Observability of Quantum Phase Fluctuations in Cuprate Superconductors

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We study the order parameter phase fluctuation effects in cuprate superconductors near T = 0, using a quasi-two-dimensional *d*-wave BCS model. An effective phason theory is obtained which is used to estimate the strength of the fluctuations, the fluctuation correction to the in-plane penetration depth, and the pair-field susceptibility. We find that, while the phase fluctuation effects are difficult to observe in the renormalization of the superfluid phase stiffness, they may be observed in a pair tunneling experiment which measures the pair-field susceptibility.

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Underdoped cuprates exhibit large deviations from the predictions of BCS mean-field theory, including a large gap Δ which does not scale with the transition temperature T_c , and a "pseudogap" feature in the normal state [1]. These facts, together with the empirical scaling of the superfluid phase stiffness (SPS) and T_c with doping [2], have motivated a picture of the cuprates in which the discrepancies are attributed to strong phase fluctuations [3]. The estimated phase fluctuation energy scale is smaller than the gap (pairing) energy scale in these materials, and within this picture T_c is determined by the small SPS rather than the gap energy. The pseudogap is ascribed to a precursor pairing amplitude whose phase coherence is destroyed above T_c [4].

As many other explanations for these effects have been put forward, it is of great interest to devise tests of the phase fluctuation scenario which distinguish it from others. Some evidence for *thermal* phase fluctuation effects was provided by Corson *et al.* [5], who observed unusual conductivity resonances in underdoped Bi₂Sr₂CaCu₂O_{8+ δ} (Bi2212) near T_c and analyzed their data using twodimensional Kosterlitz-Thouless-Berezinskii dynamics in the terahertz range. It has been claimed that phase fluctuations are the dominant excitations even at low temperatures $T \ll T_c$ [6], determining the well-known linear-*T* dependence of the penetration depth [7]. However, Millis *et al.* [8] have shown that *quantum* phase fluctuations cannot account for this behavior. Quantum phase fluctuations do have important consequences for the superconductor-insulator transition [9], and the c-axis optical conductivity [10]. The above probes provide indirect observation of the phase fluctuation effects through electronic observables which are modified by the phase fluctuations. In this paper, we search for more direct probes of phase fluctuations within a model of quasi-two-dimensional d-wave BCS superconductors with interlayer Josephson coupling near optimal doping.

The in-plane SPS can be expressed as $D_{ab} =$ $n_{s,ab}\hbar^2/4md$ where $n_{s,ab}$ is the planar superfluid electron density, m is the effective mass of the quasiparticle, and d is the interplanar spacing. Because D_{ab} is determined by the quasiparticle properties which are not strongly renormalized by the phase fluctuations, we find that although the renormalization of the Debye-Waller factor $\langle e^{i\phi} \rangle$ is relatively strong, both the SPS at T = 0 and its temperature corrections are weakly renormalized, in contrast to the case of a Josephson junction array (JJA) model [6,11]. We also consider an experiment to measure the excess current in a tunnel junction, which can be directly related to the pair-field susceptibility [12] $\chi = -i\theta(t)\langle [\Delta(\mathbf{r},t), \Delta^{\dagger}(\mathbf{0},0)] \rangle$. For the cuprates we find that this current is experimentally observable, due to the combination of large phase fluctuations and a low-lying *c*-axis plasmon mode. We predict a pronounced peak in the excess current at the *c*-axis Josephson plasma frequency ω_c .

We begin with a continuum BCS model with *d*-wave pairing symmetry in an isolated two-dimensional layer at temperature $T = 1/\beta$ in the superconducting state:

$$S_{2D} = \int_{0}^{\beta} d\tau \left\{ \int d^{2}x \sum_{\sigma} c_{\sigma}^{\dagger} \left(\partial_{\tau} - \frac{\nabla^{2}}{2m_{0}} - \mu \right) c_{\sigma} + \int d^{2}R d^{2}r \left[\Delta(\mathbf{R}, \mathbf{r}, \tau)c_{\uparrow}^{\dagger}(\mathbf{R} + \mathbf{r}/2, \tau)c_{\downarrow}^{\dagger}(\mathbf{R} - \mathbf{r}/2, \tau) + \text{H.c.} + \frac{1}{g} \left| \Delta(\mathbf{R}, \mathbf{r}, \tau) \right|^{2} \right] \right\}, \quad (1)$$

with the interaction strength g > 0. Here and throughout the paper we set $\hbar = c = k_B = 1$ for convenience except when numerical values are estimated. The gap has the property that $\int d^2r e^{-i\mathbf{p}\cdot\mathbf{r}}\Delta(\mathbf{R},\mathbf{r},\tau) = \Delta(\mathbf{R},\tau) (p_x^2 - p_y^2)/p^2$. Then we factor the pairing field

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as $\Delta(\mathbf{x}, \tau) = |\Delta(\mathbf{x}, \tau)| e^{i\phi(\mathbf{x}, \tau)}$; in what follows we will assume that the amplitude of the order parameter is constant, $|\Delta(\mathbf{x}, \tau)| = \Delta$, and focus on the phase degree of freedom, ϕ . In order to decouple the ϕ field from the order parameter amplitude, we perform a singular gauge transformation

 $\psi_{\sigma}(\mathbf{x}, \tau) = c_{\sigma}(\mathbf{x}, \tau)e^{-i\phi(\mathbf{x}, \tau)/2}$, with ψ_{σ} the field operators for the transformed quasiparticle. The phasequasiparticle coupling terms are then

$$S_{I} = \int d^{2}x \int_{0}^{\beta} d\tau \left[\frac{\nabla \phi}{2} \cdot \hat{\psi}^{\dagger} \frac{i}{m} \nabla \hat{\psi} + \frac{(\nabla \phi)^{2}}{8m} \hat{\psi}^{\dagger} \hat{\tau}_{3} \hat{\psi} \right],$$
(2)

where $\hat{\psi}$ is the Nambu spinor. The wave vector of the phase fluctuations has an upper cutoff Λ_c of order $\xi_0^{-1} \sim \Delta/v_F$ since beyond this momentum scale the mean-field assumption breaks down. Here we consider only the effect of longitudinal phase fluctuations since the production of vortex pairs is energetically unfavorable near T = 0and far away from the insulator transition. In order to study a realistic model, we consider such layers of twodimensional superconductors with an interlayer distance d and a weak Josephson tunneling (J) between adjacent layers, and a three-dimensional Coulomb interaction $V(\mathbf{q}) = 4\pi e^2/\epsilon_b q^2$, where $\mathbf{q} = (\mathbf{q}_{\parallel}, q_{\perp})$, with \mathbf{q}_{\parallel} and q_{\perp} the in-plane and c-axis components of \mathbf{q} , and ϵ_b the background dielectric constant. After integrating out the fermions we can obtain the effective phase-only action; the Gaussian term is

$$S^{(2)}[\phi] = T \sum_{\omega_n, \mathbf{q}} \frac{\omega_n^2 + \omega_p^2(\mathbf{q})}{8V(\mathbf{q})} \phi(\mathbf{q}, \omega_n) \phi(-\mathbf{q}, -\omega_n)$$
(3)

with $\omega_n = 2\pi nT$ the bosonic Matsubara frequencies. The plasma frequency ω_p is defined through $\omega_p^2(\mathbf{q}) = (\omega_{ab}^2 q_{\parallel}^2 + \omega_c^2 q_{\perp}^2)/(q_{\parallel}^2 + q_{\perp}^2)$ where $\omega_{ab} =$ $\sqrt{4\pi n_{ab}e^2/\epsilon_b m d}$, n_{ab} is the planar charge-carrier density at the plasma resonance frequency, and $\omega_c = \sqrt{4\pi J d^2 e^2 / \epsilon_b}$ is the *c*-axis Josephson plasma energy. Equation (3) gives the correct plasma spectrum for a layered superconductor, with ω_{ab} the planar plasma frequency at T = 0 which is 0.44–1.4 eV [13], and ω_c is the c-axis plasma frequency which is about 0.6 meV in Bi2212 [14] (larger in other materials), with an interlayer distance $d \approx 1.5$ nm and a dielectric constant of $\epsilon_b \approx 10$ [14]. The in-plane SPS $D_{ab} = n_{s,ab}/4md$ can be read off from Eq. (3) as the coefficient of $(\nabla_{ab}\phi)^2/2$ in the $\omega \to 0$ and $|\mathbf{q}| \to 0$ limits. The quasiparticle damping term, which will broaden the plasma mode in Eq. (3), has been omitted, but the effect is known to be small even in the case of the *d*-wave quasiparticle spectrum [14,15], and it will not affect the order of magnitude estimate of the correction to the SPS in what follows.

We first estimate the strength of the phase fluctuation from Eq. (3). One measure of the strength is the Debye-Waller factor $\alpha = e^{-\langle \phi^2 \rangle}$, which for instance can be determined from the *c*-axis optical conductivity [10]. For our model, at T = 0 we have

$$\langle \phi^2 \rangle = \sum_{\mathbf{q}} \frac{2V(\mathbf{q})}{\hbar \omega_p(\mathbf{q})} \approx \frac{2\Lambda_c e^2/\epsilon_b}{\hbar \omega_{ab}}.$$
 (4)

From this expression we see that the size of the quantum phase fluctuations is determined by the ratio of the Coulomb energy of a Cooper pair to the plasma energy; in the cuprates the short coherence lengths and small superfluid densities conspire to enhance these fluctuations. For instance, assuming $\xi_0 \approx 20$ Å and with the parameters given above, we estimate from Eq. (4) that $\langle \phi^2 \rangle$ ranges from 0.1 to 1, which is a sizable number compared to, for instance, 10^{-3} in Pb. In this paper we study the effect of these strong phase fluctuations in the BCS model given in Eq. (1), which is more appropriate near optimal doping, far away from the insulator transition.

Since we are interested in the renormalization of the SPS, we need to go beyond the quadratic expansion in Eq. (3). For instance, in the JJA model, the effective SPS can be substantially renormalized due to the non-trivial potential of the form $\cos(\phi_i - \phi_j)$ [11]. In our model, higher-order terms can be determined by expanding Eq. (2) and integrating out the fermions with a *d*-wave gap. Each *n*-point vertex of the $\nabla \phi$ field is a fermion loop (see Fig. 1). Therefore, the renormalization of the SPS and its temperature dependence is determined by the magnitudes of the fermion loops. From the new effective theory of the phase fields thus obtained,

$$S_{\text{eff}}[\phi] = \sum_{n} \int d^{4}x_{1} \cdots \int d^{4}x_{2n} \Gamma^{(2n)}(x_{1}, x_{2}, \dots, x_{2n}) \\ \times \nabla_{1}\phi(x_{1})\nabla_{2}\phi(x_{2}) \cdots \nabla_{2n}\phi(x_{2n}),$$
(5)

where $\Gamma^{(2n)}$ are the 2*n*th-order phason vertices, we can estimate the renormalization of the in-plane SPS by using the one loop expansion in ϕ as in Fig. 2. At T = 0, we can show that the diagrams in Figs. 2(a), 2(b), and 2(c) cancel one another in the limit of zero external momentum and frequency by using the identity $i\partial_{\omega}\hat{G}(\mathbf{p},\omega) = \hat{G}^2(\mathbf{p},\omega)$ and performing the integration by parts in ω , where $\hat{G}(\mathbf{p},\omega)$ is a Gorkov Greens function. Consequently, the only contribution comes from 2(d). The correction to the SPS is found to be

$$\frac{\delta D_{ab}}{D_{ab}} \approx \frac{e^2 \Lambda_c^3}{6n_{s,ab} \epsilon_b \hbar \omega_{ab}}.$$
 (6)

Assuming that the in-plane penetration depth is $\lambda_{ab} = \sqrt{mc^2 d/4\pi n_{s,ab}e^2} \approx 2000 \text{ Å}$, we estimate that $\delta D_{ab}/D_{ab} \leq 10^{-1}$. This should be compared to a 40% reduction obtained using the JJA model [11]. In our model, the phason vertices are determined by the *d*-wave



FIG. 1. The schematic diagram of the quasiparticle-phase field coupling and the effective action of the phase. The solid lines represent the quasiparticle and the dashed lines the phase field.



FIG. 2. Correction to the superfluid phase stiffness in one-loop expansion in the phase field.

quasiparticle fermion loops, which are smaller than the vertices of a JJA model; consequently the renormalization of the SPS is smaller.

Next we study the temperature dependence of the SPS. The BCS theory gives a linear temperature dependence due to thermal excitations of quasiparticles, $D_{ab}(T) \approx D_{ab}(0) - aT$ where $a = v_F \ln(2)/4\pi v_\Delta d$. Here v_Δ is the slope of the gap at the node in momentum space. We find that the one-loop correction to a is $\delta a/a \approx \frac{e^2 v_F \Lambda_c^2}{\pi \epsilon_b \hbar \omega_{ab}^2} \sim 10^{-2}$. The increase in the slope a due to phase fluctuations is therefore hardly a measurable quantity, in agreement with the JJA result [11]. Additional temperature dependence can be obtained from considering classical phase fluctuations or by coupling the phason to a heat bath [6]. However, classical phase fluctuation effects are not relevant to our model and the coupling to the heat bath leads to only a subleading T^2 correction to the SPS [8].

Unlike quasiparticle properties represented by the SPS as discussed above, the phase fluctuation effects on Cooper pair properties can be significant. Here we propose an experiment which can measure the strength and the form of the phase fluctuations via the pair-field susceptibility. We consider a *c*-axis tunnel junction between two cuprate superconductors as illustrated in Fig. 3 [16]. The Josephson coupling between the phases (denoted by ϕ and ϕ') of the two superconductors will lead to the usual Josephson current oscillating at a frequency of $2eV/\hbar$ if there is a potential difference V across the junction. There will also be a quasiparticle tunneling current. In addition, an excess current will flow due to the Josephson coupling of the superconducting pair field of one superconducting electrode to the *fluctuating* pair field of the other. To isolate the excess current a small magnetic field is applied parallel to the junction to suppress the Josephson current, and the quasiparticle tunneling current must be modeled and subtracted [12]. The excess current is interesting because it can be related to the pair-field susceptibility at a frequency 2 eV/ \hbar [17] and can thus provide information about the spectrum of phase fluctuations.

To specialize the experiment to our case, we suppose that both of the electrodes are identical with a gap Δ and ignore the fluctuations in the amplitude of the order parameter assuming that $2eV \ll \Delta$. For simplicity, we consider a junction in the *ab* plane (at z = 0) of dimensions $L_x \times L_y$, with a magnetic field H_y in the *b* direction, and we will work at zero temperature. If we assume that the thickness of the electrodes is larger than λ_c , the *c*-axis penetration



FIG. 3. Excess pair tunneling current where $\omega_c = 0.6 \text{ meV}$ and $\alpha = 0.75$. The inset illustrates a pair tunneling experiment with two cuprate superconducting electrodes SC and SC'. We assume a junction contact area of 10^{-8} m², a normal state junction resistance $R_N \approx 30 \Omega$, and an in-plane magnetic field of $H \sim 0.3$ G. [See Eq. (13).]

depth, the Josephson coupling Hamiltonian for a phase difference $\delta \phi(\mathbf{r}, t) \equiv \phi(\mathbf{r}, t) - \phi'(\mathbf{r}, t)$ is

$$H_J = \frac{E_J}{2S} e^{-i\omega t} \int d^3 r \, e^{iq_x x} e^{i\delta\phi(\mathbf{r},t)}\delta(z) + \text{H.c.}\,,\quad(7)$$

where E_J is the Josephson coupling energy, $S = L_x L_y$ is the junction contact area, $q_x \approx 4eH_y\lambda_c/\hbar c$ [17], and $\omega = 2 \text{ eV}/\hbar$. The current through the junction is

$$I = -\frac{2eE_J}{\hbar S} \operatorname{Im} e^{-i\omega t} \int d^3 r \, e^{iq_x x} \langle e^{i\delta\phi(\mathbf{r},t)} \rangle \delta(z) \,. \tag{8}$$

If we calculate the current to zeroth order in H_J , and carry out the averages with respect to the Gaussian action in Eq. (3), we obtain the Josephson current with a critical current $\tilde{I}_c = (2eE_J/\hbar)\alpha$ which is renormalized by phase fluctuations through the Debye-Waller factor α (there is no quasiparticle current in our model). By calculating the current to first order in H_J (linear response), we obtain the excess current,

$$I_{\rm ex}(\omega, q_x) = \frac{eE_J^2}{S\hbar^2} \,{\rm Im}D^R(q_x, q_y = 0, \omega; z = 0)\,, \quad (9)$$

where the retarded pair-field susceptibility is

$$D^{R}(\mathbf{r},t) = -i\theta(t) \langle [e^{i\delta\phi(\mathbf{r},t)}, e^{-i\delta\phi(\mathbf{0},0)}] \rangle.$$
(10)

For $\omega > 0$ we have $\text{Im}D^{R}(\mathbf{q}, \omega) = \text{Im}D(\mathbf{q}, \omega)$, where $D(\mathbf{q}, \omega)$ is the Fourier transform of the time-ordered correlation function, which for a Gaussian action is

$$D(\mathbf{r},t) = -i\alpha^2 e^{2\langle T[\phi(\mathbf{r},t)\phi(\mathbf{0},0)]\rangle}$$

$$\approx -i\alpha^2 \{1 + 2\langle T[\phi(\mathbf{r},t)\phi(\mathbf{0},0)]\rangle\}, \quad (11)$$

where the factor of 2 comes from the two sides of the junction and T is the time-ordering operator. Equation (11)

assumes an expansion in the small parameter $\ln \alpha$. If we neglect the boundary effects near the junction, we can obtain the propagator for the phase fields from the action in Eq. (3); after an analytic continuation, we have

$$\langle \phi(\mathbf{q},\omega)\phi(-\mathbf{q},-\omega)\rangle = \frac{-4iV(\mathbf{q})}{\omega^2 - \omega_p^2(\mathbf{q}) + i0^+}.$$
 (12)

For an *isotropic* plasma frequency ω_p it can be shown that the excess current consists of a series of δ functions at integer multiples of ω_p . This result is somewhat academic since in the known isotropic superconductors the plasma energy is much larger than the gap energy, and we would expect amplitude fluctuations and quasiparticle damping to completely obliterate this effect. For an *anisotropic* plasma frequency these resonances become broadened—since the junction is localized at z = 0, we must integrate over q_{\perp} , which results in a plasma frequency that ranges from ω_c (when $q_x/q_{\perp} \rightarrow 0$) to ω_{ab} (when $q_{\perp}/q_x \rightarrow 0$). The result of the calculation is

$$I_{\rm ex}(\omega, q_x) \approx \left(\frac{2\pi e \tilde{I}_c^2}{q_x \epsilon_b L_y L_x \hbar}\right) \frac{\theta(\omega - \omega_c) \theta(\omega_{ab} - \omega)}{\sqrt{(\omega^2 - \omega_c^2)(\omega_{ab}^2 - \omega^2)}}.$$
(13)

To quench the background Josephson current, a field may be chosen such that $q_x = 2\pi/L_x$; a result is shown in Fig. 3.

The excess current exhibits a sharp onset at $\omega = \omega_c$ with a peak of the form $(\omega^2 - \omega_c^2)^{-1/2}$. This peak would be rounded by the small quasiparticle damping which we have not considered here. This experiment would serve as a direct observation of the phase fluctuations and as an alternative way to measure the *c*-axis Josephson plasma energy [14] lying below the maximum gap. In addition, it would provide a measure of the strength of the phase fluctuations. The gapless collective modes studied in Refs. [12,17] are due to the order parameter fluctuations which are rendered visible near the transition temperature, which are in principle observable in any superconductors, whereas the quantum phason modes that we studied are observable as a result of the intrinsically strong phase fluctuations and the quasi-two-dimensionality of cuprate materials. Therefore, the result shown in Fig. 3 is a special zero-temperature property of cuprate superconductors.

We have shown that in the case of the quasi-twodimensional d-wave BCS model that we used here, the resulting correction to the absolute value and the temperature dependence of the in-plane SPS is minute despite the strong phase fluctuations. Since quasiparticle interaction effects can obscure the fluctuation corrections to the SPS, it may be difficult to observe the effects of phase fluctuations on the penetration depth. As a more direct measurement, we have proposed a pair tunneling experiment which can probe the strength and spectrum of the quantum phase fluctuations. We expect that the pair-field susceptibility will show a pronounced peak at $\omega = \omega_c$. It will be also interesting to explore the role of the order parameter fluctuations at the superconductor-insulator transition in the underdoped regime via the suggested experiment. In the underdoped regime, the simple BCS model fails, especially at the superconductor-tor-insulator transition which is one extreme example of the renormalization of the SPS, and the physics of doped Mott insulators needs to be taken into account.

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