Electron Spectral Function and Algebraic Spin Liquid for the Normal State of Underdoped High T_c Superconductors

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We propose to describe the spin fluctuations in the normal state (spin-pseudogap phase) of underdoped high T_c cuprates as a manifestation of an algebraic spin liquid. Within the slave boson implementation of spin-charge separation, the normal state is described by massless Dirac fermions, charged bosons, and a gauge field. The gauge interaction, as an exact marginal perturbation, drives the mean-field free-spinon fixed point to a new spin-quantum fixed point—the algebraic spin liquid. Luttinger-liquid-like line shapes for the electron spectral function are obtained in the normal state, and we show how a coherent quasiparticle peak appears as spin and charge recombine.

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Introduction.—The key property of high T_c superconductors is their Mott insulator property at half filling. After integrating out the excitations above the charge gap at half filling, the system is described by a generalized t-J (GtJ) model

$$H = \sum_{(ij)}\left[J\left(\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} n_i n_j\right) - t(c_{\alpha i}^{\dagger} c_{\alpha j} + \text{H.c.})\right] + \ldots,$$

which may contain long range and multiple spin couplings indicated by "...". Upon doping, the charge carriers form a new non-Fermi-liquid metallic state. Understanding this new metallic state is the key to understanding high T_c superconductors. For underdoped high T_c superconductors, the metallic state has two striking properties. First, the Fermi surface does not form a closed loop. Second, the electron spectral function contains no sharp quasiparticle peak. Although we cannot derive the above properties from the GtJ model, we find, in the slave-boson approach [1] to the GtJ model, that a metallic state described by one of the slave-boson mean-field states—the staggered flux (sF) state (which is also called d-wave paired state)—has a Fermi surface which does not form a closed loop [2]. The sF state can also explain many other unique properties of underdoped high T_c superconductors, such as the positive charge and the low density of the charge carriers. Therefore in this paper we use the sF state as our starting point to study the electron spectral function in underdoped high T_c superconductors. The effective theory of the sF state is given in Refs. [2,3], which contains spinons, holons, and a U(1) gauge field as low energy excitations.

The electron spectral function obtained at the mean-field level [ignoring the U(1) gauge interaction] [2] has a line shape different from the one measured in experiments. In this paper, we include the gauge fluctuations in our calculation of the electron spectral function. We find that the U(1) gauge interaction does not confine the spinons and holons (at least above a certain energy). The U(1) gauge interaction turns out to be an exact marginal perturbation that drives the mean-field spinon fixed point described by free massless Dirac fermions to a new spin-quantum fixed point, which in turn produces a Luttinger-liquid-like line shape for the spinon spectral function and the electron spectral function [4], at least in the very low doping limit. We call this new spin-quantum fixed point—the algebraic spin liquid (ASL).

We also show how the opening of a gap in the gauge field spectrum yields spin-charge recombination and restoration of a coherent peak in the electron spectral function, which has been observed in the superconducting phase of the cuprates [5–9]. The mechanism of the gap formation is as yet not well understood theoretically. It can be due to either boson condensation or confinement caused by instantons [10,11]. We find that analyzing doping dependent angle-resolved photoemission spectroscopy (ARPES) results can help to clarify this issue. If the gap in the gauge field is due to boson condensation (the Higgs mechanism), the sharp quasiparticle peak will appear only in the superconducting phase [8]. The weight of the sharp quasiparticle peak will increase as the superfluid density increases, $Z \propto x(\rho_s)^{2\alpha}$ [8,9]. On the other hand, if the gap of the gauge field is opened via the instanton effect, the weight of the sharp quasiparticle peak will be proportional to the doping, $Z \propto x$, and the peak may appear above T_c.

Dirac spectrum in high T_c superconductors.—Our experimentally motivated starting point is the staggered flux state where the mean-field degrees of freedom are free fermionic spin carrying particles (spinons) and charged bosons (holons). The question of interest to us is whether the mean-field spinons survive the inclusion of fluctuations, in particular, the gauge fluctuations, around the mean-field state. In order to analyze this problem we have mapped the lattice effective theory for the sF state (at zero doping) onto a continuum theory of massless Dirac spinors coupled to a gauge field [12], whose Euclidean action reads

$$S = \int d^3x \sum_{\mu} \sum_{\sigma=1}^{N} \bar{\Psi}_\sigma v_{\sigma,\mu}(\partial_\mu - i a_\mu)\gamma_\mu \Psi_\sigma\,, \tag{1}$$

where $v_{\sigma,0} = 1$ and $N = 2$, but in the following we treat N as an arbitrary integer. In general $v_{\sigma,1} \neq v_{\sigma,2}$.

However, for simplicity we assume $v_{\sigma,i} = 1$ here. The Fermi field Ψ_σ is a 4×1 spinor which describes lattice spinons with momenta near $(\pm\pi/2, \pm\pi/2)$. The 4×4 γ_μ matrices form a representation of the Dirac algebra $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ $(\mu, \nu = 0, 1, 2)$. The dynamics for the U(1) gauge field arises solely due to the screening by bosons and fermions, both of which carry gauge charge. In the low doping limit, however, we include only the screening by the fermion fields [13], which yields

$$Z = \int Da_\mu \exp\left(-\frac{1}{2}\int \frac{d^3q}{(2\pi)^3} a_\mu(\vec{q})\Pi_{\mu\nu}a_\nu(-\vec{q})\right)$$

$$\Pi_{\mu\nu} = \frac{N}{8}\sqrt{\vec{q}^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^2}\right). \tag{2}$$

Spectral function, normal state.—We have analyzed the gauge invariant spinon Green's function of the above model in a large N expansion. The details of the calculation will be described elsewhere [14]. Here we state just the result

$$G(\vec{k}) = -iC\frac{k_\mu\gamma^\mu}{k^{2-2\alpha}} \qquad \alpha = \frac{16}{N}\frac{1}{3\pi^2}\bigg|_{N=2} = 0.27\,, \tag{3}$$

where C is determined by the energy range over which our effective theory is supposed to be valid. Note that the above value of α is for the $v_{\sigma,1} = v_{\sigma,2} = v$ case. α will take a different value if $v_{\sigma,1} \neq v_{\sigma,2}$. Comparing this dressed propagator with the free spinon Green's function $G_0 = \frac{-ik_\nu\gamma^\nu}{k^2}$, we see that the inclusion of the gauge fluctuations has destroyed the coherent quasiparticle pole by changing the exponent of the algebraic decay. An important result coming out of this calculation is that the gauge interaction does not generate any mass and/or chemical potential terms for the spinons. Since the conserved current (that couples to a_μ) cannot have any anomalous dimension, the gauge fluctuation represents an exact marginal perturbation whose inclusion at the mean-field free-spinon fixed point yields a new phase with novel algebraic behavior. This new quantum fixed point for the spins is the algebraic spin liquid mentioned above [15]. We see that the ASL state contains no free quasiparticles at low energies. It is not the confined phase of the U(1) gauge field, however, which would bind the spinons into a spin wave. The ASL is closer to the deconfined phase even though there are no *free* spinon quasiparticles at low energies. We still say that there is spin-charge separation in the ASL.

We remark that despite many similarities, there is a difference between our ASL proposal and the quantum-critical-point (QCP) approach to high T_c superconductors [17]. We do not assume or require a nearby quantum phase transition which gives rise to a QCP. The ASL can exist as a phase despite the fact that its gapless excitations interact even at lowest energies.

In the following we determine the behavior of the physical electron spectral function from correlations in the ASL. By virtue of the spin-charge separation implemented in the slave-boson theory, the physical electron operator is a product of a holon and a spinon. As mentioned above at the mean-field level these 2 degrees of freedom propagate as *free* particles and, in particular, since the mean-field boson condensation temperature $T_c \sim 4\pi xt \sim 4000$ K (where $t \sim 400$ meV and the hole doping concentration $x \sim 0.1$), we may consider the bosons to be nearly condensed in the low energy effective theory. The electron spectral function being a product of charge and spin propagators is then simply determined through the spinon correlations. Mapping the continuum fields back onto the lattice fields we can utilize the result for the dressed spinon propagator in the ASL to see the effect of the gauge fluctuations on the physical electron propagator. We find for the electron spectral function [14]

$$A_+ = \theta(\omega)\left\{\frac{xC}{4\pi}\sin(\pi\alpha)\theta(\omega - E_f)\frac{\omega + \epsilon_f}{[\omega^2 - E_f^2]^{1-\alpha}}\right\},$$

$$A_- = \theta(-\omega) \times \left\{\frac{xC}{4\pi}\sin(\pi\alpha)\theta(-\omega - E_f)\frac{-\omega - \epsilon_f}{[\omega^2 - E_f^2]^{1-\alpha}}\right\}, \tag{4}$$

where $E_f \equiv \sqrt{\epsilon_f^2 + \eta_f^2}$, $\epsilon_f(\mathbf{q}) = -2\tilde{J}\chi[\cos(q_x a) + \cos(q_y a)]$, and $\eta_f(\mathbf{q}) = -2\tilde{J}\Delta[\cos(q_x a) - \cos(q_y a)]$. C is determined by noting $\int d\omega\, d^2q/(2\pi)^2 A_\pm \sim x$. Even though the momenta in the expressions for the spectral functions run over all of the Brillouin zone, strictly speaking they should be restricted to the vicinity of the four Fermi points $(\pm\pi/2, \pm\pi/2)$ where the lattice fermions are well approximated by massless Dirac fermions.

In Fig. 1(a) we plot the spectral functions for two momenta along the zone diagonal. The main point to note is the lack of coherent quasiparticles in the spectrum which is in good agreement with experimental results for the cuprates above the transition temperature. We stress here

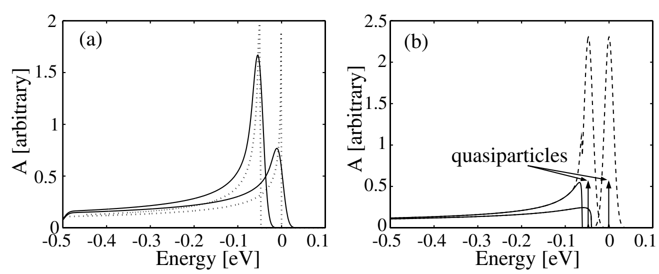

FIG. 1. (a) Two spectra along the (π, π) direction at $\mathbf{q} = (0.47, 0.47)$ and at the node $\mathbf{q} = (0.5, 0.5)$ in units of π/a. $\alpha = 0.27$ is used. The solid line is obtained on smearing the dotted line with a Gaussian of $\sigma = 10$ meV. The important point to note is the lack of a coherent quasiparticle pole which agrees well with ARPES line shapes in the normal state of the cuprates. (b) The solid line shows spectra at $(0.5, 0.5)$ and $(0.47, 0.47)$ in the superconducting state with $m = 40$ meV. The dashed line shows the delta function smeared with a Gaussian of $\sigma = 10$ meV which leads to a break in the line shape at $(0.47, 0.47)$ as opposed to a dip. The arrows indicate the position of the quasiparticle pole.

again that it is spin-charge separation combined with the ASL phase for the spin sector that yields the above spectra without the need for 1D phenomenology. The incoherent electron spectral function was also obtained using 1D physics in the stripe model for the high T_c superconductor [18].

Spectral function, superconducting state.—In marked contrast to the normal state, the superconducting phase has been shown to have coherent quasiparticles everywhere in momentum space below the superconducting gap. Explaining the development of this coherent behavior out of the incoherence of the normal state is one of the big challenges in revealing the high T_c physics.

In the spin-charge separation picture, the superconducting state can be obtained through boson condensation in the spin pseudogap phase. The gauge field a_μ obtains a Higgs mass m which implies that the gauge field is in the confinement phase [10]. Thus the spinons and holons are confined in the superconducting phase. Because of the confinement (which is referred to as spin-charge recombination) we expect a well defined quasiparticle and a sharp peak in the electron spectral function to appear in the superconducting state. We assume that after gaining a mass gap due to boson condensation (or instanton effects), the gauge effective theory is described by Eq. (2) with $\Pi_{\mu\nu} = \frac{N}{8}\sqrt{\vec{q}^2 + m^2}(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^2})$. In the boson condensation picture, m is related to the superfluid density ρ_s (the density of condensed holons): $\frac{N}{8}m \approx \rho_s/2m_h$, where m_h is the holon mass. The electron spectral function then takes the form

$$A_+(\omega, \mathbf{q}) = \Theta(\omega)C\,\frac{x}{4}\left\{(m^2)^\alpha u_f^2 \delta(E_f - \omega) + \Theta(\omega^2 - E_f^2 - m^2)\,\frac{\sin(\pi\alpha)}{\pi}\,[\omega^2 - E_f^2 - m^2]^\alpha\,\frac{\omega + \epsilon_f}{\omega^2 - E_f^2}\right\}$$

$$A_-(\omega, \mathbf{q}) = \Theta(-\omega)C\,\frac{x}{4}\left\{(m^2)^\alpha v_f^2 \delta(E_f + \omega) - \Theta(\omega^2 - E_f^2 - m^2)\,\frac{\sin(\pi\alpha)}{\pi}\,[\omega^2 - E_f^2 - m^2]^\alpha\,\frac{\omega + \epsilon_f}{\omega^2 - E_f^2}\right\}, \quad (5)$$

where $v_f^2 \equiv \frac{E_f(\mathbf{q}) - \epsilon_f(\mathbf{q})}{2E_f}$ and $u_f^2 \equiv \frac{E_f(\mathbf{q}) + \epsilon_f(\mathbf{q})}{2E_f}$ are the well known Bogoliubov coherence factors. In Fig. 1(b) we have plotted the spectra for the same momenta as in Fig. 1(a). We can clearly see the two distinct contributions to the spectral function, the delta function quasiparticle peak and the broad incoherent weight, respectively. An alternative interpretation of the peak-hump structure was given in Ref. [19].

As mentioned earlier, there are *different* ways in which the gauge field acquires its mass. In one picture, the gauge field becomes massive when the bosons acquire phase coherence via the Higgs mechanism. This way the mass generation of the gauge field is tied to the appearance of the superconducting order. Even without the boson condensation, however, the gauge field can acquire a mass via instantons [20], which is referred to as the confinement regime. In this case the gauge field can be massive even in the normal state. We stress that both boson condensation and instanton effect lead to the same phase where the gauge field is gaped and spin and charge recombine. The two pictures, with different dynamical properties, just represent two different limits of the same phase [10].

If the mass comes from boson condensation, then m will be proportional to the superfluid density. If the mass arises due to instantons, m will be the energy scale below which the instantons become important. Thus phenomenologically we may put $m = m_0 + C_1\rho_s$ to cover both boson condensation and the instanton limit. In the weak coupling limit, the mass induced by the instanton, m_0, is very small and the gauge field obtains a noticeable mass $C_1\rho_s$ only after boson condensation. In the strong coupling limit, the gauge field can obtain a large mass merely through the instanton effect.

In the SU(2) slave boson model, the gauge dynamics and its coupling constant is obtained through the screening with the fermions and is of order 1. It is hard to determine from theory if the confinement is caused by boson condensation or via the instanton effect. We can see from the expression for the spectral function (5) that the separation of the coherent particle peak from the midpoint of the leading edge of the incoherent background is given by $\Delta\omega = \sqrt{E_f^2(\mathbf{q}) + m^2} - E_f$ which simplifies for the spectrum at the node to $\Delta\omega = m$. Thus measuring the above mentioned separation for the spectrum at the node as a function of doping and superfluid density via ARPES might give us a clue as to which mechanism is responsible for the opening of the gap in the gauge spectrum.

Conclusion.—We have shown how the physics of spin-charge separation, gauge fluctuations, and the algebraic spin liquid give a consistent way of interpreting the Luttinger-liquid-like line shapes seen in the normal state of the cuprates without resorting to 1D phenomenology. We have seen how the gauge fluctuations destroy the free spinon mean-field phase and drive it to a new fixed point—the ASL. On entering the superconducting phase this ASL is destroyed through the opening of a mass gap in the gauge fluctuations via either the Higgs mechanism or instantons. This causes spin-charge recombination.

We believe that the ASL is a more general phenomenon where gapless excitations interact even at lowest energy scales. This paper discussed only a particular realization of the ASL through a slave-boson theory. It would be interesting to find other realizations of the ASL so that one can check which one fits experiments better.

It should be emphasized that in the spin-charge separation picture adopted in this paper, the spectral weight in the energy window up to $-4J \sim -0.5$ eV (where $-4J$ is the lower band edge of the mean-field spinons) is mainly determined by the spinon sector (and

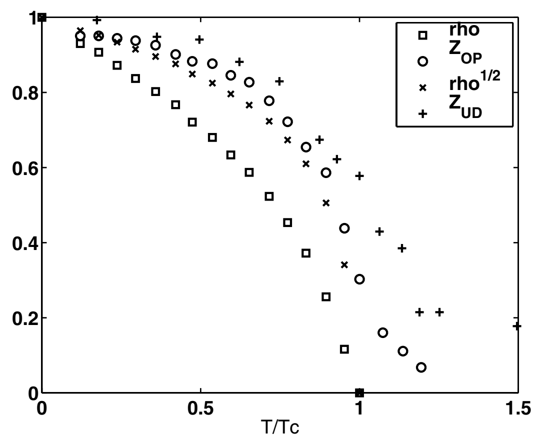

FIG. 2. Temperature dependence of Z_{UD}, Z_{OP}, ρ_s, and $\rho_s^{1/2}$.

the coherent holons) and is predicted to take the form $\int_{-4J}^{0} A_{-}(\omega, \vec{q}) d\omega\, d^2q/(2\pi)^2 = a + bx$, where x is the doping concentration, $b \sim 1$ and $a \sim 0.1$. The constant term a arises from the incoherent holon spectral weight [which is not included in Figs. 1(a) and 1(b)]. We can estimate a by noting that the total mean-field spectral weight for the holons is stretched out from 0 to $-8t \sim -3$ eV and normalized to $\frac{1}{2}$ [2,3]. This is important when extracting the doping dependence of the weight of the quasiparticle peak from ARPES measurements.

In the boson condensation picture, $m \propto \rho_s$ and the weight Z of the sharp quasiparticle peak can be determined from the superfluid density ρ_s, $Z \propto x(\rho_s)^{2\alpha}$. From this we can determine the temperature dependence of the weight of the quasiparticle peak. Furthermore, the $T = 0$ weight is $Z \propto x^{1+2\alpha}$. Under the instanton picture, $m \sim m_0$ and we have $Z \propto x$ if $m_0 \gg C_1\rho_s$.

In Fig. 2 we compare $Z(T/T_c)$ for underdoped Bi_2Sr_2-$Ca_1Cu_2O_{8+\delta}$ (Bi2212) (taken from [9]), optimally doped Bi2212 (taken from [8]) with $\rho_s^{2\alpha}(T/T_c) \sim \rho_s^{1/2}(T/T_c)$ [$\rho_s(ab)$ for optimally doped Bi2212 taken from [21]; we wished that Z and ρ_s were obtained from the same sample]. We observe that Z does not go to zero at T_c and is larger in the underdoped case (with a small T dependence above T_c) which points to mass generation via instantons. Below T_c we can see how the weight approaches $Z \propto x(\rho_s)^{2\alpha}$, where x is independent of temperature which suggests that the main contribution to the mass arises through the Higgs mechanism in this temperature regime.

Finally let us note that the behavior of the holons is still poorly understood. In this paper we have assumed that the holons have a small energy scale of order T_c in order to carry out our calculations. Although the normal state electron spectral function may not depend on the details of the holons, many other physical properties, such as normal state charge transport and the transition to the superconducting state, require a good understanding of these degrees of freedom.

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