Generation of Photon Number States on Demand via Cavity Quantum Electrodynamics

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Many applications in quantum information or quantum computing require radiation with a fixed number of photons. This increased the demand for systems able to produce such fields. We discuss the production of photon fields with a fixed photon number on demand. The first experimental demonstration of the device is described. This setup is based on a cavity quantum electrodynamics scheme using the strong coupling between excited atoms and a single-mode cavity field.

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In recent years there has been increasing interest in systems capable of generating photon fields containing a preset number of photons. This has chiefly arisen from applications for which single photons are a necessary requirement, such as secure quantum communication [1–3] and quantum cryptography [4]. Fock states are also useful for generating multiple atom entanglements in systems such as the micromaser. The generated field and the pumping atoms are in an entangled state, this entanglement can be transferred by the field to subsequent atoms, leading to applications such as basic quantum logic gates [5]. In the current experiment the micromaser employed a cavity with a Q value of 4×10^{10} corresponding to a photon lifetime of 0.3 s which is the largest ever achieved in this type of experiment and more than 2 orders of magnitude greater than in related setups [5]. In this cavity, Fock states can be used to entangle a large number of subsequent atoms. A source of single photons or, more generally, arbitrary Fock states is also a useful tool for further fundamental investigations of the atom-field interaction. It can be used to obtain the reconstruction of purely quantum states of the radiation field as represented by the Fock states [6].

Many sources for single photons have been proposed. These include single-atom fluorescence [7], single-molecule fluorescence [8], two-photon down-conversion [9] and Coulomb blockade of electrons [10], state reduction [11], and using cavity quantum electrodynamics [3,12–14]. On the other hand, only one source presented recently, involving the transfer of atoms between a magneto-optical trap and dipole trap [15], is, in principle, able to produce n atoms. However, a reliable and deterministic source of Fock states (or even single photons) has not yet been demonstrated.

Using the one-atom maser or micromaser we present the first experimental evidence for the operation of a reliable and robust source of photon Fock states, which by virtue of its operation also produces a predefined number of atoms in a particular state. These atoms are entangled with the generated field and, as mentioned above, can be further entangled with subsequent atoms.

A basic requirement for *reliably* preparing a field in a pre-set quantum state is the ability to choose the field state in a controllable manner. Trapping states provide this control. Under trapping-state conditions a quantum feedback between the atoms and the field acts to control the cavity photon number. Using trapping states, one is therefore able to provide photons on demand. This provides the additional benefit of eliminating the need to detect the atoms leaving the cavity, thus making these atoms available as a source for further experiments. The method we describe here is, in principle, also applicable to optical cavities [16] and is therefore of broad use.

Under ideal conditions the micromaser field in a trapping state is a Fock state; however, when the micromaser is operated in a continuous wave (cw) mode, the field state is very fragile and highly sensitive to external influences and experimental parameters [17,18]. However, contrary to cw operation, under pulsed operation the trapping states are more stable and more practical, and usable over a much broader parameter range than for cw operation.

The cw operation of the micromaser has been studied extensively both theoretically [19] and experimentally. It has been used to demonstrate quantum phenomena such as, for example, sub-Poissonian statistics [20], the collapse and revival of Rabi oscillations [21], and entanglement between the atoms and cavity field [22].

The micromaser setup used for the experiments has been described previously [17]. Briefly, a beam of ^{85}Rb atoms is excited to the $63P_{3/2}$ Rydberg level by single-step laser excitation ($\lambda = 297$ nm). The excited atoms enter a high-Q superconducting microwave cavity housed in a ^{3}He-^{4}He dilution refrigerator which cools the cavity to 300 mK, corresponding to a thermal photon number $n_{\mathrm{th}} = 0.03$. The cavity is tuned to a 21.456 GHz transition from the $63P_{3/2}$ upper state to the $61D_{5/2}$ lower state of the maser transition.

The emission probability, P_g, of a $63P_{3/2}$ upper level atom entering the cavity is given by

$$P_g = \sin^2(\sqrt{n + 1}\, g t_{\mathrm{int}})\,, \tag{1}$$

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where n is the number of photons in the cavity, g is the effective atom-field coupling constant ($\approx$41 kHz), and $t_{\rm int}$ is the interaction time. We note that $P_g = 0$ when

$$\sqrt{n+1}\, g t_{\rm int} = k\pi\,, \tag{2}$$

where k is an integer number of Rabi cycles. This is the trapping-state condition. When it is fulfilled, the emission probability is zero and the field has reached an upper bound, thus preventing atoms from emitting. Trapping states are denoted by the number of photons n and an integer number of Rabi cycles k for which the emission probability is zero, they are labeled (n,k) [18]. It is this mechanism that controls the emission probability of atoms entering the cavity when the interaction time is tuned to a trapping state where the Fock state is produced and stabilized by the trapping condition. For short pulse lengths the lower-state atom number will be the same as the photon number. For simplicity we will concentrate here on the preparation of a one-photon Fock state although the method can also be used to generate Fock states with higher photon numbers.

For useful comparisons between experiment and theory, Monte Carlo simulations [23] are used to calculate the rate of production of lower-state atoms rather than the production of photons in the cavity. As pulse lengths are rather short ($\tau_{\rm pulse} = 0.02\tau_{\rm cav}$), there is little dissipation and the probability of finding a one-photon state in the cavity following the pulse is very close to the probability of finding an atom in the lower state.

To demonstrate the principle, Fig. 1 shows a simulation of a sequence of 20 pulses of the pumping atoms, in which an average of seven excited atoms per pulse are present. Two operating conditions are presented comparing conditions outside trapping conditions ($gt_{\rm int} = 1.67$) with the $(1,1)$ trapping state ($gt_{\rm int} = 2.2$). Below the pulse sequences, two distributions show the probability of finding 0–5 atoms (and hence photons) per pulse in the cavity. Under the trapping condition only a single emission event occurs, producing a single lower-state atom which leaves a single photon in the cavity. Since the atom-cavity system is in the trapping condition, the emission probability is reduced to zero and the photon number is stabilized. The variation of the time when an emission event occurs during the atom pulses in Fig. 1 is due to the Poissonian spacing of upper-state atoms entering the cavity and the stochasticity of the quantum process. In Fig. 1a the broader photon number distribution is due to the absence of a feedback stabilization.

Figures 2(a)–2(c) present three curves obtained from the computer simulations, which illustrate the behavior of the maser under pulsed excitation as a function of interaction time for the same parameters as Fig. 1. The simulations show the probability of finding the following: no lower-state atom per pulse, $P^{(0)}$, related to the $n = 0$ Fock component; finding exactly one lower-state atom per pulse, $P^{(1)}$, related to the $n = 1$ Fock component; a correlation parameter, $P^{(>1;1)}$, given by the conditional probability of

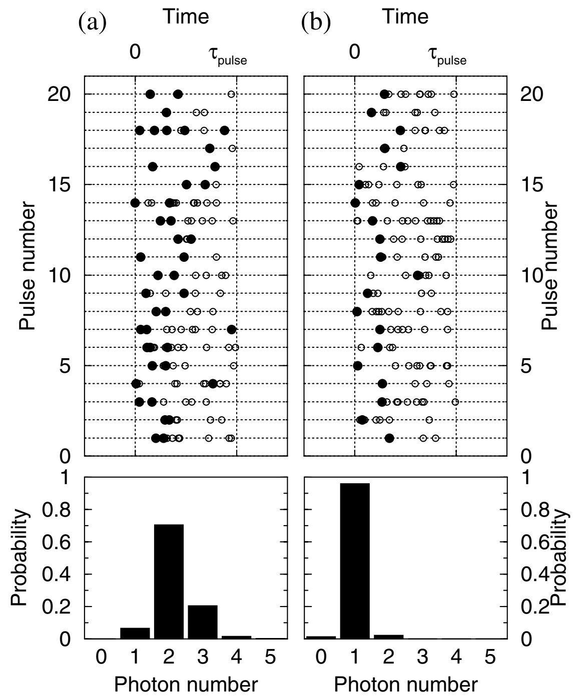

FIG. 1. A simulation of a subset of twenty subsequent pulses of the excitation laser and the associated probability distribution for photons or lower-state atom production (filled circles represent lower-state atoms and open circles represent excited-state atoms). The start and finish of each pulse is indicated by the vertical dotted lines marked 0 and $\tau_{\rm pulse}$, respectively. The two operating conditions are (a) outside the trapping-state conditions ($gt_{\rm int} = 1.67$) with a broad field distribution, and (b) the $(1,1)$ trapping state ($gt_{\rm int} = 2.2$) with a near-Fock-state distribution. Both distributions are sub-Poissonian but they are readily distinguishable experimentally. The parameters are indicated as crosses in Fig. 2. The size of the atoms in this figure is exaggerated for clarity. With the real atomic separation there are 0.06 atoms in the cavity on average (i.e., well into the one-atom regime). The other parameters are $\tau_{\rm pulse} = 0.02\tau_{\rm cav}$, $n_{\rm th} = 10^{-4}$, and $N_a = 7$ (see also Ref. [23]).

finding a second lower-state atom in a pulse already containing one. The value of $P^{(>1;1)}$ represents the sum of all Fock components $n > 1$, reaching a maximum value of 1 when there is no remaining population in the $n = 1$ or $n = 0$ states and a value of 0 when there is no population above the $n = 1$ state. This is the reason for its strong suppression at the $n = 1$ trapping state. The correlation parameter is not defined at the vacuum trapping state, where the exact one-to-one correspondence between lower-state atoms and the field is lost. In the vacuum trapping state, lower-state atoms can be produced only in the presence of thermal photons or other noise effects [17].

$P^{(>1;1)}$ is insensitive to the absolute values of the atomic detection efficiency and can therefore be measured unambiguously in an experiment. It is therefore treated as the most stable and useful observable of the interaction. The probability $P^{(1)}$ can be evaluated by using the formula

$$P^{(1)} = N_g(1 - P^{(>1;1)})\,, \tag{3}$$

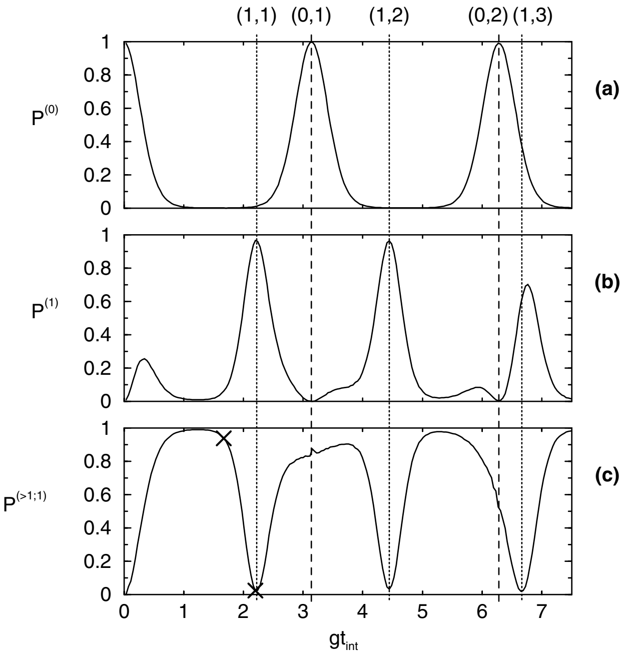

FIG. 2. The probability of finding (a) no lower-state atoms per pulse $P^{(0)}$, (b) exactly one lower-state atom per pulse $P^{(1)}$, and (c) a second lower-state atom, if one has already been detected $P^{(>1;1)}$. The crosses mark the positions of simulations in Fig. 1. The parameters are $\tau_{\text{pulse}} = 0.02\tau_{\text{cav}}$, $N_a = 7$ atoms, and $n_{\text{th}} = 10^{-4}$. The maximum value of $P^{(1)}$ is 97% for the $(1,1)$ trapping state.

where N_g is the lower-state atom probability. As the probabilities must sum to 1, the three Fock components $n = 0$, $n = 1$, and $n \geq 2$ can be determined uniquely.

It follows from Fig. 2 that with an interaction time corresponding to the $(1,1)$ trapping state, both one photon in the cavity and a single atom in the lower state are produced with nearly 97% probability. Note that at no time in this process is a detector event required to project the field; the field evolves to the target photon number state, when a suitable interaction time has been chosen so that the trapping condition is fulfilled.

To maintain the 97% probability of emission, a minimum number of atoms is required in each pulse. In fact, for a given average number of atoms per pulse, there is an upper bound to the probability of finding a single lower-state atom per pulse. This is given by

$$P^{\max} = 1 - e^{-P_g N_a}. \tag{4}$$

where N_a is the average number of atoms (of any type) per pulse and is considered the most important parameter when comparing different operating conditions, noting that the atomic beam intensity must be chosen to avoid violation of the one-atom-at-a-time condition.

The inherent stability of the single-photon–single-atom source is quite remarkable. Simulations show that stable operating conditions extend from those considered here to thermal photon numbers as high as $n_{\text{th}} = 0.1$ or for t_{int} fluctuations up to 10%. While both of these values are considerably higher than the current experimental parameters, Fock states can still be prepared with an 80%–90% fidelity [23]. This is an astounding result as it shows that Fock state production is much more stable than was previously suspected and the highly stable low thermal photon conditions required for cw trapping states [17,18,24] are not specific requirements. For this reason this source is already being considered for use in such fundamental applications as phase diffusion measurements [19] and quantum state reconstruction [6].

Experimentally, the correlation parameter $P^{(>1;1)}$ is obtained via atom pair correlations [25],

$$P^{(>1;1)} = \frac{N_{gg}}{N_{gg} + N_{eg} + N_{ge}}, \tag{5}$$

where, for example, N_{eg} is the probability of detecting a pair of atoms containing first an upper-state atom (e) and then a lower-state atom (g) within a pulse. The number of three atom events detected is negligible and can be effectively ignored as a contributing factor. Equation (5) provides a value both appropriate to the existent correlation and equal to the total probability of finding more than one lower-state atom per pulse (and thus more than one photon in the cavity). Although $P^{(>1;1)}$ is insensitive to the absolute detector efficiency it does depend on the relative detector efficiencies and the probability that a given atomic level is detected in the wrong detector (miscounts).

The present setup of the micromaser was specifically designed for cw operation. Nevertheless, the current apparatus does permit a comparison between theory and experiment in a relatively small parameter range.

A cavity pump rate of 60 atoms per cavity decay time (usually called N_{ex}) was obtained for a short range of interaction times around the maximum in the Maxwell-Boltzmann velocity distribution. This happens to correspond to the interaction time for the $(1,1)$ trapping state. A pulse length of $\tau_{\text{pulse}} = 0.066\tau_{\text{cav}}$ leading to an average of 4 atoms per pulse was chosen as a compromise between considerations of dissipative losses, the effect of external influences, and a reasonable average atom number per pulse. Figure 3 shows the results of the comparison of theory and experiment for an experimental and theoretical evaluation of $P^{(>1;1)}$ around the $(1,1)$ trapping state. Also presented is a theoretical curve representing the parameter $P^{(1)}$. The theoretical plot of $P^{(>1;1)}$ is calculated for the experimental conditions with no fit parameters. Shown are conditions of no detector miscounts and measured detector miscounts of 7% in the lower-state detector and 2% in the excited-state detector. When the miscounts are incorporated into the data there is an excellent match between experiment and theory.

For the current experimental conditions Eq. (4) gives a maximum probability for emitting into the mode of $P^{\max} = 92\%$ and a one-photon Fock state probability of 83.2%. This is in agreement with the lower-state atom-atom probability $N_g = 0.90 \pm 0.1\%$ per pulse, obtained when absolute detector efficiencies ($\approx$30%), detector miscounts, detector dark counts, and the finite lifetime of the

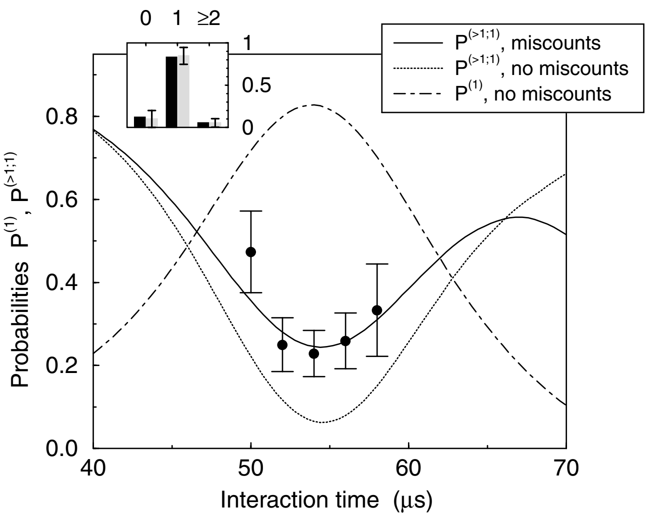

FIG. 3. Comparison between theory and experiment. The experimental data are evaluated using Eq. (5). The theoretical curves are $P^{(1)}$ and $P^{(>1;1)}$. $P^{(>1;1)}$ is presented for no detector miscounts and miscounts of 7% in the lower-state detector and 2% in the excited-state detector. Inset: a comparison of the experimental (grey) and theoretical (black) Fock components at the $(1,1)$ trapping state for the experimental conditions. The calculation is described in the text. The experimental parameters were $\tau_{\mathrm{cav}} = 300$ ms, $\tau_{\mathrm{pulse}} = 0.066\tau_{\mathrm{cav}}$, pulse spacing of 1 s, $n_{\mathrm{th}} = 0.03$, and $N_a = 4$.

atoms are taken into account. The error stems from uncertainties in these parameters.

Using Eq. (3) the distribution of emission events within a pulse was extracted from the experimental data at the position of the trapping state and is shown graphically in the inset of Fig. 3, along with the theoretical distribution. Following the evaluation above we get a success rate of Fock state production of 85%. The length of the pump pulse permits some dissipative losses, allowing a second emission event to occur. This accounts for a small proportion of observed two-atom events.

In this paper we have used trapping states to prepare a one-photon Fock state with a success rate of $85 \pm 10\%$, to be compared with the theoretical value under the present experimental conditions of 83.2%. By improving the experimental parameters, we can expect to reach conditions for which nearly 97% of the pulses prepare single-photon Fock states and a single atom in the lower state. Englert and Walther [26] showed recently that, by using only micromaser trapping states, it is possible to create Greenberger-Horne-Zeilinger (GHZ) states [27] between the atoms and cavity field. A thermal atomic source would require postselective measurements while the current apparatus can supply a sequence of single atoms with a very small error in arrival times to a second cavity, preparing the GHZ entangled states on demand. As the characteristic time scale of the apparatus is the cavity decay time, an arrival time error of $0.02\tau_{\mathrm{cav}}$ is effectively a δ function. Cavities with the appropriate properties for this measurement are already in use in the micromaser, making such a measurement possible in the near future. Further possible applications are the realization of quantum logic gates and, as mentioned above, the reconstruction of the wave function of a Fock state.

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