Implications of Muon $g - 2$ for Supersymmetry and for Discovering Superpartners Directly

Lisa L. Everett, Gordon L. Kane, Stefano Rigolin, and Lian-Tao Wang
Michigan Center for Theoretical Physics, University of Michigan, Ann Arbor, Michigan 48109
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We study the implications of interpreting the recent muon $g_\mu - 2$ deviation from the standard model prediction as evidence for virtual superpartners, with very general calculations that include effects of phases and are consistent with all relevant constraints. Assuming that the central value is confirmed with smaller errors, there are upper limits on masses: at 1.5σ, at least one superpartner mass is below about 450 GeV (550 GeV) for $\tan\beta = 35$ (50) and may be produced at the Fermilab Tevatron in the upcoming run.

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Introduction.—Quantum corrections to the magnetic moments of the electron and the muon have played major roles historically for the development of basic physics. The recent report [1] of a 2.6 standard deviation value of the muon anomalous moment $a_\mu \equiv (g_\mu - 2)/2$ from its standard model (SM) value $a_\mu^{\rm exp} - a_\mu^{\rm SM} = 426(165) \times 10^{-11}$, assuming it is confirmed as the experiment and SM theory are further developed, is the first evidence that the standard model must be extended by new physics that must exist on the electroweak scale. Other data that imply physics beyond the SM (the matter asymmetry of the universe, cold dark matter, and neutrino masses) could all be due to cosmological or very short distance phenomena (though all have supersymmetric explanations), but a deviation from the SM value of $g_\mu - 2$ must be due to virtual particles or structure that exist on the scale of 100 GeV. Their contribution is as large as or larger than the effects of the known gauge bosons W and Z, so it must be due to particles of comparable mass and interaction strength [2].

Taking into account the stringent constraints on new physics from both direct searches and precision electroweak tests, there are several logical possibilities to consider. Presumably the possibility of muon substructure can be immediately disfavored as effects would already have been observed in processes involving more highly energetic muons at LEP, HERA, and the Tevatron. Effects on $g_\mu - 2$ have been studied in extensions of the standard model such as low energy supersymmetry (SUSY) [3–10] or $(\mathrm{TeV})^{-1}$ scale extra dimensions [11,12]. However, it has been argued [11] that the effects due to large extra dimensions are generally small compared to possible effects within SUSY models for typical parameter ranges. With this information in hand as well as the strong theoretical motivation for SUSY due to its resolution of the gauge hierarchy problem, gauge coupling unification, and successful mechanism for radiative electroweak symmetry breaking, we presume that the $g_\mu - 2$ deviation is due to supersymmetry and proceed to study it in that context.

The supersymmetry contribution to $g_\mu - 2$ is not automatically large. In models such as the MSSM (the minimal supersymmetric extension of the SM with two electroweak Higgs doublets and conserved R parity), it depends on the superpartner masses and other quantities that are not yet compellingly predicted by any theory, just as the muon mass itself is not yet understood. The most important quantity involved besides masses is called $\tan\beta$. It is the ratio of the two vacuum expectation values $v_{1,2}$ of the Higgs fields that breaks the electroweak symmetry and gives masses to the SM particles; the superpartners also get mass from these sources as well as from supersymmetry breaking. At the unification/string scale where a basic effective Lagrangian for a four dimensional theory is written all particles are massless. At the electroweak phase transition the Higgs fields get vacuum expectation values (VEVs), and the quarks and leptons get masses $m_i = Y_i v_{1,2}$ via their Yukawa couplings Y_i to the Higgs fields. If the heaviest particles of each type, the top quark, the bottom quark, and the tau charged lepton, have Yukawa couplings of order gauge couplings, as can happen naturally in certain string approaches and in grand unified theories larger than SU(5) [13], then the masses and the VEVs are proportional such that the ratio of the VEV v_2 that gives mass to the top quark to the VEV v_1 that gives mass to the bottom quark is $\tan\beta \equiv v_2/v_1 \simeq m_{\rm top}/m_{\rm bottom}$.

Supersymmetric theories are (perturbatively) consistent for any value of $\tan\beta$ between about 1 and 50; values of $\tan\beta$ very near 1 are already ruled out from direct Higgs searches at LEP [14]. A large value of $\tan\beta$ has theoretical motivation both from the unification of the Yukawa couplings just given, and also that 115 GeV is a natural value for the mass of a Higgs boson if $\tan\beta$ is in this range [15] (this of course is the recently reported value for which there is evidence from LEP [16]). For $\tan\beta$ larger than about 5 the supersymmetric contribution to a_μ is essentially proportional to $\tan\beta$, as explained below. In minimal supersymmetric theories with small $\tan\beta$ it is very difficult to get a Higgs boson mass as large as 115 GeV, so we think the correlation between the Higgs mass and $g_\mu - 2$ is significant. Since supersymmetry is a decoupling theory, i.e., its contributions decrease as the superpartner masses increase (see, e.g., [17]), a nonzero contribution to $g_\mu - 2$ puts an *upper* limit on the superpartner masses that give the main contributions, the sleptons (the

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smuon and muon sneutrino) and the lighter chargino and/or neutralino.

In the following we study the one-loop supersymmetric contributions to $g_\mu - 2$ with general amplitudes, allowing in particular the full phase structure of the theory, and we check that the results are consistent with all relevant constraints. Although $g_\mu - 2$ in the context of supersymmetry has been studied extensively in the previous literature, if we assume the new experimental results will be confirmed and have errors 2–3 times smaller soon, an analysis of the data yields the first *independent upper limits* on slepton and chargino/neutralino masses, along with a *lower bound* on $\tan\beta$. We also demonstrate explicitly the effects of the relevant phase combination on $g_\mu - 2$ in the large $\tan\beta$ regime [shown later in Eq. (1)]. In much of the parameter space, $g_\mu - 2$ constrains only the combination $\tan\beta\cos\tilde{\varphi}$ (giving the previously unknown result that a nonzero value for this phase can only decrease the SUSY effect for a given $\tan\beta$).

Theoretical framework.—The one-loop contributions to $a_\mu = (g_\mu - 2)/2$ in supersymmetric models include chargino-sneutrino $(\tilde{\chi}^+ - \tilde{\nu}_\mu)$ and neutralino-smuon $(\tilde{\chi}^0 - \tilde{\mu})$ loop diagrams in which the initial and final muons have opposite chirality. Other contributions are suppressed by higher powers of the lepton masses and are negligible. As previously stated, the SUSY contributions to a_μ have been studied extensively by a number of authors [3–9], where the expressions for these amplitudes can be found.

Note that the majority of these studies assume simplified sets of soft breaking Lagrangian parameters based on the framework of minimal supergravity. However, in more general SUSY models the soft breaking parameters and the supersymmetric mass parameter μ may be complex. Several of the relevant phases are severely constrained by the experimental upper limits on the electric dipole moments (EDMs) of the electron and neutron, although the constraints can be satisfied by cancellations [18–21]. The phases, if non-negligible, not only affect CP-violating observables but also can have important consequences for the extraction of the MSSM parameters from experimental measurements of CP-conserving quantities, since almost none of the Lagrangian parameters are directly measured [22]. The effects on $g_\mu - 2$ due to the phases have recently been studied in [9], where the general expressions for the amplitudes including phases are presented (but analyzed mainly for small $\tan\beta$).

The general results of these studies have shown that the SUSY contributions to a_μ can be large for large $\tan\beta$ and have either sign, depending on the values of the SUSY parameters. In particular, it is well known that for large $\tan\beta$ the chargino diagram dominates over the neutralino diagram over most of the parameter space [4–6], and is linear in $\tan\beta$. This effect can be seen most easily in the mass insertion approximation, where the main contribution arises from the chargino diagram in which the required chirality flip takes place via gaugino-higgsino mixing rather than by an explicit mass insertion on the external muon [4–6]. In this case, the chargino contribution to $g_\mu - 2$ can be written as proportional to

$$a_\mu^{\tilde{\chi}^+} \simeq a_\mu^{\rm SUSY} \propto (m_\mu^2/\tilde{m}^2)\tan\beta\cos(\varphi_\mu + \varphi_2), \quad (1)$$

in which φ_μ is the phase of the supersymmetric Higgs mass parameter μ, and φ_2 is the phase of the SU(2) gaugino mass parameter M_2; the reparametrization invariant quantity is $\tilde{\varphi} \equiv \varphi_\mu + \varphi_2$ (note in the case of zero phases the sign of $a_\mu^{\rm SUSY}$ is given in this limit by the relative sign of μ and M_2 [4–6]). Also note that $a_\mu^{\rm SUSY}$ depends on m_μ^2 because this diagram involves one power of the muon Yukawa coupling due to the coupling of the right-handed external muon with the higgsino, and the definition of a_μ is $a_\mu = -F_2(0)/2m_\mu$ [where $F_2(q^2)$ is the form factor]. This expression can be compared with the expression for the chargino contribution to the electron EDM in the mass insertion approximation [20], as the electric dipole moment is proportional to the imaginary part of $M_2\mu$ while the anomalous magnetic dipole moment is proportional to the real part. Therefore, the electron EDM can be obtained from Eq. (1) after replacing $m_\mu \to m_e$ and $\cos(\varphi_\mu + \varphi_2) \to \sin(\varphi_\mu + \varphi_2)$. Similar expressions hold for the neutralino sector [6,20]; while they are generally suppressed due to the smaller neutral current coupling, they can be relevant for cases in which $m_{\tilde{l}}, M_1 \ll M_2, \mu$.

The fact that $a_\mu^{\rm SUSY}$ may have either sign at first may seem counterintuitive, given the well-known result [9,23,24] that $a_\mu^{\rm SUSY}$ exactly cancels $a_\mu^{\rm SM}$ in the unbroken SUSY limit, with the cancellations taking place between the chargino and the W, the massless neutralinos and the photon, and the massive neutralinos and the Z. (The general statement [23] is that any magnetic-transition operator vanishes in this limit, due to the usual cancellation between fermionic and bosonic loops in SUSY theories.) As $a_\mu^{\rm SM}$ is known to be positive [25], $a_\mu^{\rm SUSY}$ is negative in this limit. However, the limit with unbroken SUSY but broken electroweak symmetry requires the supersymmetric Higgs mass parameter $\mu = 0$ and $\tan\beta = 1$, as can be seen from the Higgs potential when the soft breaking parameters are zero: $V = |\mu|^2(v_1^2 + v_2^2) + (g^2 + g'^2)(v_2^2 - v_1^2)^2/2$. At low (but $\gtrsim 1$) values of $\tan\beta$ and nonzero μ, the chargino and neutralino contributions are comparable in magnitude but opposite in sign; however the neutralino diagram dominates as the parameters deviate from the unbroken SUSY limit since the contribution from the (nearly) massless neutralinos is much larger than that of the massive neutralinos and charginos (recall this contribution cancels the much larger photon contribution in the exact SUSY limit). At larger values of $\tan\beta$ the chargino diagram begins to dominate and the sign of $a_\mu^{\rm SUSY}$ can flip depending on the relative sign (or phase) of μ and M_2. In the traditional convention in which M_2 is chosen to be real and positive, the sign of $g_\mu - 2$ is given by the sign of μ. We pause here to comment on the sign of μ as it relates to the $b \rightarrow s\gamma$ decay [24,26]. In the literature it is often claimed that the constraints on the SUSY parameter space due to the requirement of canceling the various SUSY contributions to the $b \rightarrow s\gamma$ branching ratio place a strong constraint on the sign of μ. In SUSY models with general phases, the branching ratio actually involves a reparametrization invariant combination of phases $\varphi_\mu + \varphi_{A_t}$ (which reduces to the relative sign for zero phases). The relative sign from $b \rightarrow s\gamma$ thus is not the same combination as that constrained by $g_\mu - 2$; however, in the usual conventions with M_2 and A_t real and positive, both processes favor positive μ.

Results and discussion.—We calculated the complete one-loop MSSM contribution to $g_\mu - 2$, taking into account the possibility of nontrivial CP-violating phases for the μ term and the soft breaking parameters (see [27] for general formulas and conventions). We have made a few simplifying assumptions which do not have a significant impact on superpartner upper limits. First, we have assumed that the masses of the charginos and neutralinos are heavier than 100 GeV (allowing the parameters M_1, M_2, and μ to range between 100 GeV and a few TeV). This assumption will be easily verified for the charginos as soon as LEPII reports its final results on new searches at $\sqrt{s} = 208$ GeV. LEPII will not be able to provide such a limit on the neutralinos. There are constraints on neutralino masses but not general ones, so somewhat lighter neutralinos may be allowed. The neutralino contribution is usually small compared to the charginos so the assumption that neutralinos are not very light does not affect the upper bounds we put on superpartner masses (it does affect the $\tan\beta$ bound, as explained below). In addition, we assumed a common slepton mass $m_{\tilde{l}} > 100$ GeV for the left and right smuons and for the muon sneutrino. For very light masses that is not a good approximation, but it does not change the numerical results for upper limits on masses. We assumed also that $|\mu|\tan\beta \gg |A_\mu|$, which is a reasonable assumption for $\tan\beta > 3$. Moreover, as the smuon mass matrix enters only in the suppressed neutralino contribution, the details of the charged slepton mass matrix are almost irrelevant in the numerical analysis (except at certain exceptional points in parameter space). Thus, the most important parameter in the slepton sector is the sneutrino mass, which is likely to have a LEPII lower limit. However, as the sneutrino mass enters only in the loop integrals as a suppression, relaxing this assumption does not change our general conclusions.

In Fig. 1 we show the $m_{\tilde{\chi}_1} - m_{\text{slepton}}$ range allowed at 1.5σ by the $g_\mu - 2$ measurement for five different values of $\tan\beta$ ($\tan\beta = 5, 10, 20, 35$, and 50), where $m_{\tilde{\chi}_1}$ denotes the lightest chargino or neutralino, and m_{slepton} denotes the smuon or muon sneutrino (to connect more closely with physical objects we use mass eigenstates instead of the Lagrangian mass parameters). We think 1.5σ is a good compromise for showing what the implications will be if the results are confirmed by improved data and SM theory. The $g_\mu - 2$ data will provide the first phenomenological *upper* limit on superpartner masses. The region above the lines is excluded because the masses are too heavy to account for the experimentally observed δa_μ difference.

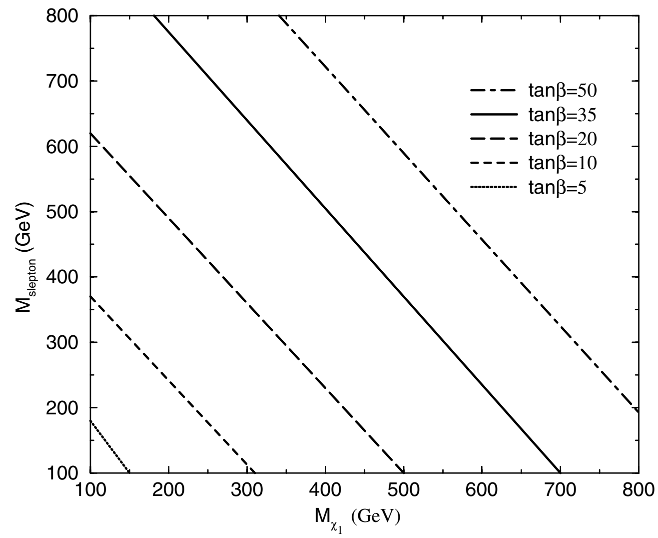

FIG. 1. In this figure m_{χ_1} denotes the lightest chargino or neutralino, and m_{slepton} the lightest smuon or muon sneutrino. $\tan\beta$ is the ratio of the two Higgs vacuum expectation values. The regions above a given $\tan\beta$ line are excluded. We require agreement with experiment within 1.5σ (see text). Below $\tan\beta \approx 5$ no allowed region remains (except perhaps for very light neutralinos; see text). Thus the figure implies related upper limits on the lightest chargino/neutralino and slepton masses.

These regions are obtained for zero phases. In the regions of parameter space in which the chargino diagram dominates, $g_\mu - 2$ depends on $\tan\beta\cos(\varphi_\mu + \varphi_2)$ [see Eq. (1)], such that nonzero phases only decrease $g_\mu - 2$ for a given $\tan\beta$. In addition, our combined analysis for the electron EDM shows that in this region $\varphi_\mu + \varphi_2$ is severely constrained to be less than 10^{-2}, due to the fact that cancellations are more difficult to achieve for large $\tan\beta$ (and two-loop contributions which we have neglected here may become important [28]). In certain exceptional points of parameter space in which the neutralino and chargino diagrams are comparable (i.e., with light sleptons and $M_1 \ll m_2, \mu$), the EDM cancellations must be reconsidered for large $\tan\beta$.

From Fig. 1, important constraints on the MSSM parameter space can be obtained. There is a maximum range allowed for the lightest chargino, neutralino, and slepton masses. For $\tan\beta = 35$, values of $m_{\tilde{\chi}_1} > 700$ GeV and $m_{\tilde{l}} > 900$ GeV are disfavored by $g_\mu - 2$ measurements at 1.5σ (note if one is near the limit the other is much lighter). For lower $\tan\beta$ allowed masses are always smaller. "Effective supersymmetry" scenarios [29], characterized by multi-TeV mass first and second generation squarks and sleptons, are ruled out.

Further, the $g_\mu - 2$ measurement implies a lower bound for $\tan\beta \gtrsim 5$ at 1.5σ (note nonzero phases do not affect this lower bound). Lower values of $\tan\beta$ give too small a contribution to $g_\mu - 2$ for almost all of the parameter space; however, there may be a small corner of parameter space for light neutralinos [with masses of $\approx O(50\ \text{GeV})$] where $\tan\beta$ can be as little as about 3. As improved measurements become available, $g_\mu - 2$ will determine $\tan\beta$ with increased precision. Measuring $\tan\beta$ is extremely difficult at hadron colliders [22], yet $\tan\beta$ is an extremely important parameter for supersymmetry predictions and tests. To obtain a large $g_\mu - 2$ large $\tan\beta$ is necessary, and since the size is essentially proportional to $\tan\beta$ it is immediately approximately known. It can then be determined accurately when a few superpartner masses and the soft phases (which are constrained from EDMs) are known.

Summary.—Because the reported $g_\mu - 2$ number is larger than the standard model prediction by an amount larger than the W and Z contributions, it implies several significant results. We presume the effect arises from superpartner loops; the chargino-(muon)sneutrino loop typically dominates, with the neutralino-smuon relevant in certain restricted regions of parameter space. Then, in approximately decreasing order of interest, we have as follows:

(i) One superpartner, either a chargino, neutralino, or a slepton, has to be lighter than about 450 GeV (for $\tan\beta = 35$; see Fig. 1 for more precise numbers).

(ii) The heavier one of the lightest chargino or sneutrino has to be lighter than about 900 GeV (for $\tan\beta = 35$; see Fig. 1). In general, models with heavier sleptons are disfavored.

(iii) $\tan\beta$ has to be larger than about 5 (in corners of parameter space with neutralinos of masses of order 50 GeV, which may not be ruled out by other data, $\tan\beta$ can be lower). Larger values of $\tan\beta$ are sufficient to obtain a Higgs boson mass of about 115 GeV, and also imply a number of interesting phenomenological consequences (e.g., [30]).

The $g_\mu - 2$ measurement is the first data to establish a firm *upper* limit on any superpartner masses. Over most of the allowed masses, the superpartners will be produced [31] at Fermilab in the upcoming run.

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