Stabilization of Femtosecond Lasers for Optical Frequency Metrology and Direct Optical to Radio Frequency Synthesis

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We have actively stabilized the comb of frequencies from a mode-locked femtosecond laser using a Fabry-Perot reference cavity. This technique offers the ability to synthesize a comb of highly stable radio frequencies directly from optical transitions. The measured fractional frequency instability of the components of the frequency comb relative to the reference cavity was $<5 \times 10^{-13}$ in 0.1 sec. The variation of the optical mode spacing versus frequency of the reference cavity was also directly measured using this technique.

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The use of ultrashort pulse trains for metrology has gained acceptance, thanks partly to a maturing in femtosecond technology. Since the Fourier transform of a regular time sequence of pulses is a comb of equally spaced frequencies, a mode-locked laser is a frequency ruler which can measure the frequency interval between optical sources [1–6]. When the full octave between a source, f, and its second harmonic, $2f$, is linked in this way, the absolute optical frequency can be directly measured [7–9]. A powerful technique for linking radio to optical frequencies relies on the relationship between the laser's repetition rate and the components of the frequency comb [10]. All these methods require an independent, stable radio frequency (rf) source to lock the laser repetition rate.

An optical frequency standard based on a mode-locked laser requires a stabilization scheme that (i) locks the repetition rate f_r to a stable reference and (ii) fixes the relationship between repetition rate and optical frequency. The optical frequency of a mode m of the comb is

$$f_m = mf_r + (\Delta\phi/2\pi)f_r\,, \tag{1}$$

where $\Delta\phi$ is the pulse to pulse carrier-envelope phase slip [11]. The approach generally adopted has been to stabilize the repetition rate f_r to a stable rf source, and determine $\Delta\phi$ by mixing two regions of the pulse spectrum separated by an optical octave [10,12]. The generation of an octave spanning spectrum is facilitated by "nonlinear fibers" [13] which broaden the pulse spectrum while preserving the discrete nature of the frequency comb. We present here a method to stabilize a mode-locked laser, using only a Fabry-Perot reference cavity to lock both the average frequency and the mode spacing (f_r) of the frequency comb. Our technique does not require the rf reference which is a limitation on the stability of the frequency comb [8–10]. Instead it provides a rf linked to, and synthesized directly from, the optical frequency via the Fabry-Perot reference cavity. The advantages are convenience, compactness, and a potential for improved stability.

Optically defined rf's have previously been obtained by mixing continuous wave (cw) lasers either stabilized to reference cavities [14,15] or to atomic or molecular transitions [16]. The use of fs lasers offers several advantages. First, the rf produced by the fs laser is directly linked to the optical frequency of interest via Eq. (1). The offset frequency, $(\Delta\phi/2\pi)f_r$, can be determined either by mixing spectral regions separated by an octave [10], or by using a reference cavity that has been previously calibrated. A second advantage is the high signal to noise levels for short-term stabilization compared to techniques which detect only a single mode of the comb. The reference cavity providing a near limitless array of frequencies, thousands of the laser modes can be detected simultaneously. Last, the laser mode spacing can be stabilized to the reference cavity across spectral regions of the laser bandwidth separated by many terahertz (THz), providing greatly enhanced discriminator sensitivity.

The reference cavity is used to stabilize the mode-locked laser, as opposed to recent experiments where Fabry-Perot cavities are "mode filters" transmitting every 20th comb component, such that any individual mode can be unambiguously measured with a wave meter [3,5]. Early experiments on stabilization of mode-locked lasers used short Fabry-Perot etalons to stabilize only the *average position* of the comb, employing techniques developed for single frequency lasers [17,18]. To truly transfer the stability of the reference cavity to all the modes of the fs laser, however, the position *and* spacing of the frequency comb must be locked. We demonstrate this dual locking using a reference cavity to stabilize two different regions of the laser's spectrum.

The stability of the fs comb is ultimately limited by that of the reference cavity, which, according to previous studies with cw lasers, reaches 3×10^{-16} for a time on the order of a second [19]. We use a long (62 cm) Fabry-Perot resonator made of a solid block of ultra-low-expansion quartz (ULE), with high reflectivity mirrors of the same material optically contacted on both ends.

The setup to stabilize a mode-locked fs laser to the reference cavity is shown in Fig. 1. The laser output is sent (double pass) through an acousto-optic modulator (AOM)

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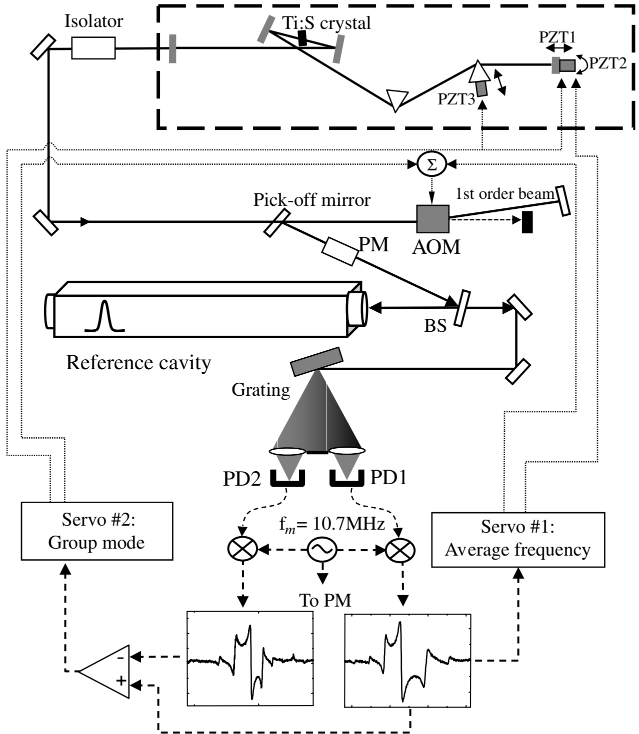

FIG. 1. Schematic of experimental setup. The laser output, protected from feedback by a Faraday isolator, is sent through an acousto-optic modulator that controls the beam average frequency before being sent through a phase modulator and mode matched to the reference cavity. AOM, acousto-optic modulator; PM, phase modulator; BS, beam splitter; PD1 and PD2, photodiodes 1 and 2; PZT1-3, piezoelectric transducers.

to control the average frequency of the comb (the laser repetition rate—hence the mode spacing—is unaffected by the AOM). A phase modulator (PM) adds sidebands at ± 10.7 MHz to each component of the frequency comb. The detectors PD1 and PD2 select two distinct spectral regions of the reflection from the reference cavity dispersed by a grating. The signals are mixed with the 10.7 MHz modulation frequency of the PM to produce error signals [20]. In contrast to previous work with single frequency lasers, each error signal is a composite from *all* longitudinal modes within the portion of the spectrum detected [21]. Because the modes of our reference cavity are not exactly equally spaced, better discrimination is obtained by selecting a portion of the spectrum ≈ 10 nm wide. The error signal from either detector can be used for locking the *average frequency* of the comb, while the difference between the error signals from PD1 and PD2 provides an amplified measurement of fluctuations in the laser repetition rate. The sensitivity of this discriminator is equal to the slope of the error signals multiplied by the number of modes between them. The slope of either error signal shown in Fig. 1 is ≈ 1 V/MHz. For an average separation of 10^5 modes (≈ 12 THz) between PD1 and PD2 we obtain a sensitivity to fluctuations in f_r of $(1\ \text{V/MHz}) \times 10^5 = 0.1$ V/Hz. This signal is used to lock the repetition rate of the laser to the reference cavity. This technique effectively locks the 100 000th harmonic of f_r. When the average frequency separation of PD1 and PD2 approaches an optical octave—this extension of the laser bandwidth can be achieved with new specialty fibers [13]—the sensitivity of this rf discriminator will approach that of the optical frequencies. This will allow the high fractional frequency stability of optical transitions, $\Delta\nu/\nu$, to be transferred to radio frequencies, $\Delta f_r/f_r$.

The error signal from servo 1 is sent to the AOM to stabilize the average frequency of the laser, providing gain out to >150 kHz. Two piezoelectric transducers (PZT's) are attached to the laser cavity end mirror to provide control of the cavity length via translation (PZT1) and control of the average group velocity of the pulse by tilting the end mirror (PZT2) [8]. A third PZT can be used to translate an intracavity prism (PZT3) for additional (slow) control of the laser cavity dispersion. Of the various permutations of actuators investigated, the preferred one is where the repetition rate is stabilized by applying the second error signal as a linear combination to the cavity length (PZT1) and AOM, in a manner to modify only the mode spacing of the frequency comb. This choice of actuators allows a larger bandwidth to be used in stabilizing the pulse repetition frequency, than can be obtained by tilting the end mirror alone. The end mirror tilt control (PZT2) is still used for slow corrections of the pulse repetition rate.

The difference between the two curves in Fig. 2 corresponds to the spectral density of noise in the repetition rate (f_r). The spectrum shows significant noise contributions out to ≈ 10 kHz. When servo 2 is turned on, this noise drops to the noise floor of the discriminator, determined by the common-mode rejection ratio of the differential amplifier, making the two curves nearly indistinguishable. However, when PZT1 is used to stabilize the average laser frequency in addition to the AOM, the noise seen in Fig. 2 is also greatly reduced. This leads us to believe that the

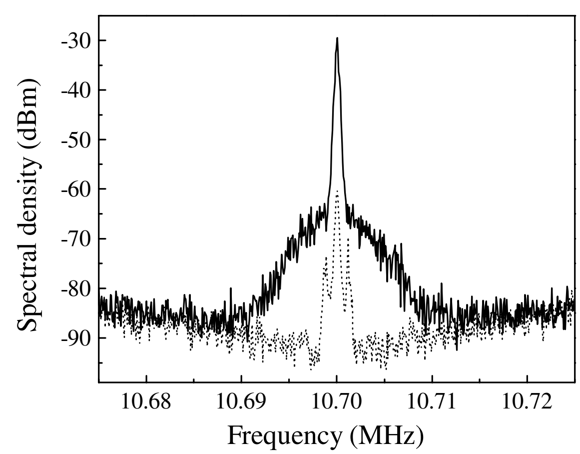

FIG. 2. Spectral density of noise for the mode-locked laser, seen at the FM carrier frequency of 10.7 MHz, from the first (lower dashed curve) and second (upper solid curve) error signals with only servo 1 locked using only the AOM. The difference between these two curves corresponds to the spectral density of noise of the laser mode spacing seen at servo 2.

dominant contributions to the spectral density of noise for f_r occur at frequencies <10 kHz, due to fluctuations in effective laser cavity length.

For the data shown here, the AOM and PZT1 are used to lock the average frequency of the laser. As mentioned above, this reduces the initial noise spectrum seen at servo 2. The repetition rate is then locked with only a combination of PZT2 and PZT3.

To measure the stability of the frequency comb relative to the reference cavity, a cw Ti:sapphire laser is simultaneously locked to a single mode of the same reference cavity. A beat note between the cw laser and modes from the fs comb is observed by sending both beams through the same single mode fiber, to ensure optimal spatial overlap, onto a photodiode. Figure 3 shows a beat note of 2 kHz width (3 dB) between the cw laser and the mth mode of the comb, where m is the mode nearest to the cw frequency. After slight amplification through a low pass filter, the beat note is counted on a HP5373A modulation domain pulse analyzer for time scales from 1 μs to 8 sec, and with a HP5345 frequency counter for 1 to 75 sec time scales.

The calculated Allan deviation from these counts is shown in Fig. 4. For time scales <1 sec, the Allan deviation averages down at roughly $\tau^{-1/2}$. The dashed line is a guide showing a slope of 20 Hz/$\sqrt{\tau}$. The instability between the two optical frequencies is as low as 4.8×10^{-13} in 0.1 sec. For longer time scales, we attribute thermal drift of the reference cavity, to which both lasers are locked, to the drift of the beat note as indicated by the positive slope in the plot. No active thermal stabilization or isolation of the reference cavity is used in the current experiment. Improving the medium term drift of the cavity should allow stabilities below 10^{-13} in 1 sec to be reached with our present system. This can provide stable rf's for times sufficient to determine optical frequencies with adequate accuracy to meet most metrological needs. We are now exploring the potential of the reference cavity stabilized fs laser to serve as a future all optical atomic clock, by locking a mode of the reference cavity (and thereby its length) to an atomic transition. As mentioned previously, the rf repetition rate could then be defined as a fraction of the optical frequency, with the same relative error.

We explore next the potential of the mode-locked laser to serve as a frequency ruler to measure the deviation of reference cavity modes from the fs comb. The beat note between the cw laser locked to a mode of the reference cavity and the nearest component of the fs comb (which serves as frequency reference) is plotted in Fig. 5. The position of the reference cavity resonances can be written as $f(m) = mf_r + \Delta(m)$ where f_r is the mode spacing of the fs comb and $\Delta(m)$ is the difference between the mth mode of the reference cavity and the nearest mode of the fs comb [given by Eq. (1)]. In this measurement it is the beat note $\Delta(m)$ that is directly measured, rather than the mode spacing $\sigma(f)$ [derivative of $\Delta(m)$ plotted as a dashed line in Fig. 5] that was recorded in similar measurements with cw lasers [22,23]. The quantity $\Delta(m)$ includes a contribution proportional to the phase shift of the mirrors, and to the index of refraction of air, at the frequency mf_r. Measurements as a function of pressure will lead to an accurate determination of the dispersion of air and of the mirrors. The main difference with the cw technique [22] is that, in the latter, the error on the mode spacing is cumulative, while the fs comb provides a direct frequency reference across the large bandwidth of the laser.

The performance of the servo loop is ultimately limited by the cavity dispersion. The measured deviation between fs comb components and the reference cavity results in an effective cavity linewidth, $\delta\nu_{\text{sum}}$, seen by detectors PD1 and PD2 (see Fig. 1) that is broadened relative to the linewidth of a single cavity resonance, $\delta\nu_o$. This results in a tradeoff between allowing a greater portion of the fs comb bandwidth to reach the detectors to maximize the signal to noise, and limiting this bandwidth to prevent

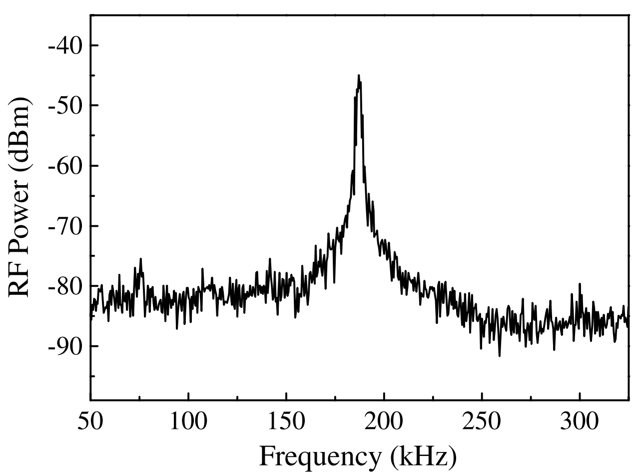

FIG. 3. Beat note between cw laser and mth mode of the frequency comb showing greater than 35 dB signal to noise ratio. Resolution bandwidth is 1 kHz; sweep time 7.5 sec.

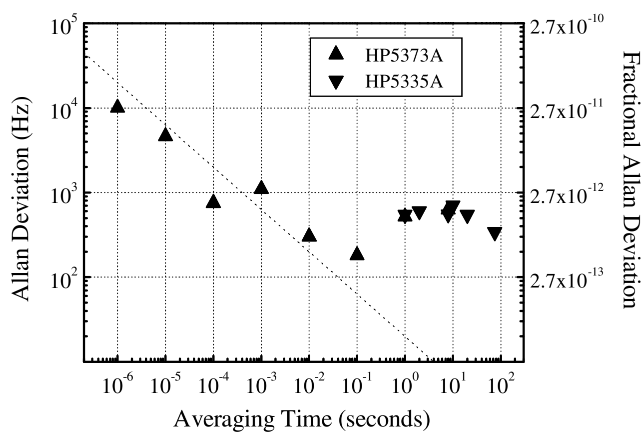

FIG. 4. Allan deviation of beat note between cw laser and frequency comb. The right ordinate shows the fractional Allan deviation of the 375 THz optical frequency assuming all noise present comes from either the cw laser or the fs comb. Up triangle, data collected with HP5373A Modulation Domain Pulse Analyzer; down triangle, data collected with HP5335A Universal Frequency Counter.

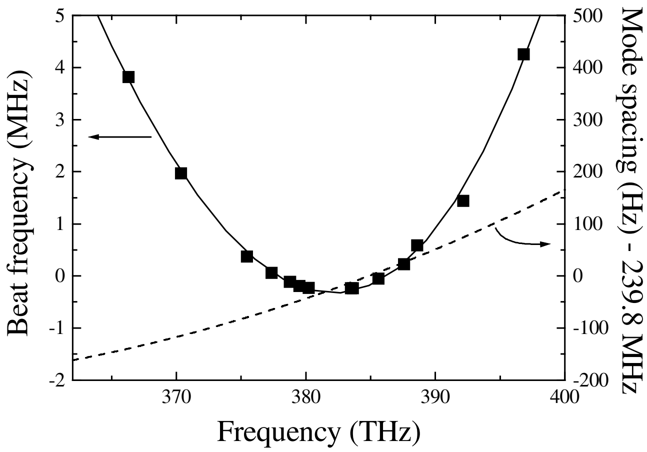

FIG. 5. Measured deviation of the reference cavity longitudinal modes from the equidistant positions of the frequency comb (left ordinate; data points) and the calculated mode spacing of the reference cavity versus frequency (right ordinate; dashed curve). The original data points were fit to a third order polynomial, and the mode spacing calculated from the relation $\sigma(f) = \sigma_0(1 + \partial\Delta/\partial f)$.

degradation of the error signal. For a single frequency laser locked to a cavity, the shot noise limited minimum laser linewidth is [24] $\Delta\nu_{\min} \propto \delta\nu_o^2/P$, where P is the power incident on the cavity. One can estimate from this expression the optimum spectral bandwidth b allowed to reach the detector. The incident power on the detector will be proportional to b under the assumption that the total laser power is distributed approximately equally among all its modes. For small changes the effective linewidth is $\delta\nu_{\text{sum}} \approx \delta\nu_o + \alpha b$ with $\alpha = \partial\Delta(f)/\partial f$ evaluated at the center frequency of the detector. Under these assumptions, the optimum bandwidth required to minimize the laser linewidth $\Delta\nu_{\min}$ is $b = \delta\nu_o/\alpha$. Using the measured cavity dispersion, and with the detectors centered at 377 and 386 THz, the optimum spectral bandwidth for our cavity corresponds to $\approx$6 THz ($<$13 nm). This bandwidth agrees with our experimental observation.

In conclusion, we have demonstrated a simple technique to stabilize the frequency, phase, and repetition rate of an ultrashort pulse laser system with a Fabry-Perot reference cavity. By detecting the difference in two separate portions of the laser's spectrum reflected by the reference cavity, the repetition rate of the laser is locked independently of its average frequency. The stability of the frequency comb locked to the reference cavity has been measured from the beat note between a component of the frequency comb and a second cw laser locked to the same cavity. The exceptional stability of the fs comb will allow precise linking of optical to radio frequencies without the need for auxiliary atomic time standards to control the laser's repetition rate. Long term stabilization obtained through locking the optical frequency to an atomic resonance will lead to a highly stable radio frequency source. Such a system would be very attractive as a combined time and length standard. Stabilization of mode-locked lasers is not only important for time and frequency standards, but also for metrology applications involving mode-locked ring and linear lasers [25,26]. Differential spectroscopy between two trains of pulses can measure small displacements, rotations [27], and magnetic fields [28].

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Note added.—In the course of submission of this manuscript, we have become aware of related work with a stabilized fs comb [29].

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