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Signal Velocity, Causality, and Quantum Noise in Superluminal Light Pulse Propagation

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We consider pulse propagation in a linear anomalously dispersive medium where the group velocity exceeds the speed of light in vacuum (c) or even becomes negative. A signal velocity is defined operationally based on the optical signal-to-noise ratio, and is computed for cases appropriate to the recent experiment where such a negative group velocity was observed. It is found that quantum fluctuations limit the signal velocity to values less than c.

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It is well known that the group velocity v_g of a light pulse can exceed c in an anomalously dispersive medium [1–4]. If there is no appreciable loss or gain and the dispersion is linear, an incident pulse described by a sufficiently smooth envelope (an analytic signal) $E(t)$ becomes simply $E(t - L/v_g)$ after propagating a distance L. As discussed many years ago by Sommerfeld and Brillouin [1], a group velocity greater than c does not violate causality because it is not the velocity of signal transmission. They noted that the "front velocity," the velocity at which an infinitely sharp step-function-like disturbance of the light intensity propagates, should be used as the velocity of information transmission [1,3,4].

On the other hand, for a smoothly varying pulse that is an analytic continuation of the incident pulse $E(t - L/c)$ [5], the signal velocity is not well defined. Because an analytic signal is entirely determined by its very small leading edge, there is no new information being carried by the peak. Furthermore, this leading edge of the pulse can in principle extend infinitely far back in time, making it impossible to assign a point marking the onset of a signal.

These considerations are not immediately applicable in the laboratory. There is first of all the impossibility in principle of realizing the infinite bandwidth associated with a step-function "front." But more subtle questions arise from the fact that a tiny leading edge of a smooth pulse determines the entire pulse. For one thing, it is not obvious how to define the "arrival time" of the signal [6]. In practice, one cannot extend the "arrival time" to any time before the detection of the *first* photon. Furthermore, if the tiniest leading edge of a smooth "superluminal" pulse determines the entire pulse, we must account for the effect that quantum fluctuations at the leading edge might have on the detection of the pulse [7,8].

We suggest here an operational definition of the signal velocity and apply it to the recently observed superluminal propagation of a light pulse in a gain medium [9]. This experiment showed not only that a superluminal group velocity is possible without any significant pulse distortion, but also demonstrated that this can occur with no appreciable absorption or amplification [9,10]. Previous considerations of quantum noise in this context focused on the motion of the peak of a wave packet, and on the observability of the superluminal velocity of the peak at the one- or few-photon level [7,8]. Here we show, based on operation definition of signal velocity, that quantum noise associated with the amplifying medium acts in effect to retard the observed signal. Hence, in order to achieve a given signal-to-noise ratio (SNR) at the output of an amplifying medium, a larger signal is required, resulting in a retardation of the signal. This retardation is found in numerical simulation to be larger than the propagation time reduction due to anomalous dispersion, leading to a signal velocity $\leq c$. The operational definition given and the conclusions reached here are independent of the intensity of the input pulse.

The experimental situation of interest is illustrated in Fig. 1 [9]. A gas of atoms with a Λ-type transition scheme is optically pumped into state $|1\rangle$. Two cw Raman pump beams tuned off-resonance from the $|1\rangle \rightarrow |0\rangle$ transition with a slight frequency offset $2\Delta\omega$, and a pulsed probe beam acting on the $|0\rangle \rightarrow |2\rangle$ transition, propagate collinearly through the cell. The common detuning Δ_0 of the Raman and probe fields from the excited state $|0\rangle$ is much larger than any of the Rabi frequencies or decay rates involved, so that we can adiabatically eliminate all off-diagonal density-matrix terms involving state $|0\rangle$. Then we obtain the following expression for the linear susceptibility as a function of the probe frequency [9,10]:

$$\chi(\omega) = \frac{M}{\omega - \Delta\omega + i\gamma} + \frac{M}{\omega + \Delta\omega + i\gamma}, \quad (1)$$

where $\gamma > 0$ and $M > 0$ is a two-photon matrix element whose detailed form and numerical value are not required for our present purposes. We note only that the dispersion relation (1) satisfies the Kramers-Kronig relations and therefore that a medium described by it is causal.

Consider now the detection of a signal corresponding to a light pulse as indicated in Fig. 1(a). In a binary state communication scheme, we assign a time window T centered about a prearranged time t_0 at the detector and monitor the photocurrent produced by the detector. In general, there is a background level of irradiation that causes a constant average photocurrent i_0 even when no light pulse is transmitted; a nonvanishing i_0 due to spontaneous emission exists whenever the medium exhibits gain. An increased photocurrent $i_1(t)$ is registered when a light pulse is received, and we assert that a signal has been received when the integrated photocurrent $\int dt\, i_1(t)$ rises above the background level by a certain prescribed factor. The time at which this preset level of confidence is reached is then defined to be the time of arrival of this signal as recorded by an ideal detector.

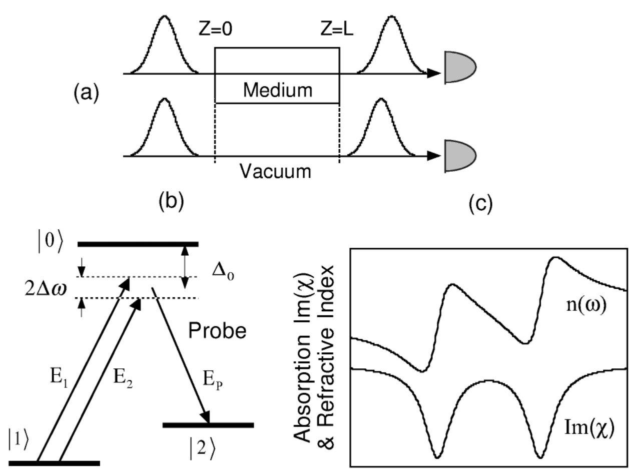

FIG. 1. (a) Schematic of the setup to create transparent anomalous dispersion; (b) atomic transition scheme for double-peaked Raman amplification; (c) refractive index and gain coefficient as a function of probe beam frequency.

The observable corresponding to this definition of the arrival time is the integrated photon number

$$\hat{S}(L,t) = \eta \int_{t_0 - T/2}^{t} dt_1\, \hat{E}^{(-)}(L,t_1)\hat{E}^{(+)}(L,t_1), \quad (2)$$

where $\hat{E}^{(+)}(L,t_1)$ and $\hat{E}^{(-)}(L,t_1)$ are, respectively, the positive- and negative-frequency parts of the reduced electric field operator at the exit point ($z = L$) of the medium. $t_0 = T_c + L/c$ where T_c is the time corresponding to the pulse peak. $T/2$ is half the time window assigned to the pulse, typically a few times the pulse width. η is a constant containing the quantum efficiency, and will be taken as unity for the rest of the analysis. The expectation value $\langle \hat{S}(L,t)\rangle$ is proportional to the number of photons that have arrived at the detector at the time t. If $\langle \hat{S}_1(L,t)\rangle$ and $\langle \hat{S}_0(L,t)\rangle$ are, respectively, the expectation values of $\hat{S}(L,t)$ with and without an input pulse, then the photocurrent difference for an ideal detector is $\langle \hat{S}_1(L,t)\rangle - \langle \hat{S}_0(L,t)\rangle$. Since the second-order variance of the integrated photon number, $\langle \Delta^2 \hat{S}(L,t)\rangle$, characterizes the noise power due to quantum fluctuations, we define an optical signal-to-noise ratio in accord with standard signal detection practice [11]

$$\mathrm{SNR}(L,t) = \frac{(\langle \hat{S}_1(L,t)\rangle - \langle \hat{S}_0(L,t)\rangle)^2}{\langle \Delta^2 \hat{S}(L,t)\rangle}. \quad (3)$$

As discussed above, we define the arrival time t_s of a signal as the time at which $\mathrm{SNR}(L,t)$ reaches a prescribed threshold level determined by the allowed error rate.

The positive-frequency part of the reduced electric field operator can be written as

$$\hat{E}^{(+)}(z,t) = \frac{e^{-i\omega_o(t - z/c)}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega\, \hat{a}(\omega) e^{-i\omega(t - z/v_g)}, \quad (4)$$

where ω_o is the carrier frequency of the pulse, and $[\hat{a}(\omega), \hat{a}^\dagger(\omega')] = \delta(\omega - \omega')$. Equation (4) assumes plane-wave propagation in the z direction and that the group-velocity approximation is valid.

In the experiment of interest the anomalously dispersive medium is a phase-insensitive linear amplifier for which [12]

$$\hat{a}_{\mathrm{out}}(\omega) = g(\omega)\hat{a}_{\mathrm{in}}(\omega) + \sqrt{|g(\omega)|^2 - 1}\, \hat{b}^\dagger(\omega), \quad (5)$$

where $\hat{a}_{\mathrm{in}}$ and $\hat{a}_{\mathrm{out}}$ refer, respectively, to the input ($z = 0$) and output ($z = L$) ports of the amplifier and the operator $\hat{b}(\omega)$ is a bosonic operator $[[\hat{b}(\omega), \hat{b}^\dagger(\omega')] = \delta(\omega - \omega')]$ that commutes with all operators $\hat{a}_{\mathrm{in}}(\omega)$ and $\hat{a}_{\mathrm{in}}^\dagger(\omega)$

and whose appearance in Eq. (5) is required among other things to preserve the commutation relations for the field operators $\hat{a}_{\text{out}}$ and $\hat{a}^{\dagger}_{\text{out}}$. $|g(\omega)|^2$ is the power gain factor given by Eq. (1).

Now we derive a rather general expression for the optical signal-to-noise ratio. Consider first the case of propagation over the distance L in a vacuum where $g(\omega) = 1$. We assume that the initial state $|\psi\rangle$ of the field is a coherent state such that $\hat{a}(\omega)|\psi\rangle = \alpha(\omega)|\psi\rangle$ for all ω, where $\alpha(\omega)$ is a c number. For such a state we may write $\hat{E}^{(+)}(0,t)|\psi\rangle = \alpha(t)e^{-i\omega_0 t}|\psi\rangle$, where $\alpha(t) \equiv \pi^{-1/4}(N_p/\tau)^{1/2}\exp[-(t - T_c)^2/2\tau^2]$, N_p is the average number of photons in the initial pulse of duration τ. We obtain after a straightforward calculation that

$$\text{SNR}_{\text{vac}}(L,t) = \langle \hat{S}_1(L,t)\rangle_{\text{vac}} = \text{SNR}_{\text{vac}}(0, t - L/c)\,. \tag{6}$$

Clearly, the point $\text{SNR}_{\text{vac}}(L,t) = \text{const}$ propagates at the velocity c without excess noise.

Next we treat the case of pulse propagation over the distance L in the anomalously dispersive medium, using Eq. (5) with $g(\omega) \neq 1$ and the same initially coherent field. We obtain in this case

$$\langle \hat{S}_1(L,t)\rangle - \langle \hat{S}_0(L,t)\rangle = |g(0)|^2\langle \hat{S}_1(0, t - L/v_g)\rangle_{\text{vac}}\,, \tag{7}$$

where $\langle \hat{S}_0(L,t)\rangle = (1/2\pi)\int_{t_0 - T/2}^{t} dt_1 \int d\omega\,[|g(\omega)|^2 - 1]$ is the photon number in the absence of any pulse input to the medium. The fact that $\langle \hat{S}_0(L,t)\rangle > 0$ is due to amplified spontaneous emission (ASE) [11]; in the experiment of interest the ASE is due to a spontaneous Raman process.

For a probe pulse with sufficiently small bandwidth, the gain factor becomes

$$|g(0)|^2 = e^{4\pi M\gamma/(\Delta\omega^2 + \gamma^2)\cdot L/\lambda}, \tag{8}$$

and the effective signal $\langle \hat{S}_1(L,t)\rangle - \langle \hat{S}_0(L,t)\rangle$ is proportional to the input signal $\langle \hat{S}_1(0, t - L/v_g)\rangle_{\text{vac}}$ with time delay L/v_g determined by the group velocity v_g. In the anomalously dispersive medium $v_g = c/(n + \omega dn/d\omega)$ and can be $>c$ or even negative, resulting in a time delay

$$\frac{L}{v_g} = \left[1 - \omega_0 M\,\frac{\Delta\omega^2 - \gamma^2}{(\Delta\omega^2 + \gamma^2)^2}\right]\frac{L}{c}\,, \tag{9}$$

which is shorter than the time delay the pulse would experience upon propagation through the same length in vacuum, or can become negative. In other words, the effective signal intensity defined here can be reached sooner than in the case of propagation in vacuum.

In order to determine with confidence when a signal is received, however, one must evaluate the SNR. Again using the commutation relations for the field operators, we obtain for the fluctuating noise background

$$\begin{aligned}\langle \Delta^2\hat{S}(L,t)\rangle \equiv \langle \hat{S}^2(L,t)\rangle - \langle \hat{S}(L,t)\rangle^2 = {}& |g(0)|^2\langle \hat{S}_1(0, t - L/v_g)\rangle_{\text{vac}} + \langle \hat{S}_0(L,t)\rangle \\ &+ 2|g(0)|^2\,\text{Re}\left[\int_{t_0 - T/2}^{t} dt_1 \int_{t_0 - T/2}^{t} dt_2\,\alpha^*(t_1 - L/v_g)\alpha(t_2 - L/v_g)F(t_1 - t_2)\right] \\ &+ \int_{t_0 - T/2}^{t} dt_1 \int_{t_0 - T/2}^{t} dt_2 |F(t_1 - t_2)|^2.\end{aligned} \tag{10}$$

Here

$$F(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\,[|g(\omega)|^2 - 1]e^{-i\omega t} \tag{11}$$

is a correlation function for the amplified spontaneous emission noise. The four terms in Eq. (10) can be attributed to amplified shot noise, spontaneous emission noise, beat noise, and ASE self-beat noise, respectively [13]. Figure 2 shows the evolution of these noise terms within the time window T. Clearly, amplified shot noise dominates when the input pulse is strong.

Using Eqs. (7) and (10), we compute $\text{SNR}_{(\text{med})}(L,t)$ for the propagation through the anomalously dispersive medium. In Fig. 3 we plot the results of such computations for $\text{SNR}_{(\text{med})}(L,t)$ as a function of time on the output signal. For reference we also show SNR for the identical pulse propagating over the same length in vacuum. It is evident that the pulse propagating in vacuum always maintains a higher SNR. In other words, for the experiments of interest here [9,10], the signal arrival time defined here is

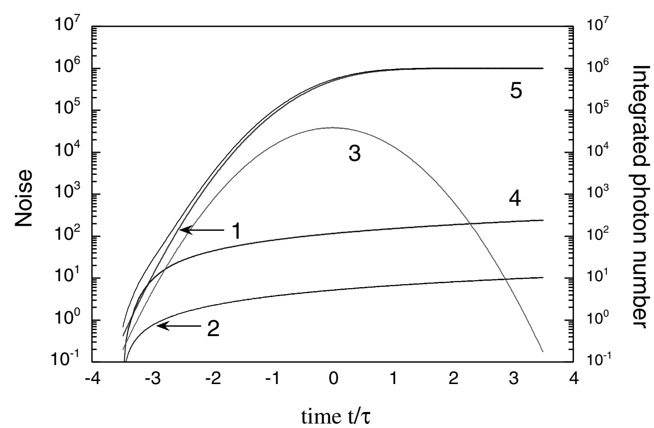

FIG. 2. Evolution of quantum noise terms. Curves 1 to 5 indicate noise associated with terms 1 to 4 in Eq. (10), and the total noise, respectively. Parameters used in the figure are adopted from the experiments reported in Refs. [9] and [10]. There are 10^6 photons per pulse. Noise retards the detection of the signal by reducing the SNR.

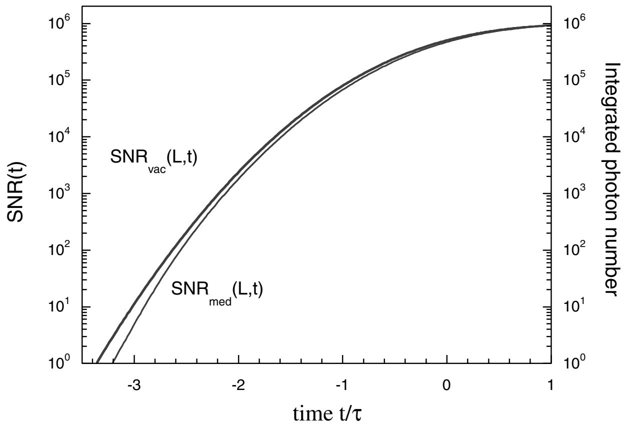

FIG. 3. Signal-to-noise ratios for light pulses propagating through the gain-assisted anomalous dispersion medium $\mathrm{SNR}_{\mathrm{med}}(L,t)$, and through the same distance in a vacuum $\mathrm{SNR}_{\mathrm{vac}}(L,t)$.

delayed, even though the pulse itself is advanced compared with propagation over the same distance in vacuum.

To further examine the signal velocity, we require that at a time t' the SNR of a pulse propagating through the medium be equal to that of the same pulse propagating through a vacuum at a time t:

$$\mathrm{SNR}_{(\mathrm{med})}(L,t') = \mathrm{SNR}_{(\mathrm{vac})}(L,t)\,. \tag{12}$$

Hence, we obtain a time difference $\delta t = t' - t$ that marks the retardation due to quantum noise. $\Delta t = t' - t + L/c$ gives the propagation time of the light signal, and $L/\Delta t$ gives the signal velocity. In Fig. 4 we plot $\delta t = t' - t$ as a function of gain for $(t - T_c)/\tau = -3$ and -1. This corresponds to cases where the signal point is set at 3 and 1 times the pulse width on the leading edge of the pulse. We also plot for reference the pulse advance L/v_g. It is evident that the retardation in the SNR far exceeds the pulse advance. In other words, the quantum noise added in the process of advancing a signal effectively impedes the detection of the useful signal defined by the signal-to-noise ratio.

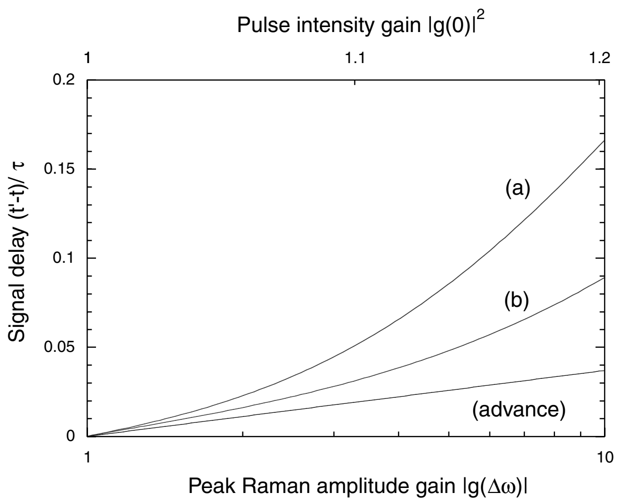

FIG. 4. Delay in signal arrival time $\delta t = t' - t$ as a function of the gain coefficient. Curves (a) and (b) are for $(t - T_c)/\tau = -3$, and -1, respectively. Curve (a) is delayed more because at the early stage of the pulse, ASE-self-beat noise produces noise much greater than the shot noise level.

In this Letter, we have presented what in our opinion is a realistic definition, based on photodetection, of the velocity of the signal carried by a light pulse. We analyzed this signal velocity for the recently demonstrated superluminal light pulse propagation, and found that while the pulse and the effective signal are both advanced via propagation at a group velocity higher than c, or even negative, the signal velocity defined here is still bounded by c. The physical mechanism that limits the signal velocity is quantum fluctuation. Namely, because the transparent, anomalously dispersive medium is realized using closely placed gain lines, amplified quantum fluctuations introduce additional noise that effectively reduces the SNR in the detection of the signals carried by the light pulse. This is related to the "no cloning" theorem [14,15], which was attributed to the quantum fluctuations in an amplifier, and which is a direct consequence of the superposition principle in quantum theory.

Finally, we note that it is perhaps possible to find other definitions of a "signal" velocity for a light pulse, different from that we presented here. But such a definition should in our opinion satisfy two basic criteria. First, it must be directly related to a known and practical way of detecting a signal. Second, it should refer to the *fastest* practical way of communicating information. While it may be hard to prove that any definition meets the second criterion, it can be hoped that the recent interest in quantum information theory might lead to a generally accepted notion of the signal velocity of light pulses.

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