

## New Mechanism of X-Ray Radiation from a Relativistic Charged Particle in a Dielectric Random Medium

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Based on the diffusional scattering of pseudophotons, we propose a new mechanism of x-ray radiation from a relativistic charged particle moving in a system consisting of microspheres distributed randomly in a dielectric material. The mechanism leads to a stronger dependence of radiation intensity on the particle energy,  $\gamma = E/mc^2$ , than that predicted by the transition radiation, and explains recent experimental data from a detector which contains randomly distributed superconducting granules.

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Radiation from charged particles moving in random inhomogeneous dielectric media is a problem of current interest. One of the applications is to utilize this property to determine the energy of a relativistic charged particle since radiation of this nature strongly depends on the particle energy,  $\gamma = E/mc^2$ . The feature of  $\gamma$  dependence makes the detection sensitivity increase as the particle energy goes up. This sensitivity is much better than that of the currently existing detectors, such as ionization chamber, Cerenkov detector, etc., which determine the particle energy by measuring the velocity,  $\beta = v/c$ , and lose the sensitivity in the relativistic regime, where  $\beta$  approaches 1. One type of the cryogenic detectors is qualified as a random system. It consists of type-I superconducting microspheres randomly distributed in dielectric material such as wax or varnish, etc. The superconducting microspheres introduce randomness of dielectric property in the medium to form a system for x-ray radiation. In the meantime, the superconducting phase transition is utilized to detect the x-ray radiation energy deposited on the grain. The detection of neutrinos,  $\gamma$ -ray, x-ray, neutrons, etc., by such a type of detector has been under intensive investigations [1,2].

Recently, Yuan *et al.* carried out an experiment utilizing the superconducting granule detector to investigate the x-ray radiation from a relativistic charged particle passing through it. They want to investigate the application of such a device as a Transition Radiation (TR) detector [3]. The detector consists of superconducting Sn microspheres randomly distributed in paraffin wax to form a cylindrical bar of 0.3 cm in diameter and 1.2 cm in length. The microsphere is 35  $\mu\text{m}$  in diameter. What is shown in Fig. 1 is the experimental result. The data points are presented by the solid circles showing the number of grain transitions as a function of the incident beam energy,  $E_{\text{beam}}$ . An abrupt rising in the number of grain transitions appears in the region with  $E_{\text{beam}} > 4$  GeV. The fitting solid curve going through the data points in this region is expressed by  $N_{ab} = 2.7 \times 10^{-6} \gamma^{1.46} - 0.047$ . The behavior of  $\gamma^a$  dependence with the phenomenological exponent  $a \sim 1.46$  cannot be explained by TR or any other known processes.

The constant term in the above fitting expression reflects the background energy deposition resulting from known processes such as TR or ionization which show much less beam energy dependence than the one under investigation.

By the conventional TR theory [4], the charged particle will emit photons when passing across the interface between the granule surface and the surrounding medium. One can calculate the radiation intensity or the number spectrum of the emitted photons by summing the contributions from the interfaces. This method of calculation takes each local interface effect into account as commonly applied in the conventional thin foil TR detector [5]. According to the TR theory, the intensity emitted is linearly

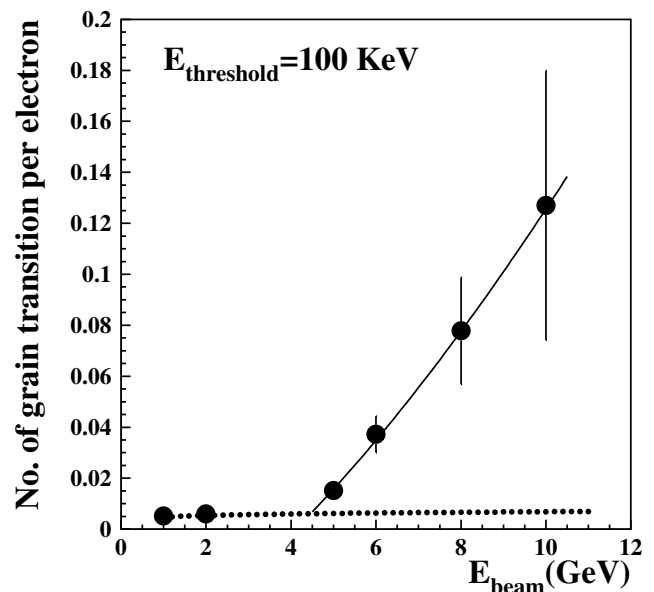


FIG. 1. The experimental data points and fitting curve for diffusional radiation. The solid circles are data points from the experiment by Yuan *et al.* [3]. The solid curve is the fitting going through the data points in the diffusional radiation region at  $E_{\text{beam}} > 4$  GeV. The dotted line shows  $\ln(\gamma)$  dependence for TR, where  $\gamma = E_{\text{beam}}/mc^2$ .  $E_{\text{threshold}}$  is the detection threshold for the radiated x-ray photon.

proportional to  $\gamma$ , while the number of photons generated, proportional to  $\ln(\gamma)$  [6,7]. The dotted line in Fig. 1, showing  $\ln(\gamma)$  dependence, does not describe the behavior of the data points at  $E_{\text{beam}} > 4$  GeV. The radiation intensity as explained by the TR is therefore much weaker than that from experiment. Here we explain the result of the experiment carried out by Yuan *et al.* [3] based on a new radiation mechanism, the radiation by diffusional scattering of pseudophotons associated with the charged particle passing through a randomly inhomogeneous medium. This is different from the other well known radiation processes such as synchrotron radiation, bremsstrahlung radiation, transition radiation, etc., considered in the textbook [8,9]. Calculation of the radiation intensity by single scattering of the pseudophoton reproduces the result of the TR theory. On the other hand, the multiple scattering, or the diffusional scattering, of the pseudophotons in a random medium to be presented below suggests that the number of x-ray photons emitted is proportional to  $\gamma^x$ ,  $0.75 \leq x \leq 1.5$ . This is consistent with the phenomenological exponent of 1.46 obtained by the experiment.

The method of pseudophoton was first suggested by Fermi [6]. The idea is to view the electromagnetic field associated with a moving charged particle as a collection of pseudophotons, which can be scattered in an inhomogeneous medium to become real photon and detected in an experiment. The conversion of pseudophoton into radiated photon is more efficient in the appropriate region of multiple scattering than in the single scattering region due to the fact that one takes into account the correlation effect of the random dielectric for the multiple scattering instead of considering the local single scattering interaction only. Since the total number of pseudophotons is  $\gamma^2$  dependence as obtained in Eq. (1) below, the  $\gamma^x$  dependence of multiple scattering simply reflects the fact that it converts a good proportion of pseudophoton into real photon than single scattering does with  $\ln(\gamma)$  dependence. One can estimate the number of pseudophotons associated with the moving charge as follows:

$$N_{\text{ps}} \sim \int \frac{d\vec{q}}{(2\pi)^3} A^2(\vec{q}) \sim \frac{e^2}{c} \gamma_m^2 L_z, \quad (1)$$

where  $\vec{A}(\vec{q})$  is the vector potential of the electric field created by the moving charge,  $\gamma_m = (1 - \beta^2 \epsilon)^{-1/2}$ , the Lorentz factor of the moving charge in the medium in which we believe that  $v\sqrt{\epsilon} < c$  is always the case, and  $L_z$ , the length of the system. The Maxwell's equation for the vector potential,  $\vec{A}$ , has the form,  $\nabla^2 \vec{A} + (\epsilon \omega^2 / c^2) \vec{A} = -4\pi \vec{j}(\vec{r}, \omega) / c$ , where  $\vec{j}(\vec{r}, \omega) = (e\vec{v}/v) \delta(x) \delta(y) e^{i\omega z/v}$  is the current of a charged particle moving uniformly in the  $z$  direction with velocity  $v$ , and  $\epsilon$  is the average dielectric constant of the system. In the derivation, we utilize the symmetry of the problem that the vector potential is directed along the  $z$  direction,  $A_i = \delta_{zi} A$ , to obtain

$$A(\vec{q}) = \frac{-8\pi^2 e}{c} \frac{\delta(q_z - \omega/v)}{k^2 - q^2}, \quad (2)$$

where  $k = \omega\sqrt{\epsilon}/c$  is the wave number of the pseudophotons. The finite size of the system has also been taken into account by replacing  $\delta(0)$  with  $L_z/2\pi$  in the above expression to obtain Eq. (1).

A simple sketch of the superconducting granule detector is presented in Fig. 2. The pseudophoton field is moving together with the charged particle as plotted. Once the charged particle along with the pseudophotons enter the system, they experience the variation of the dielectric constant introduced by the grains distributed in the wax and show scattering effect. In terms of x-ray radiation effect, one can describe the granule system by the model system consisting of a homogeneous medium with dielectric constant  $\epsilon_0(\omega)$  in which parallel plates of thickness  $a$  and dielectric constant  $\epsilon_p(\omega)$  are randomly spaced in the  $z$  direction. In considering x-ray radiation, the modeling is appropriate since the wavelength of the photon,  $\lambda$ , is much smaller than the grain diameter,  $a$ , and the charged particle is extremely relativistic to define a 1D-like property. This is a system with dielectric constant of 1D randomness as proposed by Gevorkian [10]. Experimentally, however, the superconducting microparticles have to be spherical in order to guarantee the existence of the superheated superconducting state which is necessary to provide the energy detection mechanism. Scattering of pseudophotons in such a system is characterized by the scattering mean free path in the  $z$  direction, which is related to the randomness of the media via the correlation function of the dielectric constant. The derivation of the mean free path for the x-ray scattering, hence the radiation intensity, is within the framework of Born approximation. With this approximation,  $ka|\sqrt{\epsilon_p/\epsilon_0} - 1| \ll 1$ , the mean free path is estimated as  $l(\omega) = 4c^2/[na^2(\epsilon_p - \epsilon_0)^2\omega^2]$ , where  $n$  is the concentration of plates or grains in the  $z$  direction.

The multiple scattering, which is the focused point in this investigation, occurs under the condition that  $\lambda \ll l(\omega) \ll L$ ,  $l_{\text{in}}(\omega)$ .  $L$  is the characteristic dimension of the system, and  $l_{\text{in}}$  is the inelastic mean free path of pseudophoton. This condition leads to the diffusion of pseudophoton in the  $z$  direction. The radiation intensity in the diffusion scattering region, integrated over the angles, is estimated as follows:

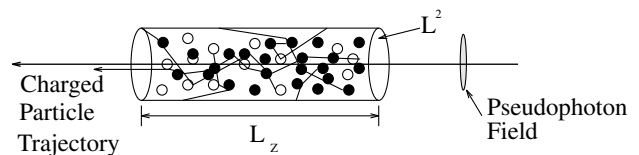


FIG. 2. Superconducting granule detector. Solid circles represent the Sn grains in the superheating state, while the open ones are for those in the normal state. The grain diameter is  $a$ . Trajectories for the scattered pseudophotons are plotted.

$$I(\omega) \sim N_{\text{ps}} \frac{1}{l(\omega)} \frac{L^2}{l^2(\omega)}. \quad (3)$$

It is proportional to the number of pseudophotons,  $N_{\text{ps}}$ , the probability of pseudophoton scattering,  $1/l(\omega)$ , and the mean number of scatterings in the random medium,  $L^2/l^2$ . By substituting Eq. (1), the diffusion radiation (DR) intensity in Eq. (3) becomes

$$I(\omega) \sim \frac{e^2}{c} \gamma_m^2 \frac{L_z}{l(\omega)} \frac{L^2}{l^2(\omega)}. \quad (4)$$

In the emission of x ray, the plasma formula for the dielectric constant applies.

$$\begin{aligned} \epsilon_p(\omega) &\approx 1 - \frac{\omega_p^2}{\omega^2}, \\ \epsilon_0(\omega) &\approx 1 - \frac{\omega_{p0}^2}{\omega^2}, \end{aligned} \quad (5)$$

where  $\omega_p$  is the plasma frequency of the plate or the grain, and  $\omega_{p0}$  is of the homogeneous medium. The average dielectric constant of the system,  $\epsilon = \epsilon_0 + na(\epsilon_p - \epsilon_0)$ , then becomes  $\epsilon(\omega) \approx 1 - [(1 - na)\omega_{p0}^2 + (na)\omega_p^2]/\omega^2$ , and the Lorentz factor in the medium has the following form accordingly:  $\gamma_m = \gamma[1 + \omega_c^2/\omega^2]^{-1/2}$ . In the above equation, one defines the characteristic frequency

$$\omega_c = \gamma \sqrt{(na)\omega_p^2 + (1 - na)\omega_{p0}^2}. \quad (6)$$

This characteristic frequency is a result of the overall dielectric inhomogeneity and the particle energy,  $\gamma$ . Substituting the plasma formulas, Eq. (5), into the condition of Born approximation, one obtains

$$\omega \gg \frac{a(\omega_p^2 - \omega_{p0}^2)}{2c} \equiv \omega_b. \quad (7)$$

The elastic mean free path of the pseudophoton in the random system, then, takes the form  $l(\omega) \approx \frac{1}{n}(\omega/\omega_b)^2$ .

In the diffusional scattering region, the condition of weak absorption limit,  $l \ll l_{\text{in}}(\omega)$ , is implied. In this limit, weak absorption of pseudophotons can be taken into account in similarity to the case of wave propagation [11]. Then, the quantity  $\sqrt{l_{\text{in}}}$ , instead of  $L$ , becomes the effective size of the system provided that  $\sqrt{l_{\text{in}}} \leq L$ . Therefore, the radiation intensity becomes

$$I(\omega) \sim \frac{e^2}{c} \frac{\gamma^2}{(1 + \frac{\omega_c^2}{\omega^2})} \frac{L_z l_{\text{in}}(\omega)}{l^2(\omega)}. \quad (8)$$

The ratio,  $l_{\text{in}}/l$ , is the mean number of pseudophoton scatterings before the absorption. Note that DR appears because of the randomness of the dielectric property of the medium. The concept of effective refractive index does not apply here, however. It is valid only in dealing with the average electric field, while the radiation intensity is obtained from the average of the square of the electric field; see Ref. [10]. For the electron with high enough

energy, above the GeV level, the x-ray radiation in the detector system under consideration becomes dominant for radiation energy loss. The inelastic mean free path of the pseudophoton in the x-ray region is determined by the absorption mainly via photoelectric effect,

$$l_{\text{in}}^{-1}(\omega) = (1 - na)N_0\sigma_{\text{ph}}^0(\omega) + (na)N_p\sigma_{\text{ph}}^p(\omega), \quad (9)$$

where  $N_0$  and  $N_p$  are the atom numbers in unit volume of the homogeneous medium and of the grain, respectively.  $\sigma_{\text{ph}}^0(\omega)$  and  $\sigma_{\text{ph}}^p(\omega)$  are the corresponding photoelectric cross sections.

The frequency dependence of the radiation intensity in different frequency regimes is derived as follows. In the region,  $\omega_b \ll \omega \ll \omega_L$ , the photoabsorption cross section is hydrogenlike,  $\sigma_{\text{ph}}(\omega) \sim \omega^{-3.5}$ . We obtain the frequency dependence of radiation intensity,  $I(\omega)$ , and the number of photons emitted from the detector,  $N(\omega) = I(\omega)/\hbar\omega$ , from Eqs. (8) and (9),

$$\begin{aligned} I(\omega) &\sim \frac{e^2}{c} \gamma^2 \frac{\omega^{-1/2}}{1 + \frac{\omega_c^2}{\omega^2}}, \\ N(\omega) &\sim \alpha \gamma^2 \frac{\omega^{-3/2}}{1 + \frac{\omega_c^2}{\omega^2}}, \end{aligned} \quad (10)$$

where  $\alpha = e^2/\hbar c$  is the fine structure constant. For  $\omega > \omega_L$ , the diffusion trajectory is cut on the system characteristic size,  $L$ . We have the formula for  $I(\omega)$  and  $N(\omega)$  in this frequency range as

$$\begin{aligned} I(\omega) &\sim \frac{e^2}{c} \gamma^2 \frac{\omega^{-6}}{1 + \frac{\omega_c^2}{\omega^2}}, \\ N(\omega) &\sim \alpha \gamma^2 \frac{\omega^{-7}}{1 + \frac{\omega_c^2}{\omega^2}}. \end{aligned} \quad (11)$$

From the above equations, by integrating over frequencies and taking into account that  $\omega_c \propto \gamma$ , one obtains the energy dependence of the total radiation intensity,  $I_T(\gamma)$ , and the total number of emitted photons,  $N_T(\gamma)$ , coming out of the pseudophoton field in the following two frequency ranges.

For  $\omega_c < \omega_L$ ,

$$\begin{aligned} I_T(\gamma) &\sim \gamma^{2.5}, \\ N_T(\gamma) &\sim \gamma^{1.5}. \end{aligned} \quad (12)$$

For  $\omega_c > \omega_L$ , the radiation reaches the saturation region,

$$\begin{aligned} I_T(\gamma) &\sim \text{const}, \\ N_T(\gamma) &\sim \text{const}. \end{aligned} \quad (13)$$

By definition,  $\omega_L$  is a signature at which the effective size of the system equals the system size. The radiation intensity of the charged particle saturates once the pseudophoton field is cut spatially by the system size. The saturation region is therefore determined by the system size,  $L$ , and the particle energy,  $\gamma$ . From

$l(\omega_L)l_{in}(\omega_L) = L^2$ , together with  $l_{in}(\omega_L) \propto \omega_L^{3.5}$  and  $l(\omega_L) \propto \omega_L^2$ , we have  $\omega_L \propto L^{4/11}$ .

In formulas (12) and (13), we have found the energy dependence of the emitted radiation intensity and the number of emitted photons. On the other hand, in the experiment [3], the number of absorbed photons is measured. These two are related to each other if we consider the photon emitted from the pseudophoton field as well as the photon absorbed by the system at the same time. Since the effective size of the system is  $\sqrt{l_{in}}$ , there exist many such subsystems in the medium along the path of moving electron in the  $z$  direction. After real photons are formed with a spatial extent of  $\sqrt{l_{in}}$ , they propagate through the rest of the system with part of them absorbed by the system. Under the condition of weak absorption, the number of real photons absorbed is proportional to the total number of real photons and the absorption probability of a diffusing photon is determined by  $\sqrt{l_{in}}/l_{in}$  in a system of effective size  $\sqrt{l_{in}}$ . Therefore, one can evaluate the total number of absorbed photons as

$$N_{ab} \leq \int_{\omega_b}^{\omega_c} d\omega \left[ \frac{l(\omega)}{l_{in}(\omega)} \right]^{1/2} N_T(\omega). \quad (14)$$

By taking into account the frequency dependence of  $\omega_c$ ,  $l(\omega)$ ,  $l_{in}(\omega)$ , and  $N_T(\omega)$ , we obtain from Eq. (14),  $N_{ab} \sim \gamma^x$ , with  $x \geq 3/4$ . Also, from Eq. (12), the upper limit of the exponent is 1.5. So, the number of grain transitions has a  $\gamma$  dependence to the power within the interval,  $0.75 \leq x \leq 1.5$ .

To make a better comparison with the experiment, some characteristic energies are evaluated from the detector parameters. The volumetric filling factor of Sn microspheres, 35  $\mu\text{m}$  in diameter mixed in the wax, is 13%. This gives the linear concentration of the grains,  $n \sim 130 \text{ cm}^{-1}$ . From this information, along with the plasma frequencies,  $\omega_p \sim 50 \text{ eV}$  for Sn and  $\omega_{p0} \sim 20 \text{ eV}$  for wax, one obtains the characteristic energies,  $\omega_b \sim 120 \text{ keV}$  and  $\omega_c \sim (37 \text{ eV})\gamma$ . For  $\omega_b$ , it is a characteristic energy determined by the dielectric plasma frequencies and the grain size; see Eq. (7). The region of interest for DR is at the photon energy above  $\omega_b$ . The other characteristic energy,  $\omega_c$ —see Eq. (6)—characterizes the interaction of the high energy charge with the medium and possesses similar meaning as in the TR theory. The DR is therefore significant for the experiment in the photon energy range  $\omega_b < \omega_{DR} < \omega_c$ . According to this description, the DR photon would show the effect of enhancement radiation at the beam energy,  $E_{\text{beam}} > 2.4 \text{ GeV}$ , which corresponds to  $\omega_c > \omega_b = 120 \text{ keV}$ . This is consistent with the experiment that the DR appears approximately at  $E_{\text{beam}} \sim 4 \text{ GeV}$ ; see Fig. 1. So, the theoretical result obtained here describes the correct radiation dependence of the experiment in the DR region as well as the onset beam energy of the DR region.

In conclusion, we have suggested a new mechanism of x-ray radiation from a relativistic charged particle moving in a random medium based on the multiple scattering of pseudophoton. Strong energy dependence of the number of superheated superconducting grains preceding the phase transition to the normal state, observed in the experiment [3], is a manifestation of this radiation mechanism. In the diffusional region,  $\lambda \ll l \ll L, l_{in}$ , the x-ray radiation is greatly enhanced. A detector working on this would then have a promising application in measuring the energy,  $\gamma$ , of an extremely relativistic charged particle. Furthermore, the radiation enhancement effect by diffusional scattering makes the random system, rather than a periodic layer system [12–15], a better x-ray radiator, since high radiation intensity can be achieved from a charged particle with a relatively low energy. In astrophysics, the diffusional radiation mechanism can play an important role to explain the x-ray emission spectra from an astrophysical object like an active galactic nucleus [16].

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