Convection in Highly Fluidized Three-Dimensional Granular Beds

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Free, buoyancy-driven convection has been observed experimentally in three-dimensional highly fluidized granular flows for the first time. Positron emission particle tracking was used to determine the position of a tracer grain in a vibrofluidized bed, from which packing fraction distributions as well as the velocity fields could be determined. The convection rolls, although small compared to the magnitude of velocity fluctuations (<5%), were consistently observed for a range of grain numbers and shaker amplitudes. Density variations are a signature of free convection and, with negative temperature gradients also present, were interpreted as the mechanism by which the convection rolls were initiated.

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The behavior of granular materials has been the subject of much interest for over 150 years [1], but it is only over the past 20 years that our understanding of some of the fundamental aspects of granular flow behavior has advanced substantially. This has principally been due to the theoretical, experimental, and numerical efforts to focus on the relationship between the motion of *individual* grains and the resultant macroscopic phenomena that can be more readily observed [2,3]. Theoretically, significant progress has been made in constructing dissipative Euler-like equations in analogy with the usual hydrodynamic expressions [4]. The kinetic theories used to construct this continuum approach have, to some degree, been validated in a series of complementary two-dimensional experiments performed in the past ten years. For example, the experiments of Warr *et al.* [5,6] indicated that a vibrofluidized granular gas behaved in a similar fashion to a thermal fluid, and a rigorous analysis of the self-diffusion coefficient showed that the Chapman-Enskog kinetic theory was broadly valid up to packing fractions of about 0.6, at least for the reasonably elastic collision restitution coefficient of $\varepsilon = 0.92$ [7].

Convection can be both beneficial and malignant. For example, convection is often used to increase the distribution of one species throughout another, but this same effect can lead to unwanted segregation. Explanations for low vibration amplitude convection of granular materials have been proposed by a number of authors. Savage suggested that a form of acoustic streaming drove the convection rolls [8], while Knight *et al.* [9,10] have suggested that the nature of the collisions with the walls creates a ratcheting effect that causes the convection. These experiments have been complemented by a description of convection that relies on an analysis of the void probabilities near the walls [11], and predicts many of the features observed in Ref. [9]. Bourzutschky and Miller [12] and Hayakawa *et al.* [13] have attempted to solve the conservation equations for vibrated beds numerically. In each case, though, they have investigated dense, weakly excited systems, which is clearly not the case for highly fluidized beds [5,7]. Molecular dynamics simulations have shown that convection is expected to occur for a fully enclosed system, when the dissipation exceeds a threshold value [14]. As the wall-particle interactions were elastic, this convection was essentially independent of the material properties of the wall. The convection acts to reduce the temperature gradient within the system; granular temperature differences are seen to be reduced following the onset of convection.

The purpose of this Letter is to present what we believe to be the first experimental observations of convection within a three-dimensional bed undergoing high amplitude vibration. There are important differences between the high amplitude case considered here, and the low amplitude vibration systems described earlier [9,10]: For example, the interaction forces are primarily inertial rather than frictional; the packing fraction drops significantly below that for a random-packed bed, and the relevant control parameter is the base velocity rather than the base acceleration.

In order to measure the key state variables for such a system, it is necessary to use an experimental technique able to probe within the cell and with a temporal resolution comparable to the mean time between collisions (typically of the order of milliseconds). The method used in this paper, positron emission particle tracking (PEPT), is a well-established technique for tracking a single radioactively labeled particle [15], and has recently been used to measure granular temperature distributions and determine packing fraction profiles for three-dimensional vibrofluidized granular beds [16,17]. However, in these studies the effects of any convection were ignored, as the magnitude of the convection velocity was small compared to the magnitude of the velocity fluctuations (~5%). The PEPT facility at the University of Birmingham is based on a Forte dual headed gamma camera (Adac Laboratories, California). Each head contains a single crystal of NaI scintillator, 500×400 mm^2, 16 mm thick, optically coupled to an array of photomultiplier tubes. Through triangulation of successive location events, the position of the tracer particle

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can be located in three dimensions. The current maximum count rate of $4 \times 10^4\ s^{-1}$ enables an ideal temporal resolution of 2 ms with a spatial accuracy of ~1 mm.

A three-dimensional granular gas was generated using a Ling Dynamic Systems (LDS) electromagnetic shaker (see, e.g., Refs. [5–7,16]). A cylindrical cell of dimensions 145 mm diameter and 300 mm height was attached to the upper surface of the vibrating piston, which was placed between the photon detectors. The cell was constructed of polymethyl methacrylate to limit the attenuation of the gamma rays as they traveled through the experimental apparatus. The base of the cell consisted of a 25 mm layer of polymethy methacrylate topped by a 6 mm layer of glass. The cell was vibrated, sinusoidally, at a frequency of 50 Hz and at an amplitude, A_0, of 0.74, 0.94, 1.14, 1.34, and 1.54 mm. Glass ballotini balls of diameter 5.0 ± 0.2 mm (with a restitution coefficient, ε, measured using high speed photography, of 0.91 ± 0.04) were used as the granular medium. Grain-wall and grain-base restitution coefficients were measured to be 0.68 ± 0.04 and 0.88 ± 0.04, respectively. The number of grains, N, in each experiment was varied through the range 350, 700, and 1050. To reduce the influence of electrostatic effects, the cell walls were coated with electrically conducting tape to provide a discharge route for charge build-up. Variations in vibration speed across the base of the cell were checked by means of a laser vibrometer and were found to be less than 0.05%.

A number of events can lead to convection in conventional fluids, but, in general, free convection is caused by density variations. Large scale density variations in a granular system can be induced by, for example, localized extra dissipation which leads to increased densities, or indirectly via temperature gradients [14,18]. In vibrofluidized beds, the dynamics of the system and the dissipation during collisions lead naturally to a temperature gradient [5,19,20]. Figure 1 shows the granular temperature as a function of height. The open symbols show the decay in granular temperature for the case where the tracer grain has been found within a central core, defined by $r < 30$ mm, where r is the distance from the cylinder axis ($N = 1050$). This nonisothermal atmosphere has been observed in both theoretical and numerical studies [19]. The authors refer the reader to Refs. [7,16,20] for details of the techniques used to measure granular temperature in vibrofluidized beds. Here, we have used a technique based on the analysis of the short time behavior of the mean squared displacement [20].

In a vibrofluidized bed, the system can be characterized through measurement of variables such as the granular temperature, packing fraction, and the "dispersive pressure" [21]. The granular temperature, E_0, is defined in the usual manner:

$$E_0 = \frac{1}{3}(E_x + E_y + E_z) = \frac{1}{3}(m\overline{v_x^2} + m\overline{v_y^2} + m\overline{v_z^2}). \quad (1)$$

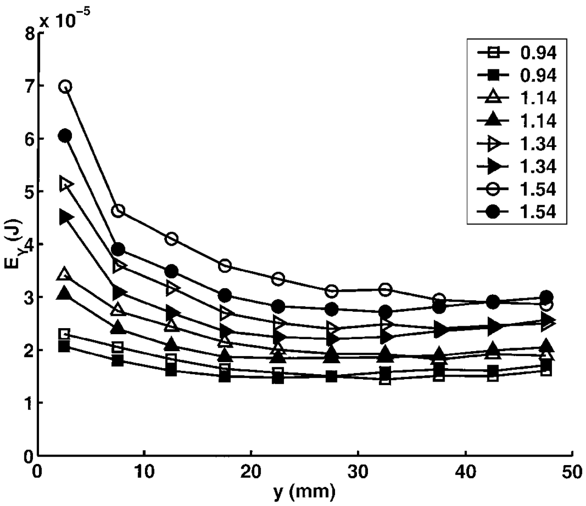

FIG. 1. y component of the granular temperature (E_y) profiles for $N = 1050$ and $A_0 = 0.94$, 1.14, 1.34, and 1.54 mm. Open symbols: $r < 30$ mm; closed symbols: $30 < r < 60$ mm.

where $\mathbf{v}$ is the particle velocity relative to the mean convective velocity, resolved in the directions x, y, or z (y being vertical); E_x, E_y, and E_z are the granular temperatures in each of the respective directions; and m is the mass of the grain (in this case, $0.19g$). In this study, the granular temperature was measured in two regions, $r < 30$ mm and $30 < r < 60$ mm, where r is the radial distance from the cylinder axis. This method of calculating the packing fraction profiles from single particle tracking data has been detailed in a previous publication [16], but has been extended to enable measurement of the packing fraction, η, throughout the whole of the cell, rather than just as a function of y. As the system may be assumed to be ergodic, the time residence fraction within a given volume element is directly proportional to the packing fraction [16]. The mean velocity field was calculated using a finite differencing procedure with a time interval of 25 ms.

Observations of slices through the cell showed that, in all cases, both the packing fraction and the velocity field were approximately symmetrical about the cylinder axis. Consequently, they were averaged over the azimuthal angle, allowing the velocity field to be displayed as a 2D image on r-y coordinates. Results for the case $N = 1050$, $A_0 = 0.94$ and 1.54 mm are shown in Figs. 2(a) and 2(b).

The convection rolls apparent from the velocity vectors in Figs. 2(a) and 2(b) are toroidal in nature: The motion of the grains is upwards along the axis and downwards along the walls. The focus of the roll is about 2–3 grain diameters from the wall. As the amplitude of vibration is increased the bed expands, as can be observed from the contour maps shown in Figs. 2(a) and 2(b). In the highest regions, the packing fraction is low and the number of grains used to determine the mean velocity is reduced. This results in significant scatter at large altitudes, and it becomes difficult to measure the convection velocity precisely. In the relatively high density regions ($y < 50$ mm),

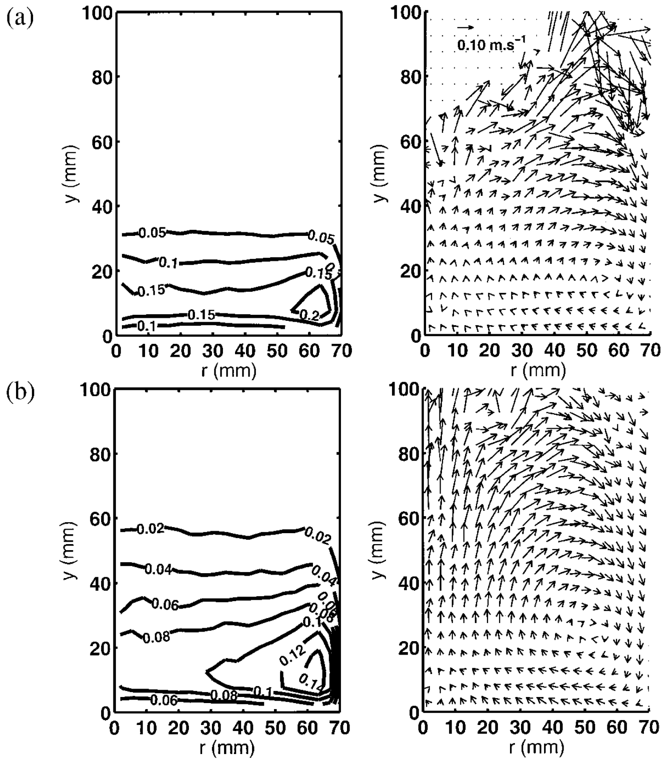

FIG. 2. Packing fraction contour maps (left) with mean velocity fields (right) for $N = 1050$; (a) $A_0 = 0.94$ mm and (b) $A_0 = 1.54$ mm. The data have been averaged around the azimuthal angle to show the r-y distributions.

however, the convection rolls are clearly visible and are consistent.

If we study the packing fraction contour maps [Figs. 2(a) and 2(b)], one can see that close to the wall there is a region of high density, about 20% denser than the regions close to the axis of the cell. Inspection of Figs. 1 and 2 confirms that two of the ingredients closely associated with free convection in convectional fluids, temperature gradient and density variations, are present. Clearly, the buoyancy forces on the grain in this region will tend to force the grains down at the wall. This will be accompanied by a tendency for the grains at the center to rise, causing convection rolls to be initiated.

An estimate of the angular velocity, Ω, was determined using

$$\Omega = \frac{\oint \mathbf{v} \cdot d\mathbf{l}}{2|\int_S d\mathbf{S}|} = \frac{\int_S \mathrm{curl}\mathbf{v} \cdot d\mathbf{S}}{2|\int_S d\mathbf{S}|}, \qquad (2)$$

where $\mathbf{S}$ is the region of interest, and $\mathbf{l}$ is the line that encircles $\mathbf{S}$. The line integral in Eq. (2) was estimated by manually locating the center of the convection roll, and performing the sum $\sum \mathbf{v} \cdot \Delta\mathbf{l}$ around the edge of a 5×5 matrix of velocity vectors, centered on this focus. As can be seen from Fig. 3, no clear trend was observed in the relationship between Ω and A_0 (for any fixed value of N), suggesting that any coupling between the peak base velocity and the magnitude of the convection rolls is weak [14].

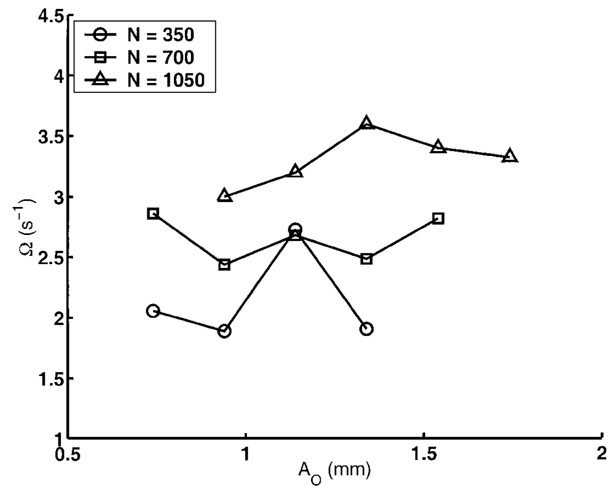

FIG. 3. Angular velocity of the convection rolls as a function of the amplitude of vibration.

In regions immediately adjacent to the sidewall, the measured packing fraction decreases significantly. The cause of this decrease is not, however, believed to be physical in origin. If the measurement error causes a grain to be assigned to a neighboring volume element, then the calculated packing fraction will be reduced in the true element. In the body of the granular gas, this inaccurate location is compensated for by the inaccurate location of grains within neighboring elements, but adjacent to the wall the errors can cause the tracer to be located outside the cell and, in these positions, grains in neighboring regions cannot compensate for the "leakage" of the packing fraction.

We suggest that the increase in density at the wall is due to a combination of two effects: (i) The presence of the wall causes the pressure at the wall to increase, as described by van der Waals [22], and (ii) the increased collision rate with the wall leads to extra dissipation and subsequent reduction in the granular temperature, leading to further increases in the density.

Analysis of the grain-grain collision probability near a boundary leads to prediction of an increase in density (and collision rate) by a factor Γ of about [22]

$$\Gamma = \frac{1 - \eta}{1 - 2\eta}, \qquad (3)$$

above that observed in regions far from the wall. This indicates that, for a packing fraction of $\eta \sim 0.08$, the density is enhanced by $\sim$10%. Clearly, this is significantly lower than the increases observed in Figs. 2(a) and 2(b), suggesting that dissipation at the wall plays a significant role in inducing density variations beyond those naturally expected due to boundary effects.

The closed symbols in Fig. 1 show the granular temperature measured in the outer annulus of the experimental cell, $30 < r < 60$ mm. Comparison between the granular temperature measurements in the outer and the inner regions (closed and open symbols, respectively) indicates that the grains nearer the wall, at heights coincident with regions of high packing fraction, are "cooler" than those in more central positions. This reduction in granular temperature provides further evidence for the suggestion that dissipation at the wall creates the large density variations that ultimately lead to the free convection observed in Figs. 2(a) and 2(b).

In conclusion, it has been demonstrated that PEPT can be used to analyze the motion within rapid granular flows, allowing measurement of the granular temperature, mean velocity field, and packing fraction distributions. The technique enabled convection rolls to be visualized in continuously vibrated granular materials at packing fractions not previously probed. Elements that are generally considered to be prerequisites to the establishment of free convection (negative granular temperature gradients and density variations) were observed. The evidence presented suggests that the convection rolls are caused by buoyancy effects initiated by enhanced dissipation at the sidewall.

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