Enhancement of the Josephson Current by an Exchange Field in Superconductor-Ferromagnet Structures

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We calculate the dc Josephson current for two superconductor/ferromagnet (S/F) bilayers separated by a thin insulating film. It is demonstrated that the critical Josephson current I_c in the junction strongly depends on the relative orientation of the effective exchange field h of the bilayers. We found that in the case of an antiparallel orientation I_c increases at low temperatures with increasing h and at zero temperature has a singularity when h equals the superconducting gap Δ. This striking behavior contrasts with the suppression of the critical current by the magnetic moments aligned in parallel and is an interesting new effect of the interplay between superconductors and ferromagnets.

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The possibility of various applications and the appearance of new interesting physics makes the experimental and theoretical study of ferromagnetic and superconducting-ferromagnetic hybrid structures a popular topic. One of the properties that has attracted a lot of interest in the last few years is a magnetoresistance due to the presence of the magnetic order [1–4]. In some structures the magnetoresistance can reach very large values. This effect has been termed "giant magnetoresistance" (GMR). First discovered in magnetic multilayers [1,2] where the typical values of MR were of order of 10%, the GMR effect can be as large as 200%–300% in Ni-Ni or Co-Co point contacts [3,4].

A typical device studied in such experiments consists of two separated ferromagnets. One measures the resistivity for different relative directions of the magnetization. The large values of the MR are due to an additional scattering of electrons at the boundary between adjacent layers (in the case of antiparallel orientation, an electron crossing this boundary goes from one subband to another and experiences a reflection from an effective potential related to the different positions of the subbands).

If the normal metals of the reservoirs are replaced by superconductors, another mechanism causes differing resistances for the antiparallel and the parallel alignment of magnetization. This mechanism is due to Andreev reflection which occurs at the superconductor/ferromagnet (S/F) interfaces, and which implies a zero spin current through them [5]. In the case of very thin magnetic layers separating the superconducting reservoirs, the resistance of the structure drops to zero and it becomes more appropriate to consider the supercurrent (or Josephson current). It was shown that if the exchange field h in the magnetic layer exceeds a certain value, the state energetically more favorable corresponds not to a zero phase difference between the reservoirs (in the absence of an external current), but to a phase difference of $\varphi = \pi$ (the so-called π junction) [6]. The predicted π state in a S/F/S Josephson junction apparently was observed by Ryazanov *et al.* [7]. The critical current decreases with increasing exchange field h in the magnetic layer, changes sign, and decays to zero while undergoing some oscillations. The superconducting properties are not so strongly reduced if the magnetization (i.e., the exchange field h) is not homogeneous [8,9].

In this Letter we demonstrate that, in contrast to the common knowledge, the exchange field can under certain conditions enhance the Josephson critical current in a S/F-I-S/F tunnel junction rather than reduce it (here I is an insulating layer). As a result, the critical current I_c may considerably exceed the critical current of the Josephson junction in the absence of the exchange field. The conditions are quite simple: one needs low temperatures and the antiparallel alignment of the magnetization in the different parts of the superconductor. At the same time, if the magnetization in the bilayers is parallel, the critical current is suppressed. This leads to a high sensitivity of the critical current to the mutual alignment of the magnetic moments and, hence, to a possibility of an experimental observation.

To be specific we consider a system consisting of two S/F bilayers (F here is a thin film) separated by a thin insulating layer (see Fig. 1), i.e., the Josephson S/F-I-F/S

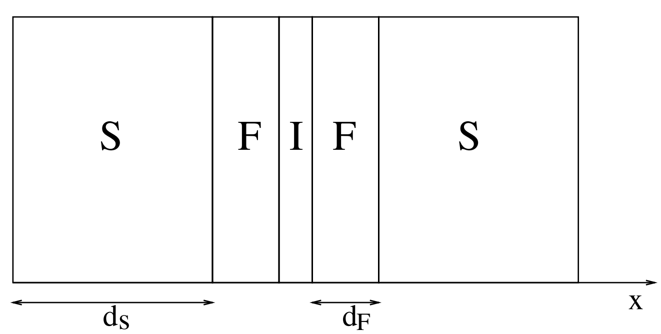

FIG. 1. The S/F-I-F/S system.

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junction. This system can be studied using quasiclassical equations [10–12] complemented with the boundary conditions [13,14]. This approach allows one to describe the system completely and was used to get the main results of the present paper.

However, the Josephson current and other thermodynamic quantities can be derived in a considerably simpler way if the thicknesses of the layers d_S and d_F in Fig. 1 are smaller than the superconducting coherence length $\xi_S \sim \sqrt{D/2\pi T_c}$ and the length of the condensate penetration into the ferromagnet $\xi_F \sim \sqrt{D/h}$, respectively. These conditions can be met experimentally.

Although, generally speaking, solutions for the superconducting order parameter Δ of the quasiclassical equations depend on the coordinates, the assumption about the thickness allows one to write solutions that do not have this dependence. In this limit, the influence of the ferromagnetic layers on superconductivity is not local and is equivalent to inclusion of a homogeneous exchange field with a reduced value. Of course, the other physical quantities characterizing the superconductor should be modified, too.

Proceeding in this way, one comes to effective values of the superconducting order parameter Δ_{eff}, of the coupling constant λ_{eff}, and of the magnetic moment h_{eff} described by the following equations:

$$\begin{aligned} \Delta_{\text{eff}}/\Delta &= \lambda_{\text{eff}}/\lambda = \nu_s d_s(\nu_s d_s + \nu_f d_f)^{-1}, \\ h_{\text{eff}}/h &= \nu_f d_f(\nu_s d_s + \nu_f d_f)^{-1}, \end{aligned} \tag{1}$$

where ν_s and ν_f are the densities of states in the superconductor and ferromagnet, respectively.

Assuming that the exchange field acts only on spin of electrons (which implies that the magnetizations are parallel to the interface) one can write the Gor'kov equations for the S/F layers

$$\begin{aligned} (i\varepsilon_n + \xi - \boldsymbol{\sigma}\mathbf{h})\hat{G}_\varepsilon + \hat{\Delta}\hat{F}^+_\varepsilon &= 1, \\ (-i\varepsilon_n + \xi - \boldsymbol{\sigma}\mathbf{h})\hat{F}_\varepsilon + \hat{\Delta}\hat{G}_\varepsilon &= 0, \end{aligned} \tag{2}$$

where $\boldsymbol{\sigma}$ are Pauli matrices and $\xi = \varepsilon(\mathbf{p}) - \varepsilon_F$, ε_F is the Fermi energy, $\varepsilon(\mathbf{p})$ is the spectrum, $\varepsilon_n = (2n+1)\pi T$ are Matsubara frequencies, and G_ε and F_ε are normal and anomalous Green functions. [We omit the subscript "eff" in Eqs. (2) and below.] Equations (2) should be complemented by the self-consistency equation

$$\Delta = \lambda T \sum_\varepsilon \text{Tr}\hat{f}_\varepsilon, \tag{3}$$

where trace Tr should be taken over the spin variables and

$$\hat{f}_\varepsilon = \frac{1}{\pi}\int \hat{F}_\varepsilon \, d\xi. \tag{4}$$

Equations (2)–(4) may describe superconductors with a homogeneous exchange field as well. We neglect the influence of the magnetic moments on the orbital electron motion, which is definitely legitimate for the thin ferromagnetic layers considered here. As soon as the S/F system is described by Eqs. (2)–(4) the Josephson current I_J can be expressed in terms of $\hat{f}$,

$$I_J = (2\pi T/eR)\,\text{Tr}\sum_n \hat{f}(h_1)\hat{f}(h_2)\sin\varphi, \tag{5}$$

where R is the barrier resistance in the normal state. This formula can easily be obtained by using the standard tunneling Hamiltonian method or boundary conditions [13,14]. h_1 and h_2 are the exchange fields to the left and to the right of the junction.

In the case of the conventional singlet superconducting pairing the matrix $\hat{\Delta}$ has the form $\hat{\Delta} = i\sigma_y\Delta$. Solving Eqs. (2) and using Eq. (4) we find easily for the function $\hat{f}_\varepsilon$

$$\hat{f}_\varepsilon = \hat{\Delta}[(\varepsilon_n + i\boldsymbol{\sigma}\mathbf{h})^2 + \Delta^2]^{-1/2}. \tag{6}$$

With Eq. (6) one can calculate the Josephson current I_J for any direction of the magnetic moments $\mathbf{h}_1$ and $\mathbf{h}_2$. The most interesting are the cases of the parallel and antiparallel alignments of the magnetic moments. In both cases computation of the current I_J in Eq. (5) is very simple, and we obtain for the parallel configuration

$$I_J^{(p)} = \frac{\Delta^2(T)4\pi T}{eR} \times \sum_\varepsilon \frac{\varepsilon_n^2 + \Delta^2(T,h) - h^2}{[\varepsilon_n^2 + \Delta^2(T,h) - h^2]^2 + 4\varepsilon_n^2 h^2}, \tag{7}$$

whereas the Josephson current $I^{(a)}$ for the antiparallel configuration takes the form

$$I_J^{(a)} = \frac{\Delta^2(T)4\pi T}{eR} \times \sum_\varepsilon \frac{1}{\sqrt{[\varepsilon_n^2 + \Delta^2(T,h) - h^2]^2 + 4\varepsilon_n^2 h^2}}. \tag{8}$$

In Eqs. (7) and (8), $\Delta(T,h)$ is the superconducting gap which depends on both the temperature T and the exchange field h (for simplicity we assume that the moduli of the exchange field are equal to each other). The value of the superconducting order parameter $\Delta(T,h)$ is determined by Eqs. (3) and (6) that can be reduced to the form

$$1 = \lambda\pi T\sum_\varepsilon \text{Re}\frac{1}{\sqrt{(\varepsilon_n + ih)^2 + \Delta^2(T,h)}}. \tag{9}$$

Equations (7)–(9) solve completely the problem of calculation of the Josephson energy and the critical current of the junction with the parallel and antiparallel alignment of the magnetic moments, and all new interesting results of the present paper are described by these equations.

It is clear without further calculations that the current $I_c^{(p)}$ of the parallel configuration is always smaller than the current $I_c^{(a)}$ corresponding to the antiparallel one. So, rotating experimentally the magnetic moment of one of the S/F bilayer one might considerably change the critical current.

Although this phenomenon is interesting on its own, Eq. (8) written for the antiparallel alignment describes at low temperatures a much more striking effect. In the limit $T \to 0$, the sums over the Matsubara frequencies can be replaced by integrals and one obtains [15,16]

$$\Delta(0,h) = \begin{cases} \Delta_0, & h < \Delta_0\,, \\ 0, & h > \Delta_0\,, \end{cases} \tag{10}$$

where Δ_0 is the BCS superconducting gap at $T = 0$ in the absence of the exchange field. There is another solution for $\Delta(h) < \Delta_0$ in the interval $1/2 < h < 1$ [15,16], but this solution is unstable.

Inserting Eq. (10) in Eq. (8) one can see that the Josephson critical current $I_c^{(a)}$ grows with increasing exchange field and even formally logarithmically diverges when $h \to \Delta_0$

$$I_c^{(a)}(h \to \Delta_0) \simeq \frac{I_c(0)}{\pi} \ln(\Delta_0/\omega_0)\,, \tag{11}$$

where $I_c(0)$ is the critical current in the absence of the magnetic moment at $T = 0$, and ω_0 is a cutoff at low energies.

At finite temperatures $\omega_0 \sim T$ but, in principle, it should remain finite also at $T = 0$. The formal divergence seen in Eq. (8) can apparently be removed by considering any damping in the excitation spectrum of the superconductors or higher orders in expansion in the tunneling rate.

The enhancement of the Josephson current by the presence of ordered magnetic moments in superconductors, Eq. (11), is the main result of our paper and is, to the best of our knowledge, a novel effect. It occurs if the magnetic moments are aligned *antiparallel*. In contrast, at finite temperature the Josephson critical current for a *parallel* alignment of the magnetic moments are always smaller than the corresponding values without the magnetic moments. At $T = 0$, the calculation of the integral over the frequencies in Eq. (7) shows that $I_c^{(p)}$ does not depend on h, coinciding with $I_c(0)$.

In principle, the dependence of the critical currents on the exchange field can be more complicated due to a possibility of a transition to the nonhomogeneous phase predicted by Larkin and Ovchinnikov [16] and Fulde and Ferrell [15] (LOFF) for the region $0.755\Delta_0 < h < \Delta_0$. Nevertheless, Eqs. (7)–(11) are applicable for $h < 0.755\Delta_0$, and a possible transition to the LOFF state would manifest itself in a drop of the critical current. Even for $h > 0.755\Delta_0$ the predicted effect may survive because the state with homogeneous Δ may exist as a metastable one.

The enhancement of the Josephson current occurs only at sufficiently low temperatures. Near the transition temperature T_c and for small h one obtains

$$\begin{aligned} I_c^{(a)} &= \pi(eR)^{-1}(\Delta^2/h)\tanh(h/2T_c), \\ I_c^{(p)} &= (\pi/2)\,(eR)^{-1}(\Delta^2/T_c)\cosh^{-2}(h/2T_c)\,, \\ I_c^{(a)}/I_c^{(p)} &= (T_c/h)\sinh(h/T_c)\,, \end{aligned} \tag{12}$$

where $\Delta = \Delta(T,h)$ is determined from Eq. (9). The dependence of T_c on h is presented in Ref. [17]. At arbitrary temperatures the dependence of the critical currents on the exchange field h can be obtained from Eqs. (7)–(9) only numerically. The results are represented in Fig. 2 for the antiparallel configuration and in Fig. 3 for the parallel one.

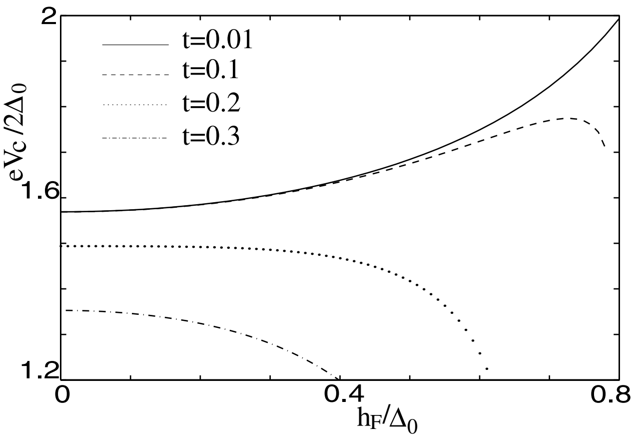

FIG. 2. Dependence of the normalized critical current on h for different temperatures in the case of an antiparallel orientation. Here $eV_c = eRI_c$, h_F is the effective exchange field, $t = T/\Delta_0$, and Δ_0 is the superconducting order parameter at $T = 0$ and $h = 0$.

If the angle α between the directions of the magnetization is arbitrary, the critical current I_c^α can be written in the form

$$I_c^\alpha = I_c^{(p)}\cos^2(\alpha/2) + I_c^{(a)}\sin^2(\alpha/2)\,. \tag{13}$$

Equation (13) shows that the singular part of the critical current is always present, and its contribution may reach 100% at $\alpha = \pi$.

All the conclusions presented above are valid also for two magnetic superconductors with uniformly oriented magnetization in each layer. Equations (7) and (8) could be obtained from formulas written in Ref. [18] for magnetic superconductors with a spiral structure. However, the effects found in our work were not discussed in Ref. [18].

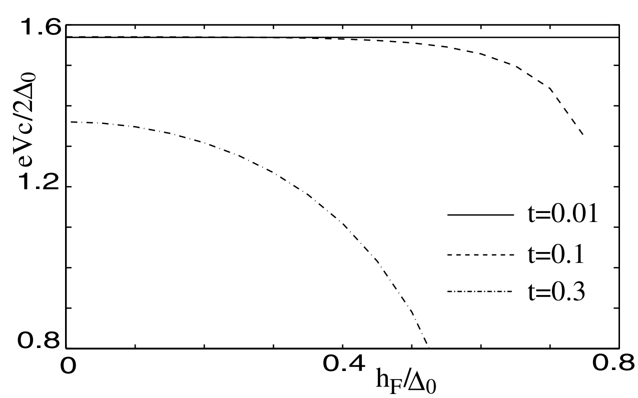

FIG. 3. The same dependence as in Fig. 2 in the case of a parallel orientation.

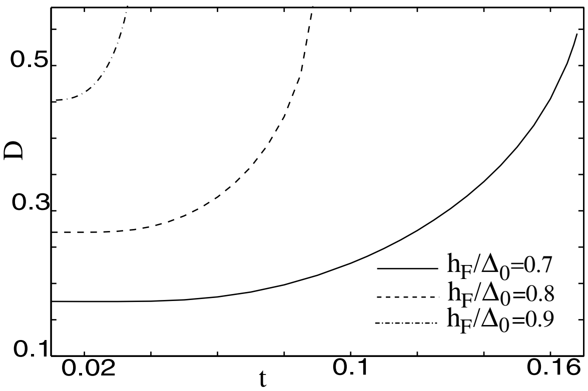

FIG. 4. Temperature dependence of the coefficient D. Here h_F is the effective exchange field and $t = T/\Delta_0$.

Experimentally, it might be convenient to measure the coefficient D

$$D = \frac{I_c^{(a)} - I_c^{(p)}}{I_c^{(p)}} \tag{14}$$

as a function of temperature. We draw in Fig. 4 several curves characterizing the temperature dependence $D(T)$ for different h. One can change h by varying the thickness of the magnetic layers. We see that the coefficient D can reach values of the order of unity. We note that at a given h ($h > 1/2$) a first order transition takes place when T reaches a certain critical value. In this case either Δ drops to a smaller value or the normal state is realized. If the S/F interface resistance per unit area $R_{S/F}$ exceeds the value $\rho_F d_F$ (ρ_F is the specific resistance of the ferromagnet), the condensate functions experience a jump at the S/F interface and a subgap $\epsilon_{sg} = (D\rho_F)_F/R_{S/F}d_F < \Delta$ arises in the ferromagnet [19]. In this case a singularity appears when $h \rightarrow \epsilon_{sg}$.

All the results presented in this paper can be obtained by using the quasiclassical Green's function technique generalized for spin-dependent interaction. The details of the calculations will be presented elsewhere. It is important to mention that the enhancement of the Josephson current by the antiparallel alignment of the magnetic moments is obtained only for the singlet pairing.

In conclusion, we have shown that in contrast to the common view, the presence of an exchange field h can increase the critical current I_c in a Josephson tunnel junction S/F-I-F/S in the case of an antiparallel alignment of the magnetization in the ferromagnets.

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