wall parallel to the xy plane is

$$\varphi = 4\arctan e^{mz/u}, \tag{12}$$

so the wall interpolates between $\varphi = 0$ at $z = -\infty$ and $\varphi = 2\pi$ at $z = \infty$. The tension of the domain wall is

$$\sigma = 8\sqrt{2a}\,uf\mu\Delta. \tag{13}$$

A good analog of this domain wall is the $N = 1$ axion domain wall, which also interpolates between the same vacuum.

Decay of the domain wall.—It is important to understand the mechanism of the decay of the wall. It has nothing to do with the decay of η meson quanta, which are due to η coupling to photons, ungapped quarks, or the gluons of the unbroken $\mathrm{SU}(2)_c$ subgroup. The domain wall is already a local minimum of the energy, and the decay of its excitations means only that the fluctuations around this minimum, corresponding to deformations of the wall, are damped.

The domain wall is not stable because the same ground state is on both of its sides: $\varphi = 0$ and $\varphi = 2\pi$ are equivalent. The instability is due to higher energy meson modes integrated out and not present in the Lagrangian (8). One can visualize the effect of these modes by considering an effective potential which depends on the magnitude $|\Sigma|$ as well as on the phase φ of the order parameter Σ. This potential has the shape of a Mexican hat, slightly tilted by an angle proportional to a. A similar picture is discussed in Ref. [3], except that the tilt of the hat is very small in our case. The domain wall is a configuration that, as a function of the coordinate perpendicular to the wall, starts from the global minimum, goes along the valley, and returns to the starting point. One can continuously deform this configuration into a trivial constant one by pulling the looplike trajectory over the top of the hat.

As in the case of the axion wall [2,12], this deformation has to be done in a finite area of the wall first, thus creating a hole. If this hole exceeds the critical size, it will expand, destroying the wall. On the rim of the hole the magnitude of $|\Sigma|$ vanishes. The field configuration around the rim is a vortex: on a closed path around the rim, φ changes by 2π. The decay of the wall is a quantum tunneling process in which a hole bounded by a closed circular vortex line is nucleated. The semiclassical probability of this process is

$$\Gamma \sim \exp\left(-\frac{16\pi}{3}\frac{\nu^3}{u\sigma^2}\right), \tag{14}$$

where ν is the tension of the vortex line in the limit of massless boson, $m = 0$. The factor $1/u$ in the exponent of Eq. (14) is due to the fact that u plays the role of light speed for the effective dynamics of the Goldstone boson [see Eq. (6)].

To find Γ we still need to compute the vortex tension ν. Since the vortex is a global string, its tension is logarithmically divergent,

$$\nu = 2\pi u^2 f^2 \ln\frac{R}{R_{\text{core}}} = 2\pi u^2 f^2 \ln(R\Delta), \tag{15}$$

where R is a long-distance cutoff to be specified later, and R_{core} is the size of the core of the vortex line, which is the short-distance cutoff. $R_{\text{core}} \sim 1/\Delta$ since Δ is the momentum scale at which the effective Lagrangian description breaks down. We are helped by the fact that the vortex tension is dominated by the region outside the core, so the effective Lagrangian (6) is sufficient for computing ν to the logarithmic accuracy. By using Eqs. (13) and (15), and taking R to be the thickness of the wall, we find the decay rate to be

$$\Gamma \sim \exp\left(-\frac{\pi^4}{3}\frac{u^3}{a}\frac{f^4}{\mu^2\Delta^2}\ln^3\frac{1}{\sqrt{a}}\right). \tag{16}$$

Since $f \sim \mu \gg \Delta$, and a decreases with increasing μ, the decay rate is exponentially suppressed at high μ. Thus, we have shown that (i) domain walls exist in the limit of large chemical potentials, and (ii) they are metastable with parametrically long lifetime. These conclusions are valid in the regime of very large chemical potentials μ, where our calculations are under control.

Calculation of the potential.—What happens at smaller μ? The most interesting possibility is that the walls persist down to $\mu = 0$ as advocated in Ref. [3] using large-N_c arguments. Another possibility is that, as the description based on the Lagrangian (8) breaks down, the walls disappear. This happens when the mass of the η excitation becomes comparable to 2Δ, the typical energy scale for higher mesons [13]. From Eq. (11), one derives the following condition when our effective Lagrangian description is under control:

$$a(\mu) \lesssim 1/\pi^2. \tag{17}$$

We shall now evaluate the function $a(\mu)$.

To compute $V_{\text{inst}}(\varphi)$, we start from the instanton-induced effective four-fermion interaction [6,14,15],

$$L_{\text{inst}} = \int d\rho\, n_0(\rho)\left(\frac{4}{3}\pi^2\rho^3\right)^2\Big\{(\bar{u}_R u_L)(\bar{d}_R d_L) + \frac{3}{32}\Big[(\bar{u}_R\lambda^a u_L)(\bar{d}_R\lambda^a d_L) - \frac{3}{4}(\bar{u}_R\sigma_{\mu\nu}\lambda^a u_L)(\bar{d}_R\sigma_{\mu\nu}\lambda^a d_L)\Big]\Big\} + \text{H.c.} \tag{18}$$

By taking the average of Eq. (18) over the superconducting state (1), one finds V_{inst}, and confirms that it is proportional to $\cos\varphi$ as in Eq. (10). In the ground state,

$$|X| = |Y| = \frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\frac{\Delta(p_0)}{p_0^2 + (|\mathbf{p}| - \mu)^2 + \Delta^2(p_0)}, \tag{19}$$

where $\Delta(p_0)$ is the momentum-dependent BCS gap. Using the perturbative result [16],

$$\Delta(p_0) = \Delta\cos\left(\frac{g}{3\sqrt{2}\pi}\ln\frac{p_0}{\Delta}\right), \qquad \Delta \lesssim p_0 \lesssim \mu, \tag{20}$$

we find

$$|X| = \frac{3}{2\sqrt{2}\,\pi}\frac{\mu^2\Delta}{g}. \tag{21}$$

Averaging Eq. (18) in the superconducting background, we find, after some calculations,

$$V_{\rm inst}(\varphi) = -\int d\rho\, n_0(\rho)\left(\frac{4}{3}\,\pi^2\rho^3\right)^2 12|X|^2\cos\varphi\,. \tag{22}$$

Using the standard formula for the instanton density at finite chemical potential [11,15]

$$n_0(\rho) = C_N\left(\frac{8\pi^2}{g^2}\right)^{2N_c}\rho^{-5}\exp\left(-\frac{8\pi^2}{g^2(\rho)}\right)e^{-N_f\mu^2\rho^2} \tag{23}$$

with

$$C_N = \frac{0.466e^{-1.679N_c}1.34^{N_f}}{(N_c-1)!\,(N_c-2)!}\,, \tag{24}$$

we arrive at the final result

$$a = 5\times 10^4\left(\ln\frac{\mu}{\Lambda_{\rm QCD}}\right)^7\left(\frac{\Lambda_{\rm QCD}}{\mu}\right)^{29/3}. \tag{25}$$

Thus $a \to 0$ when $\mu \to \infty$, so at sufficiently large μ the criterion (17) is satisfied. However, due to the large numerical constant in Eq. (25), the critical μ is quite high: $\mu_{\rm crit} \sim 6\Lambda_{\rm QCD} \sim 1$ GeV. This result should be taken with some care due to the uncertainty in the criterion (17). However, since a depends quite sensitively on μ, it is reasonable to expect that our estimate is not far from the true value.

In the estimate above we neglected the contribution from large instantons, which arise from the unbroken $SU(2)_c$ sector of the theory. This sector is governed by a pure Yang-Mills theory with the confinement scale $\Lambda'_{\rm QCD} \sim \Delta\exp[-{\rm const}\cdot\mu/(g\Delta)]$ [17]. The nonperturbative contribution of large $SU(2)_c$ instantons is of order $(\Lambda'_{\rm QCD})^4$. Since $\Lambda'_{\rm QCD}$ is exponentially small, this contribution is negligible compared to that from small $SU(3)_c$ instantons.

Inclusion of quark masses does not change the domain walls in a substantial way. The mass contribution to the potential has been found in Ref. [9],

$$V_{\rm mass} = -bm_um_d\Delta^2\cos\varphi\,, \tag{26}$$

where $b \sim 1$. Equation (26) has the same φ dependence as $V_{\rm inst}$, therefore, in all previous formulas one should replace a by $a + bm_um_d/\mu^2$.

Discussion.—It would be interesting to investigate possible astrophysical consequences of the high-density QCD walls. In particular, one would like to know if such walls can be created inside neutron stars. To describe the motion of the wall, one may need more than just the effective Lagrangian (8): the coupling of η to ungapped quarks and $SU(2)_c$ gluons could be important. The moving wall may radiate quark-hole pairs, gluons, or photons, slowing down the collapse of a closed domain-wall surface.

It is possible to generalize our results to the color-flavor-locking state of $N_f = 3$ QCD [18]. The $U(1)_A$ symmetry is also spontaneously broken in this case. The role of the η boson is played by the η' meson, which is also light at high densities [19]. The instanton-induced η' potential has a form similar to (10) [20]:

$$V_{\rm inst}(\varphi) = -a'\cdot(m_s/\mu)\mu^2\Delta^2\cos\varphi\,, \tag{27}$$

where the evaluation of dimensionless function a' is very much the same as our calculation of a and amounts to inserting the extra factor $m_s\rho$ into (22). Thus one expects the domain walls to exist and to be metastable at large μ. Because of the mixing between the neutral mesons [19], the π^0 and η fields are also nontrivial on the wall. The domain walls also exist in QCD with large isospin density [21]. This case is interesting, since it can be studied by a Monte Carlo lattice simulation.

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