R-Parity Violating Contribution to the Neutron Electric Dipole Moment at One-Loop Order

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We present the full result for the down squark mass-squared matrix in the complete theory of supersymmetry without *R* parity where all kinds of *R*-parity violating terms are admitted without bias using an optimal parametrization. The major result is a new contribution to *LR* squark mixing, involving both bilinear and trilinear *R*-parity violating couplings. Among other things, the latter leads to neutron electric dipole moment at one-loop level. Similar mechanisms lead to electron electric dipole moment at the same level. We present here a short report on major features of neutron electric dipole moment from supersymmetry without *R* parity and give the interesting constraints obtained.

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Introduction.—The minimal supersymmetric standard model (MSSM) is no doubt the most popular candidate theory for physics beyond the standard model (SM). The alternative theory with a discrete symmetry called *R* parity not imposed deserves no less attention. A complete theory of supersymmetry (SUSY) without *R* parity, where all kinds of *R*-parity violating (RPV) terms are admitted without bias, is generally better motivated than *ad hoc* versions of RPV theories. The large number of new parameters involved, however, makes the theory difficult to analyze. It has been illustrated [1] that an optimal parametrization, called the single-vacuum-expectation-value (VEV) parametrization (dubbed SVP), can be of great help in making the task manageable. Here in this Letter, we use the formulation to present the full result for the down squark mass-squared matrix. The major result is a new contribution to *LR* squark mixing, involving both bilinear and trilinear RPV couplings. The interesting physics implications of this new contribution are discussed. Among such issues, we focus here on the RPV contribution to neutron electric dipole moment (EDM) at the one-loop level.

Neutron and electron EDM's are important topics for new *CP* violating physics. Within MSSM, studies on the plausible EDM contributions lead to the so-called SUSY-*CP* problem [2]. In the domain of *R*-parity violation, two recent papers focus on the contributions from the trilinear RPV terms and conclude that there is no contribution at the one-loop level [3]. Perhaps it has not been emphasized enough in the two papers that they are *not* studying the complete theory of SUSY without *R* parity. It is interesting to see in the latter case that there is, in fact, contribution at one-loop level, as discussed below. We emphasize again that the new contribution involves both bilinear and trilinear (RPV) couplings. Since various other RPV scenarios studied in the literature typically admit only one of the two types of couplings, the contribution has not been previously identified.

The most general renormalizable superpotential for the supersymmetric SM (without *R* parity) can be written as

$$W = \varepsilon_{ab}[\mu_\alpha \hat{H}_u^a \hat{L}_\alpha^b + h_{ik}^u \hat{Q}_i^a \hat{H}_u^b \hat{U}_k^C + \lambda'_{\alpha jk} \hat{L}_\alpha^a \hat{Q}_j^b \hat{D}_k^C + \frac{1}{2}\lambda_{\alpha\beta k} \hat{L}_\alpha^a \hat{L}_\beta^b \hat{E}_k^C] + \frac{1}{2}\lambda''_{ijk} \hat{U}_i^C \hat{D}_j^C \hat{D}_k^C, \tag{1}$$

where (a, b) are SU(2) indices, (i, j, k) are the usual family (flavor) indices, and (α, β) are extended flavor indexes going from 0 to 3. In the limit where $\lambda_{ijk}, \lambda'_{ijk}, \lambda''_{ijk}$, and μ_i all vanish, one recovers the expression for the *R*-parity preserving case, with $\hat{L}_0$ identified as $\hat{H}_d$. Without *R* parity imposed, the latter is not *a priori* distinguishable from the $\hat{L}_i$'s. Note that λ is antisymmetric in the first two indices, as required by the SU(2) product rules, as shown explicitly here with $\varepsilon_{12} = -\varepsilon_{21} = 1$. Similarly, λ'' is antisymmetric in the last two indices, from $\mathrm{SU}(3)_C$.

R parity preserves the accidental symmetries of baryon number and lepton number in the SM, at the expense of making particles and superparticles having a categorically different quantum number. It is actually not the most effective discrete symmetry to control superparticle mediated proton decay [4].

Doing phenomenological studies without specifying a choice of flavor bases is ambiguous. It is like doing SM quark physics with 18 complex Yukawa couplings, instead of the 10 real physical parameters. As far as the SM itself is concerned, the extra 26 real parameters are simply redundant, and attempts to relate the full 36 parameters to experimental data will be futile. In SUSY without *R* parity, the choice of an optimal parametrization mainly concerns the four $\hat{L}_\alpha$ flavors. Under the SVP, flavor bases are chosen such that (1) among the $\hat{L}_\alpha$'s, only $\hat{L}_0$ bears a VEV, i.e., $\langle\hat{L}_i\rangle \equiv 0$; (2) $h_{jk}^e (\equiv \lambda_{0jk}) = (\sqrt{2}/v_0)\mathrm{diag}\{m_1, m_2, m_3\}$; (3) $h_{jk}^d (\equiv \lambda'_{0jk} = -\lambda_{j0k}) = (\sqrt{2}/v_0)\mathrm{diag}\{m_d, m_s, m_b\}$; (4) $h_{ik}^u = (v_u/\sqrt{2}) V_{\mathrm{CKM}}^T \mathrm{diag}\{m_u, m_c, m_t\}$, where $v_0 \equiv \sqrt{2}\langle\hat{L}_0\rangle$ and $v_u \equiv \sqrt{2}\langle\hat{H}_u\rangle$. The big advantage here is that the (tree-level) mass matrices for *all* the fermions *do not* involve any of the trilinear RPV couplings, though the approach makes *no assumption* on any RPV coupling including even those from soft SUSY breaking; all the parameters used are uniquely defined, with the exception of some removable phases. In fact, the (color-singlet) charged fermion mass matrix is reduced to the simple form

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \frac{g_2 v_0}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{g_2 v_u}{\sqrt{2}} & \mu_0 & \mu_1 & \mu_2 & \mu_3 \\ 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & m_3 \end{pmatrix}. \tag{2}$$

Squark mixing and EDM.—The soft SUSY breaking part of the Lagrangian can be written as follows:

$$\begin{aligned} V_{\rm soft} = {} & \epsilon_{ab} B_\alpha H_u^a \tilde{L}_\alpha^b + \epsilon_{ab}[A_{ij}^U \tilde{Q}_i^a H_u^b \tilde{U}_j^C + A_{ij}^D H_d^a \tilde{Q}_i^b \tilde{D}_j^C + A_{ij}^E H_d^a \tilde{L}_i^b \tilde{E}_j^C] + \text{H.c.} \\ & + \epsilon_{ab}\left[A_{ijk}^{\lambda'} \tilde{L}_i^a \tilde{Q}_j^b \tilde{D}_k^C + \frac{1}{2} A_{ijk}^{\lambda} \tilde{L}_i^a \tilde{L}_j^b \tilde{E}_k^C \right] + \frac{1}{2} A_{ijk}^{\lambda''} \tilde{U}_i^C \tilde{D}_j^C \tilde{D}_k^C + \text{H.c.} \\ & + \tilde{Q}^\dagger \tilde{m}_Q^2 \tilde{Q} + \tilde{U}^\dagger \tilde{m}_U^2 \tilde{U} + \tilde{D}^\dagger \tilde{m}_D^2 \tilde{D} + \tilde{L}^\dagger \tilde{m}_{\tilde{L}}^2 \tilde{L} + \tilde{E}^\dagger \tilde{m}_E^2 \tilde{E} + \tilde{m}_{H_u}^2 |H_u|^2 + \frac{M_1}{2} \tilde{B}\tilde{B} + \frac{M_2}{2} \tilde{W}\tilde{W} + \frac{M_3}{2} \tilde{g}\tilde{g} + \text{H.c.}, \end{aligned} \tag{3}$$

where we have separated the R-parity conserving ones from the RPV ones ($H_d \equiv \hat{L}_0$) for the A terms. Note that $\tilde{L}^\dagger \tilde{m}_{\tilde{L}}^2 \tilde{L}$, unlike the other soft mass terms, is given by a 4×4 matrix. Explicitly, $\tilde{m}_{L_{00}}^2$ is $\tilde{m}_{H_d}^2$ of the MSSM case, while $\tilde{m}_{L_{0k}}^2$'s give RPV mass mixings.

We have illustrated above how the SVP keeps the expressions for the down-quark and color-singlet charged fermion mass matrices simple. The SVP performs the same trick to the corresponding scalar sectors as well. Here we concentrate on the down squarks. We have the mass-squared matrix as follows:

$$\mathcal{M}_D^2 = \begin{pmatrix} \mathcal{M}_{LL}^2 & \mathcal{M}_{RL}^{2\dagger} \\ \mathcal{M}_{RL}^2 & \mathcal{M}_{RR}^2 \end{pmatrix}, \tag{4}$$

where $\mathcal{M}_{LL}^2$ and $\mathcal{M}_{RR}^2$ are the same as in MSSM while

$$(\mathcal{M}_{RL}^2)^T = A^D \frac{v_0}{\sqrt{2}} - m_D \mu_0^* \tan\beta - (\mu_i^* \lambda'_{ijk})_\star \frac{v_u}{\sqrt{2}}. \tag{5}$$

Here m_D is the down-quark mass matrix, which is diagonal under the parametrization adopted; $(\mu_i^* \lambda'_{ijk})_\star$ denotes the 3×3 matrix $()_{jk}$ with elements listed; and $\tan\beta = \frac{v_u}{v_0}$. Note that in the equation for $(\mathcal{M}_{RL}^2)^T$, we can write the first, A term, contribution as

$$A^D \frac{v_0}{\sqrt{2}} = A_d m_D + \delta A^D \frac{v_0}{\sqrt{2}} \tag{6}$$

with A_d being a constant (mass) parameter representing the "proportional" part. The remaining terms in $(\mathcal{M}_{RL}^2)^T$ are F-term contributions; in particular, the last term is a new "SUSY conserving" [5] but R-parity violating contributions. In fact, contributions to LR scalar mixing of this type, for the sleptons, are first identified in a recent paper [6] where their role in the SUSY analog of the Zee neutrino mass model [7] is discussed. In a parallel paper by one of the authors (O.K.) [8], a systematic analysis of the full squark and slepton masses as well as their contributions, through LR mixings, to one-loop neutrino masses are also presented. Note that the full F-term part in the above equation can actually be written together as $(\mu_\alpha^* \lambda'_{\alpha jk})_\star (v_u/\sqrt{2})$ where the $\alpha = 0$ term is the usual μ-term contribution in the MSSM case. The latter is, however, diagonal; i.e., it vanishes for $j \neq k$. We emphasize that the above result is complete—all RPV contributions are included. The simplicity of the result is a consequence of the SVP. Explicitly, the RPV A term contributions [cf., Eq. (6)] vanish as $v_i \equiv \sqrt{2}\langle \hat{L}_i \rangle = 0$.

The $(\mu_i^* \lambda'_{ijk})_\star$ term is very interesting. It involves only parameters in the superpotential and has *nothing to do with soft SUSY breaking*. Without an underlying flavor theory, there is no reason to expect any specific structure among different terms of the matrix. In particular, the off-diagonal terms ($j \neq k$) may have an important role to play. They contribute to flavor changing neutral current processes such as $b \to s\gamma$. Moreover, both the μ_i's and the λ'_{ijk}'s are complex parameters. Hence, diagonal terms in $(\mu_i^* \lambda'_{ijk})_\star$ also bear CP-violating phases and contribute to EDM's of the corresponding quarks. In particular, $\mu_i^* \lambda'_{i11}$ gives the contribution to the neutron EDM at the one-loop level, in exactly the same fashion as the A term in MSSM does. The similar term in LR slepton mixing gives rise to electron EDM. This result is in direct contrast to the impression one may get from reading the two recent papers on the subject [3]. One should bear in mind that the two papers do not put together both the bilinear and the trilinear RPV terms. Our treatment here, based on the SVP, gives, for the first time, the result of squark masses for the complete theory of SUSY without R parity. Going from here, obtaining the EDM contributions is straightforward.

The contribution to the EDM of the d quark at the one-loop level, from a gaugino loop with LR-squark mixing in particular (see Fig. 1), has been widely studied within MSSM [2,9–11]. With the squark mixings in the down sector parametrized by δ_{jk}^D (normalized by the average squark mass as explicitly shown below), we have the neutron EDM result given by

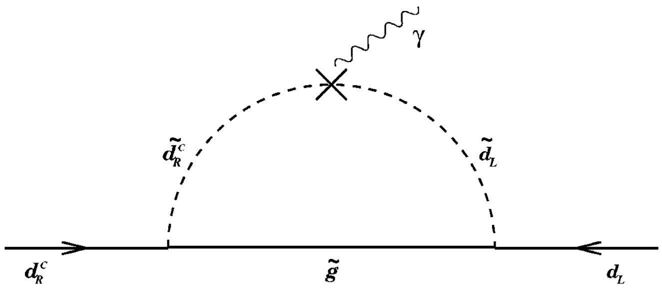

FIG. 1. EDM for d quark at one-loop.

$$d_n = -\frac{8}{27}\frac{e\alpha_s}{\pi}\frac{M_{\tilde{g}}}{M_{\tilde{d}}^2}\,\mathrm{Im}(\delta_{11}^D)F_1\left(\frac{M_{\tilde{g}}^2}{M_{\tilde{d}}^2}\right), \qquad (7)$$

where $M_{\tilde{g}}$ and $M_{\tilde{d}}$ are the gluino and down squark masses, respectively, and

$$F_1(x) = \frac{1}{(1-x)^3}\left(\frac{1+5x}{2} + \frac{2+x}{1-x}\ln x\right). \qquad (8)$$

The contribution of $\mu_i^*\lambda'_{i11}$ to δ_{11}^D is to be given as

$$-\mu_i^*\lambda'_{i11}\frac{v_u}{\sqrt{2}}\frac{1}{M_{\tilde{d}}^2}.$$

Requiring the contribution alone not to upset the experimental bound on neutron EDM: $(d_n)^{\mathrm{exp}} < 6.3\times 10^{-26}e$ cm, a bound can be obtained for the RPV parameters. Note that going from d quark EDM to neutron EDM, we assume the simple valence quark model [12]. Taking $M_{\tilde{d}} = 100$ GeV and $M_{\tilde{g}} = 300$ GeV gives the bound

$$\mathrm{Im}(\mu_i^*\lambda'_{i11}) \le 10^{-6}\ \mathrm{GeV} \qquad (9)$$

(with $v_u \sim 200$ GeV). This result is interesting. Let us first concentrate on the $i = 3$ part, assuming the $i = 1$ and 2 contribution to be subdominating. Imposing the 18.2 MeV experimental bound [13] for the mass of ν_τ still admits a relatively large μ_3, especially for a large $\tan\beta$. Reading from the results in Ref. [1], the bound is $\sim$7 GeV at $\tan\beta = 2$ and $\sim$300 GeV at $\tan\beta = 45$, while the best bound on the corresponding λ'_{311} (from $\tau \to \pi\nu$) is around $0.05 \sim 0.1$ [14].

Here an explicit comparison with the corresponding R-parity conserving contribution is of interest. From Eqs. (5) and (6), it is obvious that we are talking about $(A_d - \mu_0^*\tan\beta)m_d$ versus $-\mu_i^*\lambda'_{i11}(v_u/\sqrt{2})$. Both A_d and μ_0 are expected to be roughly at the same order as v_u. We are hence left to compare m_d with $\mu_i^*\lambda'_{i11}$. The above discussion then leads to the conclusion that the RPV part could easily be larger by one or even 2 orders of magnitude.

On the other hand, if one insists on a sub-eV mass for ν_τ as suggested, but far from mandated, by the result from the Super-Kamiokande (super-K) experiment [15], we would have $\mu_3\cos\beta \le 10^{-4}$ GeV [16]. This means that at least for the large $\tan\beta$ case, the EDM bound as given by Eq. (9) is still worth notification, even under this most limiting scenario.

The $\mu_i^*\lambda'_{i11}$ contribution to squark mixing, as well as λ'_{i11} in itself together with an A-term mixing, also gives rise to neutrino mass at one-loop. Hence, to consistently impose the super-K sub-eV neutrino mass scenario, one should also check the corresponding bound obtained. Figure 2 shows a familiar quark-squark loop neutrino mass diagram. We are interested in the case where both the λ' couplings are λ'_{311}. We have, for the R-parity conserving LR squark mixing, the familiar result

$$m_{\nu_\tau} \sim \frac{3}{8\pi^2}m_d^2\frac{(A_d - \mu_0^*\tan\beta)}{M_{\tilde{d}}^2}\lambda'^2_{311}. \qquad (10)$$

However, with the full LR mixing result as given in Eq. (5), there is an extra contribution to be given as

$$\frac{3}{8\pi^2}m_d\frac{v_u}{\sqrt{2}}\frac{\mu_i^*\lambda'_{i11}}{M_{\tilde{d}}^2}\lambda'^2_{311}. \qquad (11)$$

From Eq. (10), one can easily see that the requirement for m_{ν_τ} to be at the super-K atmospheric neutrino oscillation scale gives only a bound for λ'_{311} of about the same magnitude as the one quoted above, from the other sources. As for the contribution [Eq. (11)], the bound given by Eq. (9) itself says the contribution is smaller than the previous one. Hence, neutrino mass contributions from Fig. 2 do not change our conclusion above.

Note that the EDM bound given by Eq. (9) actually involves a summation over index i. Results from Ref. [1] indicated that while μ_1 is very strongly bounded, the bound on μ_2 could not be very strong. Moreover, the bound on λ'_{211} is no better than that on λ'_{311} [14]. Hence, the EDM bound may still be of interest there too. The story for imposing the super-K constraint is obviously the same as the above discussion for the $i = 3$ case.

One should bear in mind that the EDM and neutrino mass bounds involve different combinations of the RPV, as well as other SUSY, parameters. An exact comparison for the bounds obtained from the two sources is hence difficult. Our discussion above is aimed at illustrating the fact that the EDM bound is not completely overshadowed by the super-K neutrino mass bound. In other words, even requiring the magnitudes for the RPV parameters to satisfy the most stringently interpreted super-K bounds does not make them so small that the above discussed contribution to neutron EDM will always be satisfied.

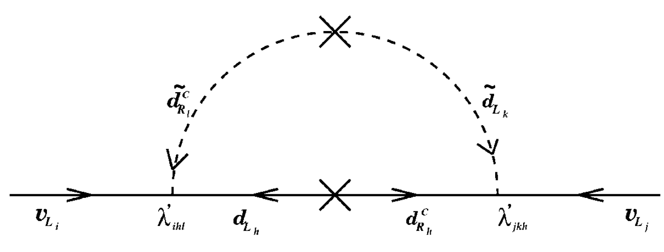

FIG. 2. Neutrino mass at one-loop.

Beyond the gluino diagram.—Similar RPV contributions on the neutron and electron EDM's are obtained through the neutralino exchange diagram. There are other one-loop contributions. In the case of MSSM, the chargino contribution is known to be competitive or even dominates over the gluino one in some regions of the parameter space [9]. Here we give the corresponding formula generalized to the case of SUSY without R parity. This is given by [17]

$$\left(\frac{d_f}{e}\right)_{\chi^-} = -\frac{\alpha_{\rm em}}{4\pi \sin^2\theta_W} \sum_{\tilde{f}'\mp} \sum_{n=1}^{5} {\rm Im}(C_{fn\mp}) \frac{M_{\chi_n^-}}{M^2_{\tilde{f}'\mp}} \times \left[\mathcal{Q}_{\tilde{f}'} B\left(\frac{M^2_{\chi_n^-}}{M^2_{\tilde{f}'\mp}}\right) + (\mathcal{Q}_f - \mathcal{Q}_{\tilde{f}'}) A\left(\frac{M^2_{\chi_n^-}}{M^2_{\tilde{f}'\mp}}\right)\right], \tag{12}$$

for f being u (d) quark and f' being d (u), where

$$\begin{aligned} C_{un\mp} &= \frac{y_u}{g_2} \mathbf{V}^*_{2n} \mathcal{D}_{d1\mp}\left(\mathbf{U}_{1n}\mathcal{D}^*_{d1\mp} - \frac{y_d}{g_2}\mathbf{U}_{2n}\mathcal{D}^*_{d2\mp} - \frac{\lambda'_{i11}}{g_2}\mathbf{U}_{(i+2)n}\mathcal{D}^*_{d2\mp}\right), \\ C_{dn\mp} &= \left(\frac{y_d}{g_2}\mathbf{U}_{2n} + \frac{\lambda'_{i11}}{g_2}\mathbf{U}_{(i+2)n}\right)\mathcal{D}_{u1\mp}\left(\mathbf{V}^*_{1n}\mathcal{D}^*_{u1\mp} - \frac{y_u}{g_2}\mathbf{V}^*_{2n}\mathcal{D}^*_{u2\mp}\right). \end{aligned} \tag{13}$$

The terms in $C_{dn\mp}$ with only one factor of $\frac{1}{g_2}$ and a λ'_{i11} gives the RPV analog of the dominating MSSM chargino contribution. The term is described by a diagram, which at first order requires a $l^-_{L_i}$–$\tilde{W}^+$ mass mixing. The latter vanishes, as shown in Eq. (2). From the full formula above, it is easy to see that the exact mass eigenstate result would deviate from zero only to the extent that the mass dependence of the B and A functions [17] spoils the GIM like cancellation in the sum. The resultant contribution, however, is shown by our exact numerical calculation to be substantial. What is most interesting here is that an analysis through perturbational approximations illustrates that the contribution is proportional to, basically, the same combination of RPV parameters, i.e., $\mu_i^* \lambda'_{i11}$. While we cannot give much of the details here (see Ref. [17]), let us list numbers from a sample point for illustration: with $A_u = A_d = 500$ GeV, $\mu_0 = -300$ GeV, $\tan\beta = 3$, a common gaugino masses at 300 GeV, $\tilde{m}_Q = 200$ GeV, $\tilde{m}_u = \tilde{m}_d = 100$ GeV, $\mu_3 = 1 \times 10^{-4}$ GeV, and $\lambda'_{311} = 0.1 \times \exp(i\pi/6)$ (being the only complex parameter), we have the resulting neutron EDM contributions from gluino, chargino(-like), and neutralino(-like) one-loop diagrams given by 2.49, 0.56, and -0.056 times $10^{-27} e$ cm, respectively.

Summary.—In summary, we have presented the complete result for LR squark mixing and analyzed its contribution to neutron EDM through the gluino diagram. The result provide interesting new bounds on RPV parameters. A brief discussion for the chargino(-like) one-loop contribution is also given, together with a sample result from exact numerical calculations, including also the neutralino(-like) loop.

There is also the analogous case for the slepton mixing and electron EDM. The issue is under investigation.

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