Radio-Frequency Bloch-Transistor Electrometer

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A quantum electrometer is proposed which is based on charge modulation of the Josephson supercurrent in the Bloch transistor inserted in a superconducting ring. As this ring is inductively coupled to a high-Q resonance tank circuit, the variations of the charge on the transistor island are converted into variations of amplitude and phase of oscillations in the tank. These variations are amplified and then detected. At sufficiently low temperature of the tank the device sensitivity is determined by the energy resolution of the amplifier, that can be reduced down to the standard quantum limit of $\frac{1}{2}\hbar$. A "back-action-evading" scheme of subquantum limit measurements is proposed.

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The single electron transistor (SET) whose operation is based on correlated electron tunneling in small-capacitance double junctions has significantly extended the possibilities of modern experiments. This remarkable device with subelectron sensitivity to the charge induced on its central electrode (island) has made it possible to study the electron transport and noise processes in various mesoscopic structures (see, for example, the review by Likharev [1]). In recent years, especially after the encouraging experiment by Nakamura *et al.* [2], the possibility of using SET electrometers for measuring the quantum state of the charge qubit (Cooper-pair box) has been extensively discussed [3]. In such measurements both the sensitivity of the detector (electrometer) to the input signals and its destructive back action on the quantum mechanical state of the box are of utmost importance. The detector's figure of merit, which takes into account the back-action effect, is the energy resolution ϵ in the unit bandwidth. According to the quantum mechanical uncertainty principle for a phase-insensitive detector, the figure $\epsilon \geq \frac{1}{2}\hbar$. Its ultimate value of $\frac{1}{2}\hbar$ (the so-called standard quantum limit—SQL) can be approached by a perfect (quantum-limited) device [4].

The normal-state metallic SET operating in the regime of sequential electron tunneling does not belong to the category of perfect devices. In the usual case of high tunneling resistance of the junctions $R_t \gg R_Q$ (here $R_Q \equiv h/4e^2 \approx 6.5\ \mathrm{k}\Omega$ is the resistance quantum), the value $\epsilon \gg \frac{1}{2}\hbar$ [5]. SQL can, in principle, be approached using SET with $R_t \gtrsim R_Q$ and operating it in the cotunneling regime at very low voltage bias [6,7]. However, in this case the output signal of the electrometer is vanishingly small so that the regime can hardly be practical.

In contrast to the SET operating on "normal carriers," i.e., electrons, its superconducting counterpart with appreciable strength of Josephson coupling E_J in the junctions, i.e., the Bloch transistor [8], can operate in the regime of a gate-controlled supercurrent at zero quasiparticle current. In this regime the charge carriers are the Cooper pairs with charge $2e$, and their transfer across the junctions takes place without power dissipation (and, hence, without shot noise) in the transistor. Read-out of the critical current value can be performed by measuring the voltage across the resistor with $R_s \ll R_Q$, shunting the transistor [9]. Although this electrometer is a quantum-limited device, its implementation still suffers from a low ($\propto R_s/R_Q$) conversion factor [10].

In this paper we propose an electrometer with the Bloch transistor inserted in a superconducting ring which is inductively coupled to the rf-driven resonance tank circuit. In contrast to the so-called rf-SET electrometer [11] based on charge-dependent dissipation in a resonance circuit containing a normal SET, the Bloch transistor controls the ac supercurrent in the loop and, hence, the effective reactance of the tank circuit. As a result, both the amplitude and phase of oscillations in the tank circuit depend on the island charge. This mode of electrometer operation is similar to that of a single-junction superconducting quantum interferometer device (SQUID) with low critical current (nonhysteretic regime) [12]. The goal of this paper is to compute the characteristics of the electrometer and demonstrate its potential for qubit measurements.

The equivalent electric diagram of the electrometer is presented in Fig. 1. The characteristic Josephson coupling energies in the first and second junctions of the transistor are assumed to be not very different, $E_{J1} \simeq E_{J2}$; both of them will, therefore, be characterized by the parameter $E_J = \Phi_0 I_{c0}/2\pi$, where $\Phi_0 \equiv h/2e \approx 2.07 \times 10^{-15}$ Wb is the flux quantum and I_{c0} is the nominal critical current of

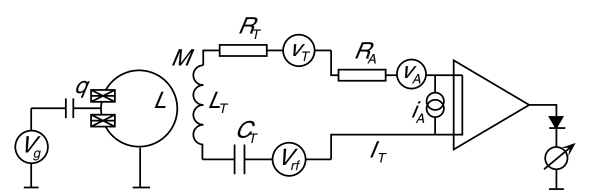

FIG. 1. Electric diagram of the rf-Bloch electrometer comprising the grounded superconducting ring including the Bloch transistor with capacitively coupled gate, the series-resonance tank circuit driven by source V_{rf}, a linear amplifier, and an amplitude (or phase) detector.

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individual junctions. The charge-sensitive element of the device is the transistor island. We assume that the total capacitance of the island $C = C_1 + C_2 + C_g$ is sufficiently small (here $C_{1,2}$ and C_g are the self-capacitances of the junctions and the coupling capacitance to the gate, respectively). The corresponding charging energy $E_c = e^2/2C$, the Josephson energy E_J, and the energy gap Δ of the superconducting material the transistor is made of should obey the condition

$$\Delta > E_c \sim E_J \gg k_B T \,, \tag{1}$$

where T is the temperature. This relation ensures blockade of quasiparticle tunneling across the junctions owing to the even-odd parity effect on the superconducting island [13].

The relation chosen, $E_J/E_c \equiv \lambda \sim 1$, first ensures substantial modulation of the supercurrent in the whole range of variation of the polarization charge on the island, $-e \leq q \leq e$ [9,14]. Second, the width of the forbidden band in the energy spectrum ($\approx E_J$ at $\lambda \lesssim 1$ [15]) is large enough to prevent thermal excitation of higher Bloch bands leading to a reduction of the resultant critical current and of the depth of its modulation by the gate. For $\lambda \approx 1$ the critical current of the transistor $I_c(q) = \alpha(q) I_{c0}$, with the value α varied in the range from 0.24 ($q = 0$) to 0.56 ($q = \pm e$) [9]. Then the supercurrent-charge-phase relation is approximated by the formula $I_s = I_c(q)\sin\varphi$ [14]. Because of finite Josephson coupling the effective capacitance of the electrometer island becomes nonlinear [9,15]. For $-\frac{1}{2}e \lesssim q \lesssim \frac{1}{2}e$ its value is $C'(q) = \beta(q) C$, where the factor $\beta(q) \gtrsim 1$ can be assumed to be constant for moderate $\lambda \lesssim 1$.

The inductance L of the superconducting ring incorporating the Bloch transistor should obey two conditions:

$$\ell \equiv 2\pi L I_c(q)/\Phi_0 < 1 \quad \text{and} \quad \Phi_0^2/2L \gg k_B T \,. \tag{2}$$

The first relation ensures the single-valued dependence of the total flux $\Phi = \Phi_e - L I_c(q) \sin(2\pi\Phi/\Phi_0)$, which threads the loop, on the external flux $\Phi_e = \Phi_{\rm dc} + \Phi_{\rm rf}$ applied to the loop (see, e.g., Ref. [16]). The constant flux $\Phi_{\rm dc}$ can be induced by dc current through an auxiliary coil (not shown in Fig. 1), while flux $\Phi_{\rm rf}$ is induced by the tank circuit current. For sufficiently small values of ℓ, the flux $\Phi \approx \Phi_e$ and the Josephson phase $\varphi = 2\pi\Phi/\Phi_0 \approx 2\pi\Phi_e/\Phi_0$. The second relation in Eq. (2) ensures an exponential smallness of thermodynamic fluctuations of flux Φ. Thus, the Josephson phase φ across the transistor behaves almost as a classical variable whose value and (small) fluctuations are determined by the current in the tank circuit.

The eigenfrequency of the tank circuit $\omega_0 = (L_T C_T)^{-1/2}$ and the frequency $\omega \approx \omega_0$ of the rf drive $V_{\rm rf} = V_\omega \cos\omega t$ are assumed to be sufficiently low, i.e., $\omega \ll \omega_J \equiv E_J/\hbar \sim E_c/\hbar$, and do not, therefore, excite the Bloch transistor by means of an alternating Josephson phase $\varphi(t)$. In our model, φ is considered a slowly varied parameter in the Hamiltonian [9] of the transistor system. The quality factor is $Q = \omega L/R_\Sigma = (\omega C_T R_\Sigma)^{-1} \gg 1$, where $R_\Sigma = R_T + R_A$ is the total series resistance of the tank circuit. The dimensionless coupling coefficient is $\kappa = M/(L L_T)^{1/2} \ll 1$, where M is the mutual inductance, while the product

$$\kappa^2 Q \ell > 1 \,. \tag{3}$$

A similar regime of operation of single-junction SQUIDs, proposed by Danilov and Likharev [17], offers a significant experimental advantage in the sense of a large transfer coefficient [18].

The noise associated with resistance R_T is represented by the voltage v_T with a power spectrum

$$S_T(\omega) = \frac{2}{\pi}\,\Theta_T R_T \,, \tag{4}$$

where $\Theta_T = (\frac{\hbar\omega}{2})\coth(\frac{\hbar\omega}{2k_B T_T})$ with T_T symbolizing the temperature of the tank circuit. The internal sources of noise in the amplifier are represented by the two uncorrelated generators v_A and i_A with the spectral densities

$$S_V(\omega) = \frac{2}{\pi}\,\Theta_A R_A \quad \text{and} \quad S_I(\omega) = \frac{2}{\pi}\,\frac{\Theta_A}{R_A} \,, \tag{5}$$

respectively, where $\Theta_A = (\frac{\hbar\omega}{2})\coth(\frac{\hbar\omega}{2k_B T_A})$ with T_A symbolizing the noise temperature of the amplifier in the classical limit $k_B T_A \gg \hbar\omega$ [4]. The (low) active input impedance of the amplifier obeys the relation $R_A = (S_V/S_I)^{1/2}$ while the "uncertainty product" $(S_V S_I)^{1/2} \geq \hbar\omega/\pi$ [19].

The three principal sources of fluctuations (v_T, v_A, and i_A) [20] determine the device's sensitivity which is derived from the equations of the electric circuit of Fig. 1 linearized with respect to a small input signal and small fluctuations. These equations are solved by the method of harmonic balance. Whereas similar equations describing the conventional (flux-sensitive) rf-SQUID were solved by this method elsewhere [16,17,21], here we focus only on the details associated with the charge origin of the input signal.

Because of large Q and weak coupling κ, the steady oscillations of the tank circuit current, $I_T = I_a \cos(\omega t + \vartheta)$, and the Josephson phase,

$$\varphi = a\cos(\omega t + \vartheta) + \varphi_0 \,, \tag{6}$$

are quasiharmonic with slowly varying parameters $a = 2\pi M I_a/\Phi_0$ and ϑ and constant phase $\varphi_0 = 2\pi\Phi_{\rm dc}/\Phi_0$. The dependence of the dimensionless amplitude a on the detuning $\xi_0 = (\omega - \omega_0)/\omega_0$ is shown in Fig. 2. At a sufficiently large amplitude V_ω this dependence is multivalued. This property allows high values (theoretically infinite) of the transfer coefficients "charge-to-amplitude" and "charge-to-phase" to be realized. Because of the shift of the resonance frequency in the tank circuit coupled

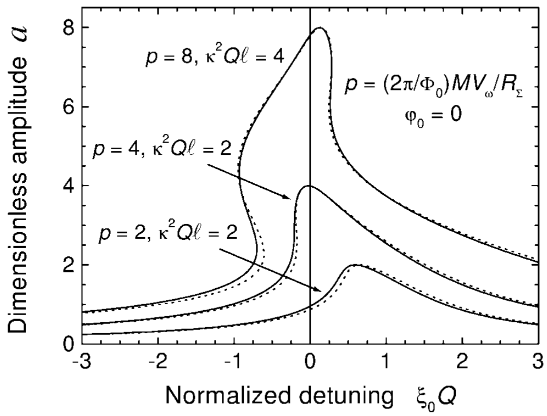

FIG. 2. Resonance curves of the tank circuit for different values of the drive amplitude V_ω and the value of product $\kappa^2 Q\ell$. The dotted lines correspond to a 10% increase in critical current $I_c(q)$ caused by a variation of charge q.

to the electrometer loop, the effective detuning is $\xi = \xi_0 - \kappa^2 \ell(q)\cos\varphi_0 J_1(a)/a$. Here, J_1 is the first-order Bessel function. These peculiar curves are typical of the rf-SQUID (see, e.g., Ref. [16]).

The coefficients $\eta_a = |\frac{\partial I_a}{\partial q}|$ and $\eta_\vartheta = I_a|\frac{\partial \vartheta}{\partial q}|$ governing the transformation of small charge variations δq into variations of two orthogonal components of ac current in the tank, δI_a and $I_a \delta\vartheta$, are expressed as $\eta_a = |\xi|\eta_0$ and $\eta_\vartheta = (2Q)^{-1}\eta_0$, respectively. Here, the factor η_0 is

$$\eta_0 = \mu \frac{M}{L_T}\left|\frac{\cos\varphi_0 J_1(a)}{(2Q)^{-2} + \xi\tilde{\xi}}\right|, \tag{7}$$

the transfer function $\mu = |\frac{\partial I_c}{\partial q}|$ and dynamic detuning $\tilde{\xi} = \xi_0 - \kappa^2 \ell \cos\varphi_0 J_1'(a)$. Equation (7) in particular shows that zero magnetic flux Φ_{dc}, giving $\varphi_0 = 0$ and, hence, $\cos\varphi_0 = 1$, ensures a maximum of either η_a and η_ϑ. The optimum amplitude of the rf drive should give the value of $a \approx 1.8$ yielding the maximum value of $(J_1)_{\max} = j_1 \approx 0.58$.

For the amplitude (phase) detection of a low-frequency signal ($\omega_s \ll \omega$) , the output resolution in bandwidth Δf,

$$\delta q_x = \langle \tilde{q}^2 \rangle^{1/2} = \eta_{a,\vartheta}^{-1}(2\pi S_{a,\vartheta}\Delta f)^{1/2}, \tag{8}$$

is determined by the spectral density of the in-phase (out-of-phase) fluctuations of the current flowing through the amplifier,

$$S_{a,\vartheta} = \frac{g_{a,\vartheta}(\xi,\tilde{\xi})}{R_\Sigma^2}[S_T(\omega) + S_V(\omega)] + S_I(\omega), \tag{9}$$

where $g_a(\xi,\tilde{\xi}) = Q^{-2}(Q^{-2} + 4\xi^2)(Q^{-2} + 4\xi\tilde{\xi})^{-2} = g_\vartheta(\tilde{\xi},\xi)$. The *output* noise in the energy representation $\epsilon_I = \langle \tilde{q}^2 \rangle/(2C'\Delta f)$ finally is

$$\epsilon_I^{(a,\vartheta)} = \frac{b d_{a,\vartheta}}{\kappa^2 Q\ell\omega}\left(\frac{R_T}{R_\Sigma}\Theta_T + \frac{R_A}{R_\Sigma}\Theta_A + \frac{R_\Sigma}{g_{a,\vartheta}R_A}\Theta_A\right). \tag{10}$$

Here, the numerical factor $b = \pi I_c/(j_1^2 \Phi_0 \mu^2 C')$, while

$$d_a = \frac{Q^{-2} + 4\xi^2}{4\xi^2} \quad \text{and} \quad d_\vartheta = \frac{Q^{-2} + 4\tilde{\xi}^2}{Q^{-2}} \tag{11}$$

for the amplitude and phase detection, respectively. Note that, owing to the large value of product $\kappa^2 Q\ell$ [Eq. (3)], the output noise figures $\epsilon_I^{(a,\vartheta)}$ (they do not include the back-action effect) may be smaller than $\frac{1}{2}\hbar$ within the limit $T_T, T_A \ll \hbar\omega/k_B$.

The electrometer back action on the source of the input charge is determined by low-frequency ($\sim\omega_s$) fluctuations of the electric potential of the transistor island $\tilde{U} = \frac{\Phi_0}{2\pi}\mu\widehat{\sin\varphi\,\tilde{\varphi}}$ [9]. Here $\tilde{\varphi}$ are fluctuations of the Josephson phase Eq. (6) and $\widehat{\ldots}$ denotes averaging over time τ: $2\pi/\omega \ll \tau \ll 2\pi/\omega_s$. Finally, the *input* noise figure $\epsilon_U = C'\langle \tilde{U}^2 \rangle/(2\Delta f)$ for either regime is given by

$$\epsilon_U^{(a)} = \epsilon_U^{(\vartheta)} = \frac{g_a \kappa^2 Q\ell}{b\omega}\left(\frac{R_T}{R_\Sigma}\Theta_T + \frac{R_A}{R_\Sigma}\Theta_A\right). \tag{12}$$

From Eq. (6) it follows that fluctuations $\tilde{U}$ are proportional to fluctuations of amplitude $\tilde{a}$; therefore, these two signals are completely correlated. Because of this fact the cross correlation $\epsilon_{IU} = |\langle \tilde{q}\tilde{U} \rangle|/2\Delta f$ in the regime of amplitude detection has the largest magnitude which is equal to the geometric mean of $\epsilon_U^{(a)}$ of Eq. (12) and $\epsilon_I^{(a)}$, with the third term omitted in Eq. (10). The energy resolution of a narrow-band signal [9,21,22]

$$\epsilon = (\epsilon_I \epsilon_U - \epsilon_{IU}^2)^{1/2} \tag{13}$$

then is equal to

$$\epsilon = \omega^{-1}\{d_a \Theta_A[(R_T/R_A)\Theta_T + \Theta_A]\}^{1/2}. \tag{14}$$

This equation shows that the electrometer figure of merit ϵ depends decisively on the amplifier parameter Θ_A. In particular, for $R_T\Theta_T \ll R_A\Theta_A$ and $|\xi| \gg (2Q)^{-1}$, the figure $\epsilon = \Theta_A/\omega$, and its value approaches the SQL of $\frac{1}{2}\hbar$ at $k_B T_A < \hbar\omega$.

We arrive at the remarkable property of the rf-Bloch electrometer: It converts an input charge into an output signal introducing only insignificant noise on the stage preceding the amplifier. This is because the device operates as a parametric converter $\omega_s \to (\omega \pm \omega_s) \to \omega_s$ (similar to the single-junction SQUID; see, e.g., Ref. [16]). In such a scheme of an electrometer (in contrast to other SET electrometers), the amplifier can be optimized as a *separate* block. In the frequency range of 100–500 MHz, the state-of-the-art narrow-band dc-SQUID-based amplifiers make it possible to almost approach the SQL [23]. For such an amplifier the impedance matching with the tank can be carried out by means of a transformer.

The set of experimental parameters for the Al transistor can be chosen as follows: $E_J \sim E_c \sim 100\ \mu$eV (corresponds to $C \sim C'/2 \sim 0.4$ fF, $R_t \sim 6$ kΩ, $I_c \sim 15$ nA and $\omega_J/2\pi \sim 25$ GHz $\gg \omega/2\pi \sim 300$ MHz),

$L \sim 20$ nH (gives $\ell \sim 0.3$ and $\Phi_0^2/2k_B L \sim 5$ K $\gg T \sim$ 20 mK), $Q \sim 300$ (bandwidth $\sim\omega/Q \sim 1$ MHz) and $\kappa^2 \sim 0.3$. These parameters yield the value $\kappa^2 Q\ell \sim 30$ that ensures a large transfer coefficient. Using the quantum-limited amplifier the charge resolution is expected to be equal to $(C'\hbar)^{1/2} \approx 3 \times 10^{-7} e/\mathrm{Hz}^{1/2}$.

Another important conclusion can be drawn with respect to a possible "back-action-evading" measurement by the rf-Bloch electrometer. For such a measurement it is assumed that one quadrature component of the internal noise is "squeezed" to less than SQL [24]. One of the ways to achieve this is to apply to the tank two driving signals with frequencies ω_1 and ω_2 which obey the relation $\omega_s = \omega_1 - \omega_2$ (see a similar proposal for rf-SQUID in Ref. [22]). In this "degenerate" mode of operation [25] the device is sensitive to a quadrature component, say $\hat{X}_1$, of the input ac signal $q = (\hat{X}_1 + i\hat{X}_2)e^{i\omega_s t}$ whose Heisenberg uncertainties obey the relation $\delta\hat{X}_1 \times \delta\hat{X}_2 \geq C'\hbar$. As a result, one side ($\delta\hat{X}_1$) of the "error box" is squeezed while another ($\delta\hat{X}_2$) is increased, with their product kept constant.

Finally, there is yet another advantage of the rf-Bloch electrometer for qubit measurements: Its transducer (the ring with transistor) is generically superconducting and the tank circuit can also be made of superconducting material. This device, when positioned near superconducting qubit, is, therefore, free from the normal-electron excitations which may significantly shorten the decoherence time of qubit.

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