Domain Walls without Cosmological Constant in Higher Order Gravity

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We consider a class of higher order corrections in the form of Euler densities of arbitrary rank n to the standard gravity action in D dimensions. We present a generating functional and an explicit form of the conserved energy-momentum tensors. We show that this class of corrections allows for domain-wall solutions despite the presence of higher powers of the curvature. The existence of such solutions no longer depends on the presence of cosmological constants. For example, the Randall-Sundrum-type scenario can be realized without bulk and/or brane cosmological constant.

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The idea of calculation of the effective quantum corrections to the energy-momentum tensors in the nontrivial gravitational background is relatively old [1,2]. If gravity is treated classically such a calculation corresponds to a one-loop result in quantum theory. If the background metric is conformally flat then there is no massless particle creation and the result for the vacuum polarization is local; therefore, the use of the effective Lagrangian is justified. These corrections are in the form of an expansion in powers of the curvature tensor. They belong to two classes. One of them ("topological") consists of terms that can be obtained from an effective action in the first order formalism without need for any metric. The second class ("nontopological") consists of terms coming from an action that necessarily involves a metric.

In this paper we are interested in the first class corresponding to the set of Euler densities of arbitrary order n (n being a power of the curvature tensor) [3]. When the dimension of the space-time D is equal to $2n$ the Euler density gives part of the conformal anomaly. This type of anomaly is unique and preserves scale invariance [4] (the other type requires introduction of a scale through regularization and vanishes for the vanishing Weyl tensor). It is important to stress that these higher order gravitational terms should always be included in the presence of the quantum matter fields—for example, in the case of the standard model fields [1,2].

There is another, independent motivation to consider terms of higher order in the curvature tensor. Such terms appear in the α' expansion of the string theory effective action and it was shown that the quadratic terms can be put in the form of the Gauss-Bonnet combination ($n = 2$ Euler density) [5].

It is interesting to note that Euler densities are the only higher order terms in the action that do not introduce ghosts and assure "Cauchy-like" evolution of the initial conditions.

From the form of the Euler densities it follows that for arbitrary D and n all the energy-momentum tensors belonging to the first class do not have derivatives acting on the curvature tensor. For the case of the conformally flat metrics we find a generating functional for these energy-momentum tensors. We analyze the domain-wall solutions in this class of models. We show that such solutions can be constructed since, even in the presence of higher powers of the curvature tensor, the type of singularity at the wall is the same as in the standard gravity. In the usual domain-wall metric the curvature has a deltalike singularity on the wall. Thus, one would naively expect that the higher powers of the curvature tensor introduce meaningless expressions (higher powers of the Dirac delta function) which cannot correspond to any sources. The Euler densities seem to be the only combinations of the higher order terms that do not lead to such meaningless expressions and which allow for the domain-wall metric with the usual sources.

The number of the domain-wall solutions with higher order terms in the action is larger than in the standard case and the cosmological constant is in general no longer crucial for their existence. It may even happen that the Randall-Sundrum-type scenario [6] can be realized without bulk and/or brane cosmological constant. The fact that there exist branelike solutions without any cosmological constant does not solve the cosmological constant problem—it is, as usual, the fine-tuning problem since the solution requires then some relations among the couplings.

In this paper we assume that all the higher order corrections to the Lagrangian are of the form of Euler densities which in D dimensions are defined (in the form notation) as

$$I^{(n)} = \frac{1}{(D-2n)!}\epsilon_{a_1a_2\cdots a_D}R^{a_1a_2}\wedge\cdots R^{a_{2n-1}a_{2n}} \wedge e^{a_{2n+1}}\wedge\cdots e^{a_D}. \tag{1}$$

For $D = 2n$ they are invariants and formally total derivatives. However, careful regularization does not allow us to discard them either in the action or in the equations of motion since they correspond to the conformal anomaly.

If we have a Lagrangian of the form

$$\mathcal{L} = -\sum_{n=0}^{n_{\max}} \kappa_n I^{(n)}, \tag{2}$$

then the equations of motion from the variation of the vielbein read [for $n \leq (D-1)/2$]

$$\sum_n \frac{\kappa_n(D-2n)}{(D-2n)!}\epsilon_{a_1 a_2 \cdots a_{D-1} a} R^{a_1 a_2} \wedge \cdots R^{a_{2n-1}a_{2n}} \wedge e^{a_{2n+1}} \wedge \cdots \wedge e^{a_{D-1}} = 0 . \quad (3)$$

We can write the curvature two-form as

$$R^{ab} = C^{ab} + \frac{1}{D-2}(e^a \wedge K^b - e^b \wedge K^a), \quad (4)$$

where C^{ab} is a two-form composed of the Weyl tensor $C_{\mu\nu\rho\sigma}$, while K^a is a one-form defined (for an invertible vielbein e^a_μ) as $K^a = K_{\mu\nu}e^{\mu a}dx^\nu = K^a_b e^b$ with

$$K_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2(D-1)} g_{\mu\nu} R . \quad (5)$$

In this paper we consider the background metrics for which the Weyl tensor vanishes. In this case the curvature two-form (4) can be expressed in terms of the Ricci tensor and the curvature scalar only.

Multiplying (3) by e^b we get

$$\begin{aligned} 0 &= e\sum_n \kappa_n \frac{2^n(D-n-1)!}{(D-2)^n(D-2n-1)!} \cdot (\delta^{c_2}_{a_2}\delta^{c_4}_{a_4}\cdots\delta^{c_{2n}}_{a_{2n}}\delta^b_a \pm \text{perm})K^{a_2}_{c_2}K^{a_4}_{c_4}\cdots K^{a_{2n}}_{c_{2n}} \\ &= e\sum_n \kappa_n (-1)^n \frac{2^n n!(D-n-1)!}{(D-2)^n(D-2n-1)!} H^{(n)}_{ab}, \end{aligned} \quad (6)$$

where $H^{(n)}$ are defined by this equality and will be described in detail below.

It is not too difficult to prove that the generating functional for the tensors $H^{(n)}_{\mu\nu}$ is given by

$$\begin{aligned} M_{\mu\nu}(t,K) &= \det(\mathbb{1} - tK)[(\mathbb{1} - tK)^{-1}]_{\mu\nu} \\ &= \sum_{n=0}^{\infty} t^n (H^{(n)})_{\mu\nu} , \end{aligned} \quad (7)$$

where from now on we use the matrix notation ($\mathbb{1}$ denotes δ^ν_μ, while K denotes K^ν_μ).

Differentiating equality $C_{\mu\nu\rho\sigma} = 0$ over x^μ and using Bianchi identity we get

$$K_{\mu\nu;\rho} = K_{\mu\rho;\nu} . \quad (8)$$

By differentiating $M_{\mu\nu}(t,K)$ over x^μ and using (8) it is straightforward to prove that tensors $H^{(n)}_{\mu\nu}$ are covariantly conserved. A more tedious calculation shows a nontrivial fact that (when the Weyl tensor vanishes) $H^{(n)}_{\mu\nu}$ are the only covariantly conserved tensors which are algebraically composed of the Ricci tensors.

From the generating functional (7) one gets the recurrence relations for $H^{(n)}_{\mu\nu}$:

$$H^{(n)}_{\mu\nu} = H^{(n-1)}_{\mu\rho}K^\rho_\nu - \frac{1}{n} g_{\mu\nu} H^{(n-1)}_{\sigma\rho}K^{\sigma\rho} \quad (9)$$

starting with $H^{(0)}_{\mu\nu} = g_{\mu\nu}$; the explicit formula (for $n > 0$) reads

$$H^{(n)}_{\mu\nu} = \sum_{\{q_1,\ldots,q_n\}} \left[\prod_{m=1}^{n} \frac{(-\mathrm{Tr}K^m)^{q_m}}{m^{q_m} q_m!}\right](K^{n-Q})_{\mu\nu} , \quad (10)$$

where $Q = \sum_{k=1}^n k q_k$ and the sum is over all sets of n non-negative integers q_k for which $Q \leq n$.

As an example we present $H^{(2)}_{\mu\nu}$ and $H^{(3)}_{\mu\nu}$ ($H^{(1)}_{\mu\nu}$ is equal to the Einstein tensor):

$$\begin{aligned} H^{(2)}_{\mu\nu} &= R_{\mu\rho}R^\rho_\nu - \frac{D}{2(D-1)}RR_{\mu\nu} + g_{\mu\nu}\left(-\frac{1}{2}R_{\sigma\rho}R^{\sigma\rho} + \frac{D+2}{8(D-1)}R^2\right), \\ H^{(3)}_{\mu\nu} &= R_{\mu\sigma}R^{\sigma\rho}R_{\rho\nu} - \frac{D+1}{2(D-1)}RR_{\mu\sigma}R^\sigma_\nu + R_{\mu\nu}\left(-\frac{1}{2}R_{\rho\sigma}R^{\rho\sigma} + \frac{D^2+3D-2}{8(D-1)^2}R^2\right) \\ &\quad + g_{\mu\nu}\left(-\frac{1}{3}R_{\tau\sigma}R^{\sigma\rho}R_{\rho\tau} + \frac{D+1}{4(D-1)}RR_{\rho\sigma}R^{\rho\sigma} - \frac{D^2+7D+2}{48(D-1)^2}R^3\right). \end{aligned} \quad (11)$$

$H^{(2)}_{\mu\nu}$ was used (for $D = 4$) in [7] to prove that the Einstein-Hilbert gravity with such a correction does not have a cosmological singularity and can describe inflation.

One can see from the definition (1) that in D space-time dimensions forms $\boldsymbol{I}^{(n)}$ are nonzero only for $0 \leq n \leq [D/2]$ {therefore also $n_{\max} \leq [D/2]$ in (2)}. The situation is different for the tensors $H^{(n)}_{\mu\nu}$. Using the generating functional (7) or the explicit formula (10) it is straightforward to show that $H^{(n)}_{\mu\nu}$ are nonzero in the larger range $0 \leq n < D$. The difference arises because the energy-momentum tensors $T^{(n)}_{\mu\nu}$ derived from $\boldsymbol{I}^{(n)}$ are proportional to $H^{(n)}_{\mu\nu}$ with coefficients vanishing for $n \geq D/2$:

$$T^{(n)}_{\mu\nu} = \kappa_n \frac{(-2)^{n-1}n!(D-n-1)!}{(D-2)^n(D-2n-1)!} H^{(n)}_{\mu\nu}. \quad (12)$$

The case with $D = 2n$ should be treated with care because it is related to the conformal anomaly.

Let us illustrate this point by considering $n = 2$. The Euler density in this case is the famous Gauss-Bonnet combination: $I^{(2)} = R_{\alpha\beta\rho\sigma}R^{\alpha\beta\rho\sigma} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$. The energy-momentum tensor obtained from this density is given by (3)

$$\begin{aligned} T^{(2)}_{\mu\nu} = \kappa_2\Big(&- 4R^{\alpha\beta}R_{\alpha\mu\beta\nu} + 2R_{\mu\alpha\beta\gamma}R_\nu^{\alpha\beta\gamma} \\ &- 4R_{\mu\alpha}R^\alpha_\nu + 2RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}I^{(2)}\Big). \end{aligned} \quad (13)$$

Let us note that this energy-momentum tensor is covariantly conserved even with the nonvanishing Weyl tensor. Relation (13) for the conformally flat metric gives

$$T_{\mu\nu}^{(2)} = -\kappa_2 \frac{4(D-4)(D-3)}{(D-2)^2} H_{\mu\nu}^{(2)}. \tag{14}$$

The coefficient in the above equality contains the factor $(D-4)$, but it is compensated by a factor $\Gamma(D-4)$ present in κ_2 due to regularization of the effective action so $T_{\mu\nu}^{(D/2)}$ should be taken into account.

Let us now use the above formalism to investigate the possibility of warped metric solutions in theories with higher order gravity terms in the Lagrangian (2).

In this paper we consider a model in D-dimensional space-time with a $(D-2)$ brane (a domain wall). Its action is the sum of the bulk and brane contributions:

$$S = S_{\text{bulk}} + S_{\text{brane}},$$
$$S_{\text{bulk}} = -\int d^D x \sqrt{-g} \sum_{n=0}^{n_{\max}} \kappa_n I^{(n)}, \tag{15}$$
$$S_{\text{brane}} = -\int d^{D-1} x \sqrt{-\tilde{g}}\, (\lambda + \cdots).$$

The metric on the brane is given by $\tilde{g}_{\mu\nu}(x^\rho) = g_{\mu\nu}(x^\rho, y = 0)$, where $y = x^D$ and from now on $\mu, \nu, \ldots = 1, \ldots, D-1$, while $M, N, \ldots = 1, \ldots, D$. In the brane action we write explicitly only the most important term—the brane cosmological constant. The bulk gravitational interactions are described by the sum of terms $I^{(n)}$ defined in (1). The first two terms are known from conventional gravity. The one with $n = 1$ is the usual Hilbert-Einstein term, $I^{(1)} = R$, and its coefficient, κ_1, depends on the fields normalization. We use normalization for which $\kappa_1 = 1$ [the other frequent choice is $(2\kappa^2)^{-1}$]. The term with $n = 0$ corresponds to the cosmological constant: $I^{(0)} = 1$, $\kappa_0 = \Lambda$. The maximal number of the higher order terms is $n_{\max} \leq [D/2]$ as previously discussed.

We obtain the equations of motion by differentiating the action (15) with respect to g^{MN}:

$$R_{MN} - \frac{1}{2} R g_{MN} = \frac{1}{2} \Lambda g_{MN} + \frac{1}{2} \lambda \tilde{g}_{\mu\nu} \delta_M^\mu \delta_N^\nu \delta(y) - \sum_{n=2}^{n_{\max}} T_{MN}^{(n)}. \tag{16}$$

We see that $T_{MN}^{(n)}$, given by (12), can be treated for $n \geq 2$ as contributions to the energy-momentum tensor coming from higher order interactions present in the action (15).

Let us now look for the domain-wall solutions in D-dimensional space-time which are flat from the $(D-1)$-dimensional point of view. The metric in this case can be written in the form

$$ds^2 = e^{-2f(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \tag{17}$$

Using the generating functional (7) we can find $H_{MN}^{(n)}$ for this metric:

$$H_{MN}^{(n)} = g_{MN} \left(\frac{D-2}{2}\right)^n \binom{D-1}{n} \left(\frac{\partial f}{\partial y}\right)^{2n-2} \cdot \left[\left(\frac{\partial f}{\partial y}\right)^2 - (1 - \delta_D^M) \frac{2n}{D-1} \left(\frac{\partial^2 f}{\partial y^2}\right)\right]. \tag{18}$$

This explicit formula shows that all $H_{MN}^{(n)}$ vanish for $n \geq D$ and that the second derivative of $f(y)$ is absent in $H_{DD}^{(n)}$ and appears only linearly in $H_{\mu\nu}^{(n)}$. This is quite surprising for a theory with higher orders of the curvature tensor in the action and, as we will see shortly, is crucial for the existence of the domain-wall solutions. This property follows from the unique structure of the Euler densities (1) and the fact that the second derivative of $f(y)$ appears only in K_{DD} and is absent in $K_{\mu\nu}$. Although higher powers of $\partial^2 f(y)/\partial y^2$ are present in separate terms on the right-hand side of Eq. (10), they exactly cancel in the full sum.

Using tensors (18) we find that the equations of motion (16) for the domain-wall metric (17) reduce to just two conditions:

$$\sum_{n=1}^{n_{\max}} p_n \left(\frac{\partial f}{\partial y}\right)^{2n} = \frac{1}{2} \Lambda, \tag{19}$$

$$\sum_{n=1}^{n_{\max}} n p_n \left(\frac{\partial f}{\partial y}\right)^{2n-2} \frac{\partial^2 f}{\partial y^2} = -\frac{D-1}{4} \lambda \delta(y), \tag{20}$$

where the coefficients p_n are equal

$$p_n = \kappa_n \frac{(-1)^{n-1}(D-1)!}{2(D-1-2n)!}. \tag{21}$$

The absence of the third or higher derivatives of $f(y)$ and/or higher powers of the second derivative $\partial^2 f(y)/\partial y^2$ in (18) allows for the solution of (16) of the same type as in the standard gravity:

$$f(y) = \sigma |y|, \tag{22}$$

where the parameter σ in this case must satisfy two algebraic conditions coming from (19),(20):

$$\sum_{n=1}^{n_{\max}} p_n \sigma^{2n} = \frac{1}{2} \Lambda, \tag{23}$$

$$\sum_{n=1}^{n_{\max}} \frac{n}{2n-1} p_n \sigma^{2n-1} = -\frac{D-1}{8} \lambda. \tag{24}$$

One of these equations can be used to determine the warp factor coefficient σ in terms of the parameters p_n and one of the cosmological constants (bulk, Λ, or brane, λ). The second equation is then the (fine-tuning) condition for the other cosmological constant.

Recently there has been a lot of interest in domain-wall solutions in theories with the space-time dimension equal to 5. The Randall-Sundrum scenario [6] corresponds to $D = 5$ and only two terms with $n = 0, 1$ ($n_{\max} = 1$) in the Lagrangian. In such a case the square of the warp factor exponent, σ^2, is just proportional to the bulk cosmological constant Λ and the brane cosmological constant λ must be appropriately fine-tuned: $\sigma\lambda = -\Lambda$.

However, Eqs. (23) and (24) have other very interesting solutions when the higher order terms are present (i.e., $n_{\max} \geq 2$). The square of the warp factor exponent σ^2 is no longer just linearly proportional to Λ. It depends also on the other Lagrangian parameters κ_n. In general, there are up to $n_{\max}$ possible values of σ^2 for a given value of Λ. This is true also for $\Lambda = 0$. Thus, it is possible to have nontrivial warped metric solutions with vanishing bulk cosmological constant for any $n_{\max} \geq 2$. There is one such solution for each positive zero of the polynomial $P(x) = \sum_{n=1}^{n_{\max}} p_n x^n$. It is also possible to solve Eqs. (23) and (24) with $\lambda = 0$. The domain wall is then supported only by higher order corrections to the action and (in distinction to the Einstein-Hilbert gravity) there is no need for conventional sources on the brane.

An even more interesting situation emerges when the zero of $P(x)$ satisfies simultaneously Eq. (24). Then, there exists such a warp factor σ that Eq. (23) is satisfied for $\Lambda = 0$ and Eq. (24) is satisfied for $\lambda = 0$. In such a case, there is a domain-wall solution which is flat from the $(D - 1)$-dimensional point of view with both cosmological constants vanishing. This, however, can happen only for $n_{\max} > 2$ (i.e., $D \geq 6$) and for some very specific values of the Lagrangian parameters κ_n. The fine-tuning of parameters κ_n replaces the fine-tuning of the brane tension λ used in the original Randall-Sundrum scenario. Therefore, it is a different formulation of the cosmological constant problem.

The domain-wall solutions with the first correction (Gauss-Bonnet term) were discussed in different contexts in the literature. The five-dimensional Gauss-Bonnet gravity with dilaton was discussed in detail in [8]. The brane solutions with vanishing bulk cosmological constant (but with nonvanishing brane cosmological constant) were discussed in gravity coupled to the dilaton (see, for example, [9,10] where the first reference does not discuss the Gauss-Bonnet term). The mechanism of "self-tuning" presented in these papers was shown later [11] to be another form of fine-tuning. The possibility of brane-type solutions with vanishing cosmological constants was also discussed in [12]. The idea of employing quadratic corrections as boundary counterterms to the AdS action was used, for example, in [13,14]. The brane configurations in the context of AdS/CFT correspondence were considered, for example, in Refs. [14,15] and in the cosmological context in [16].

In conclusion, we considered a class of models with higher order gravity corrections in the form of the Euler densities with arbitrary power n of the curvature tensor in arbitrary space-time dimension D. We found a generating functional and an explicit form of the corresponding energy-momentum tensors. The domain-wall-type solutions were constructed for the considered class of models. Such solutions are possible despite the presence of higher powers of the curvature tensor because the singularity structure at the wall is of the same type as in the standard gravity. The number of possible domain-wall solutions increases with the order of included corrections and the cosmological constant is no longer crucial for their existence (as was the case for models with $n \leq 2$ previously analyzed in the literature). We showed, for example, that the Randall-Sundrum-type scenario can be realized without the bulk and/or brane cosmological constant.

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