Enforced Electrical Neutrality of the Color-Flavor Locked Phase

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We demonstrate that quark matter in the color-flavor locked phase of QCD is rigorously electrically neutral, despite the unequal quark masses, and even in the presence of an electron chemical potential. As long as the strange quark mass and the electron chemical potential do not preclude the color-flavor locked phase, quark matter is automatically neutral. No electrons are required and none are admitted.

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If cold, dense quark matter is approximated as noninteracting up, down, and strange quarks, it has long been understood that equilibrium with respect to the weak interactions together with the relatively large mass of the strange quark imply that the strange quarks are less abundant than the other quarks. Thus, noninteracting quark matter is electrically positive and a nonzero density of electrons is required in order to obtain electrically neutral bulk matter.

Explicitly, weak equilibrium imposes

$$\mu_u = \overline{\mu} - \tfrac{2}{3}\,\mu_e, \qquad \mu_d = \mu_s = \overline{\mu} + \tfrac{1}{3}\,\mu_e\,, \tag{1}$$

where $3\overline{\mu}$ is the chemical potential for the baryon number and μ_e is that for the electron number. In the absence of interactions, the corresponding number densities are

$$N_{u,d} = \frac{1}{\pi^2}\,\mu_{u,d}^3 \qquad N_s = \frac{1}{\pi^2}\,(\mu_s^2 - m_s^2)^{3/2},$$
$$N_e = \frac{1}{3\pi^2}\,(\mu_e)^3. \tag{2}$$

(Throughout, we set $m_u = m_d = m_e = 0$ and $m_c = m_b = m_t = \infty$.) Electric neutrality requires

$$\tfrac{2}{3}\,N_u - \tfrac{1}{3}\,N_d - \tfrac{1}{3}\,N_s - N_e = 0\,. \tag{3}$$

With $m_s = 0$, this can be satisfied with $N_u = N_d = N_s$ and $N_e = 0$. Because $m_s > 0$, however, noninteracting quark matter must have $\mu_e > 0$ and an electron density $N_e > 0$ if it is to be electrically neutral. The condition (1) cannot be modified by the strong interactions among the quarks. It has long been known, however, that interactions can modify the relations (2) [1]. We argue here that interactions modify these relations *qualitatively*: they favor a state of quark matter in which $N_u = N_d = N_s$ and $N_e = 0$ even when $m_s \neq 0$ and $\mu_e \neq 0$, as long as neither is too large.

It is becoming widely accepted that at asymptotic densities the ground state of QCD with $m_s = 0$ is the color-flavor locked (CFL) phase [2–4]. In this phase, color gauge symmetry is completely broken, as are both chiral symmetry and baryon number (i.e., the material is superfluid). The effective coupling is weak (because QCD is asymptotically free), and the ground state and low-energy properties can be determined by adapting methods used in the theory of superconductivity [Bardeen-Cooper-Schrieffer (BCS) theory] [2–5]. The CFL phase persists for finite masses, and even for unequal masses, as long as the differences are not too large [6,7]. It is very likely the ground state for real QCD, assumed to be in equilibrium with respect to the weak interaction, over a substantial range of densities.

Since the CFL phase occurs at weak coupling, it might seem natural to think that, like noninteracting quark matter, CFL quark matter with $m_s \neq 0$ is electrically positive, and that its existence in bulk requires a neutralizing electron fluid. Indeed, this has been tacitly assumed in the literature. The presence of an electron fluid drastically affects the low-energy dynamics of dense matter in the CFL phase. Specifically, for example, the electron fluid dominates the low-temperature specific heat and powerfully resists the motion of magnetic field lines.

In this Letter we demonstrate that in reality the CFL phase requires equal numbers of u, d, s quarks, and is therefore automatically electrically neutral. No electrons are required. None are present, even when μ_e is nonzero.

There is a precedent for such behavior. In an ordinary superconductor, one may consider the effect of a perturbation $\delta\mathcal{L} = \delta\mu(e_\uparrow^\dagger e_\uparrow - e_\downarrow^\dagger e_\downarrow)$ that splits up and down spin energies. It has long been known that, for small $\delta\mu$, the superconducting ground state is completely unchanged by such a perturbation and, in particular, it contains equal numbers of up and down spins [8]. The analogous phenomenon was recently analyzed in QCD, in the context of pairing between two flavors of quarks with chemical potentials,

$$\mu_1 = \overline{\mu} - \delta\mu, \qquad \mu_2 = \overline{\mu} + \delta\mu\,. \tag{4}$$

For $0 < \delta\mu < \Delta_0/\sqrt{2}$, with Δ_0 being the superconducting gap, the ground state of the system is precisely that obtained for $\delta\mu = 0$ [9,10]. By introducing $m_s \neq 0$ in the Lagrangian, we are considering a rather different type of perturbation, varying the relative mass of the paired components, which changes (for example) the relative velocities within a pair. This perturbation does change the form of the superconducting ground state; nevertheless, the number of particles of different types remains equal.

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The argument relies on the fact that the CFL ordering, in weak coupling, involves pairing between quarks with equal and opposite momenta and with different flavors. For such pairing to be maximally effective, the Fermi surfaces for different flavors of quarks must occur at equal magnitudes of the *momentum* (as opposed to energy). To put it vividly, the Fermi surfaces are rigidly locked in momentum space. Since the number of occupied states, given by the occupied volume in momentum space, is now the same for all quark flavors, so is their number density. Deviation from $N_u = N_d = N_s$ would reduce the free energy in the absence of interactions; CFL pairing, however, reduces the free energy most strongly if $N_u = N_d = N_s$, and this equality is therefore enforced.

We now make the argument concrete. We work in a model in which quarks interact via a four-fermion interaction which we take to be that with the quantum numbers of single-gluon exchange [4]. The argument is sufficiently general, however, to apply qualitatively (and quantitatively if $\Delta_0/\overline{\mu}$ is small) to any model with four-fermion interactions which are attractive in the appropriate channel and to QCD at asymptotically high density, where quarks interact by single-gluon exchange.

We can demonstrate the physics of interest by focusing on pairing between, say, red up quarks and green strange quarks. Let us call these two species of quarks "1" and "2," assume they have masses $m_1 = 0$ and $m_2 = m_s$, and denote their chemical potentials as in (4). The generalization of the derivation of the gap equation for Δ_0 given in Section 4.3 of Ref. [4] to the case with $m_s \neq 0$ and $\delta\mu \neq 0$ is straightforward, although the algebra is somewhat involved. The result is

$$\frac{\Delta_0}{4G} = \int \frac{d^4p}{(2\pi)^4} \times \frac{\Delta_0 w}{w^2 - (4\overline{\mu}^2 - m_s^2)p^2 - (\overline{\mu} + ip_0 - \delta\mu)^2 m_s^2}, \tag{5}$$

where $w \equiv \Delta_0^2 + \overline{\mu}^2 + p^2 + (p_0 + i\delta\mu)^2$, where $p \equiv |\vec{p}|$, where the four-fermion coupling constant G is normalized as in Ref. [4], and where we have chosen to work in Euclidean space. This gap equation has been derived and solved explicitly with $\delta\mu = 0$ and $m_s \neq 0$ in Refs. [6,11] and with $m_s = 0$ and $\delta\mu \neq 0$ in Ref. [10]. All we will need, however, is the fact that, upon replacing the integration variable p_0 by $p_0' = p_0 + i\delta\mu$, the gap equation is seen to be independent of $\delta\mu$. This argument holds as long as the shift from p_0 to p_0' does not move a pole from the upper-half plane to the lower-half plane, or vice versa.

We wish to obtain the number densities by differentiating the free energy density Ω with respect to μ, and so must construct Ω. For noninteracting quarks with Fermi momenta ν_1 and ν_2,

$$\Omega_{\rm free}(\mu_1, \mu_2, \nu_1, \nu_2) = \frac{1}{\pi^2}\int_0^{\nu_1} p^2(p - \mu_1)\,dp + \frac{1}{\pi^2}\int_0^{\nu_2} p^2\left(\sqrt{p^2 + m_s^2} - \mu_2\right)dp\,. \tag{6}$$

We assert that the free energy of the BCS state is

$$\Omega_{\rm BCS} = \Omega_{\rm free}(\mu_1, \mu_2, \nu_1, \nu_2) + \int_0^{\Delta_0} d\Delta\left(-\frac{2\Delta}{G} + 8\int \frac{d^4p}{(2\pi)^4}\,\text{integrand}\right), \tag{7}$$

where the "integrand" is that on the right-hand side of the gap equation (5), where Δ_0 solves (5), and where

$$\nu_1 = \nu_2 = \overline{\mu} - \frac{m_s^2}{4\overline{\mu}}\,. \tag{8}$$

That is, we assert that the correct way to construct the BCS state is to first fill noninteracting quark states up to the common Fermi momentum (8), and to then pair. (The last term in (7) is the condensation energy [4].) Why is the procedure embodied in (7) correct? Thinking of ν_1 and ν_2 as variational parameters, under what circumstances does (8) minimize the free energy (7)? Note first that, if $\nu_1 = \nu_2 = \nu$ is imposed, then the choice (8) for ν minimizes $\Omega_{\rm free}$, and therefore minimizes $\Omega_{\rm BCS}$ because the condensation energy is independent of ν. Now, what about variation with respect to $\delta\nu \equiv \nu_2 - \nu_1$? Trying $\delta\nu > 0$ reduces $\Omega_{\rm free}$, but exacts a cost in reduced condensation energy: if $\nu_2 > \nu_1$ there is a region of momentum space $\nu_1 < p < \nu_2$ wherein pairing is impossible. (The pair creation operator tries to create either one quark of each type or one hole of each type, and there is already a "1" quark and a "2" hole at every point in this region of momentum space. See Ref. [10] for further analysis of such "blocking regions.") To lowest order in $m_s^2/\overline{\mu}^2$, $\delta\mu/\overline{\mu}$, and $\Delta_0/\overline{\mu}$, the free energy cost of increasing $\delta\nu$ from zero outweighs the free energy benefit if

$$\left|\frac{m_s^2}{4\overline{\mu}} - \delta\mu\right| < \Delta_0\,. \tag{9}$$

This condition can alternatively be derived via analysis of the locations of the poles in the integrand in (5): the requirement (9) *is* the requirement that shifting $p_0 \to p_0'$ in (5) not move a pole across the real axis.

The condition (9) has a simple interpretation: when the free energy gained either by converting a strange quark near the common Fermi surface into a light quark ($m_s^2/2\overline{\mu} - 2\delta\mu$) or by converting a light quark into a strange quark ($2\delta\mu - m_s^2/2\overline{\mu}$) compensates for the free energy lost by breaking a single pair ($2\Delta_0$), the paired state is unstable. Equation (9) is the criterion for the existence of the BCS phase as a *local* minimum of the free energy. To check that it is the global minimum, we must compare $\Omega_{\rm BCS}$ to that for the unpaired state:

$$\Omega_{\text{normal}} = \Omega_{\text{free}}(\mu_1, \mu_2, \mu_1, \sqrt{\mu_2^2 - m_s^2}). \qquad (10)$$

We use the gap equation to eliminate G in (7), work to lowest order in $m_s^2/\overline{\mu}^2$, $\delta\mu/\overline{\mu}$, and $\Delta_0/\overline{\mu}$, and find

$$\Omega_{\text{BCS}} - \Omega_{\text{normal}} = \frac{\overline{\mu}^2}{\pi^2}\left[\left(\frac{m_s^2}{4\overline{\mu}} - \delta\mu\right)^2 - \frac{\Delta_0^2}{2}\right], \qquad (11)$$

meaning that the BCS state is the global minimum of the free energy if

$$\left|\frac{m_s^2}{4\overline{\mu}} - \delta\mu\right| < \frac{\Delta_0}{\sqrt{2}}. \qquad (12)$$

Wherever (12) is an equality, there is a first order phase transition from the BCS state to the unpaired state. At this transition, the Fermi surfaces relax to the (separated) values favored in the absence of interaction. For $m_s = 0$, this agrees with previous results [10].

Wherever (12) is satisfied, and the paired state is favored, we now use the fact that the gap equation is independent of $\delta\mu$ to conclude that Ω_{BCS} of (7) is independent of $\delta\mu$ and therefore

$$N_1 = \frac{\partial\Omega_{\text{BCS}}}{\partial\mu_1} = \frac{\partial\Omega_{\text{BCS}}}{\partial\mu_2} = N_2. \qquad (13)$$

To lowest order in $\Delta_0/\overline{\mu}$,

$$N_1 = N_2 = \frac{(\overline{\mu} - m_s^2/4\overline{\mu})^3}{3\pi^2} - \frac{1}{2\pi^2}\frac{\partial}{\partial\overline{\mu}}\overline{\mu}^2\Delta_0(\overline{\mu})^2. \qquad (14)$$

Thus, $N_1 = N_2$ in the paired state even when $m_s \neq 0$ and $\delta\mu \neq 0$. Note that Δ_0, N_1, and N_2 all depend on m_s. The point is that $N_1 = N_2$, independent of m_s. The dependence of Δ_0 on m_s has been analyzed previously [6,7]; the only reason that these authors failed to notice that $N_1 = N_2$ is that they did not calculate N_1 and N_2.

The complete analysis of the CFL state with $m_s \neq 0$ requires the 9×9 block-diagonal color-flavor matrices given in Ref. [6]. The analysis is more involved, but the conclusion generalizes as follows: the number densities of all nine quarks (three colors and three flavors) are the same in the CFL phase, even when m_s and $\delta\mu = \mu_e/2$ are both nonzero. The excitations in the CFL phase include charged Nambu-Goldstone bosons [5], but this does not change the analysis of electrically neutral bulk matter: adding equal numbers of electrons and positively charged mesons costs free energy and is not favored. We conclude that quark matter in the CFL phase is electrically neutral in the absence of any electrons. Even an imposed μ_e cannot push electrons into the quark matter, because introducing electrons while maintaining charge neutrality and weak equilibrium costs too much pairing energy.

As an example, take $m_s = 200$ MeV and consider quark matter with $\overline{\mu} = 400$ MeV and $\mu_e = 2\delta\mu = 150$ MeV. (Contact with ordinary nuclear matter would impose $\mu_e > 0$.) According to (12), this quark matter, even with such a large μ_e, is in the electron-free CFL phase as long as $(\Delta_0/\sqrt{2}) > |25-75| = 50$ MeV. The value of Δ_0 is uncertain, but $\Delta_0 \sim 100$ MeV is not unreasonable [4]. Because the stresses imposed on the CFL phase by $m_s \neq 0$ and by $\mu_e > 0$ have opposite sign, it is more likely than previously thought that, if present, quark matter within neutron stars is in the CFL phase.

The criterion which defines the region wherein the CFL phase is favored will deviate somewhat from (12). First, there is now an electronic contribution to Ω_{normal} (but *not* to Ω_{BCS}). This contribution is only of order μ_e^4. Second, instead of comparing the CFL phase to a phase with no pairing, we should compare it to phases with less symmetric pairing. The CFL vs 2SC comparison of Ref. [6] does this at $\mu_e = 0$, and the value of m_s at which the unlocking transition occurs is in good agreement with (12), demonstrating that, at least at $\mu_e = 0$, this is not a large effect. Third, we expect that, as has been demonstrated at $m_s = 0$ [10], there is a region of (m_s, μ_e) just outside the CFL region (12) where crystalline color superconductivity, the analog of the Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) state in ordinary superconductivity [12], is favored. Outside the CFL region, very weak pairing among like flavor quarks is also possible [13]. Because both LOFF and single-flavor condensates have much less condensation energy than the CFL phase; their effect on the location of the boundary (12) is negligible.

We have demonstrated that, while CFL ordering is maintained, there will be strictly equal numbers of the three types of quarks and rigorous electrical neutrality, in the absence of any electrons. If m_s or μ_e becomes too large, however, some less symmetric phase of quark matter will have lower free energy. A first order phase transition occurs at this boundary as the rigidly locked Fermi surfaces spring free under accumulated tension.

The enforced neutrality of the color-flavor locked phase has many consequences:

In the CFL phase, there is an unbroken $U(1)_{\tilde{Q}}$ gauge symmetry and a corresponding massless photon given by a specific linear combination of the ordinary photon and one of the gluons [2,14]. With respect to the $\tilde{Q}$ charge associated with this unbroken U(1) symmetry, CFL quark matter is electrically neutral at zero temperature, and is a perfect insulator. Because of the absence of $\tilde{Q}$-charged excitations, CFL quark matter is transparent to $\tilde{Q}$ photons, and any $\tilde{Q}$-magnetic flux can move unimpeded. In contrast, because electrons have nonzero $\tilde{Q}$ charge, if they were present they would scatter $\tilde{Q}$ photons and would make the material a very good conductor in which $\tilde{Q}$-magnetic flux would be frozen in place [14]. The absence of electrons therefore changes the conclusions of Ref. [14]: quark matter in the 2SC phase in the core of a neutron star anchors magnetic fields, as described in Ref. [14]; quark matter in the CFL phase, however, is electron-free and therefore offers no resistance to the motion of $\tilde{Q}$-magnetic flux as the neutron star spins down.

Similarly, if electrons were present they would dominate the specific heat, which plays a role in the cooling of neutron stars [15,16]. In the absence of electrons, the specific heat at zero temperature is that of a neutral superfluid, much less than previously thought. The qualitative conclusion that the cooling of a neutron star with a CFL quark matter core is dominated by the (large) heat capacity of the nuclear matter mantle remains, however.

If neutron stars have CFL cores, the absence of electrons and the consequent reduction in specific heat and increase in transparency amplify the effects (described in Ref. [17]) imprinted on the time distribution of the neutrinos from a supernova by a transition from quark-gluon plasma to CFL quark matter as the hot, seconds-old protoneutron star cools. Effects of the first order nature of the transition need further investigation, however.

Schäfer realized that, if electrons were required to maintain charge neutrality, an alternative would be a condensate of negatively charged kaons in the CFL phase [18]. With $N_u = N_d = N_s$ in the CFL phase, however, we need neither kaon condensation nor a fluid of electrons.

The broader lesson is that, at temperatures which are nonzero and small compared to Δ_0, the transport and response properties of CFL quark matter, in the real world with nonzero m_s, are dominated by the lightest excitations of the CFL quark matter itself, and not by electrons as had previously been assumed. These bosonic degrees of freedom are the massless neutral Nambu-Goldstone boson associated with spontaneous baryon number violation (superfluidity) and the neutral and charged pseudo-Nambu–Goldstone bosons associated with spontaneous chiral symmetry breaking [2], which have masses of order $\sqrt{m_s m_{u,d}}\,\Delta_0/\overline{\mu}$, of order 10 MeV [4,5]. The effective field theory which describes these light degrees of freedom (and thus, we now see, the phenomenology) is known and at high enough density all coefficients in it can be determined by controlled, weak-coupling calculations [4,5].

Finally, the transition from an ordinary nuclear matter mantle to a quark matter core at some radius within a neutron star may be greatly simplified if the transition occurs directly to quark matter in the CFL phase. With a noninteracting quark matter core, one has to face the fact that, at any given $\overline{\mu}$, electrically neutral nuclear matter and electrically neutral quark matter generically have different values of μ_e. Since μ_e must be continuous across any interface, a mixed phase region is thought to form, within which positively charged nuclear matter and negatively charged quark matter with the same μ_e coexist at any given radius, with μ_e changing with radius [19]. If the quark matter is in the CFL phase, there is another possibility. At the $\overline{\mu}$ (i.e., the radius) at which $\Omega_{\rm CFL}$ crosses $\Omega_{\rm nuclear}$, an interface between bulk nuclear matter with $N_e \neq 0$ and bulk CFL quark matter with $N_e = 0$ may be stable, as long as the μ_e at the interface satisfies (12). The CFL insulator cannot admit electrons while remaining neutral, even when in equilibrium with nuclear matter with large μ_e. The description of the interface is more complicated than that of the bulk phases. Boundary layers within which local electric neutrality is not maintained and across which an electrostatic potential gradient develops are required, as at an ordinary metal-insulator boundary.

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[1] B. A. Freedman and L. D. McLerran, Phys. Rev. D **16**, 1169 (1977).

[2] M. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).

[3] R. Rapp, T. Schäfer, E. V. Shuryak, and M. Velkovsky, Ann. Phys. (N.Y.) **280**, 35 (2000); T. Schäfer, Nucl. Phys. **B575**, 269 (2000); I. A. Shovkovy and L. C. Wijewardhana, Phys. Lett. B **470**, 189 (1999); N. Evans, J. Hormuzdiar, S. D. Hsu, and M. Schwetz, Nucl. Phys. **B581**, 391 (2000).

[4] For a review, see K. Rajagopal and F. Wilczek, hep-ph/0011333.

[5] R. Casalbuoni and R. Gatto, Phys. Lett. B **464**, 111 (1999); D. T. Son and M. A. Stephanov, Phys. Rev. D **61**, 074012 (2000); **62**, 059902(E) (2000); D. K. Hong, T. Lee, and D. Min, Phys. Lett. B **477**, 137 (2000); C. Manuel and M. H. Tytgat, Phys. Lett. B **479**, 190 (2000); M. Rho, E. Shuryak, A. Wirzba, and I. Zahed, Nucl. Phys. **A676**, 273 (2000); K. Zarembo, Phys. Rev. D **62**, 054003 (2000); S. R. Beane, P. F. Bedaque, and M. J. Savage, Phys. Lett. B **483**, 131 (2000); D. K. Hong, Phys. Rev. D **62**, 091501 (2000); C. Manuel and M. Tytgat, hep-ph/0010274; R. Casalbuoni, R. Gatto, and G. Nardulli, hep-ph/0010321.

[6] M. Alford, J. Berges, and K. Rajagopal, Nucl. Phys. **B558**, 219 (1999).

[7] T. Schäfer and F. Wilczek, Phys. Rev. D **60**, 074014 (1999).

[8] A. M. Clogston, Phys. Rev. Lett. **9**, 266 (1962); B. S. Chandrasekhar, Appl. Phys. Lett. **1**, 7 (1962).

[9] P. F. Bedaque, hep-ph/9910247.

[10] M. Alford, J. Bowers, and K. Rajagopal, hep-ph/0008208.

[11] M. Alford, J. Berges, and K. Rajagopal, Phys. Rev. Lett. **84**, 598 (2000).

[12] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **47**, 1136 (1964) [Sov. Phys. JETP **20**, 762 (1965)]; P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964).

[13] T. Schäfer, Phys. Rev. D **62**, 094007 (2000).

[14] M. Alford, J. Berges, and K. Rajagopal, Nucl. Phys. **B571**, 269 (2000).

[15] D. Blaschke, T. Klahn, and D. N. Voskresensky, Astrophys. J. **533**, 406 (2000); D. Blaschke, H. Grigorian, and D. N. Voskresensky, astro-ph/0009120.

[16] D. Page, M. Prakash, J. M. Lattimer, and A. Steiner, Phys. Rev. Lett. **85**, 2048 (2000).

[17] G. W. Carter and S. Reddy, Phys. Rev. D **62**, 103002 (2000).

[18] T. Schäfer, nucl-th/0007021.

[19] N. K. Glendenning, Phys. Rev. D **46**, 1274 (1992).