Convection, Heaping, and Cracking in Vertically Vibrated Granular Slurries

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We address the situation in which vibration is applied to thin layers of granular material with water filling the pore space, but with air above. Beyond a critical drive parameter we observe heap-shaped convecting domains of nontrivial topology, which exhibit cracks as the forcing amplitude is further increased. We summarize these results in a phase diagram, understand the onset of the convecting heaps as a manifestation of the Rayleigh-Taylor instability for fluids, and measure the response of isolated convecting structures.

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The study of patterning in fluid and fluidlike systems under vertical vibration has led to an understanding of the Faraday surface instability in vibrated liquids [1,2] and the recent illumination of the dynamic properties of dry granular materials, which include a rich variety of spatiotemporal patterns [3–7] and isolated solitonlike structures [8]. Similar features were documented with clay suspensions [9], which illustrate one possible response of a fluid-particle system. Interstitial fluid (gas or liquid) is known to lead to convection in granular systems by a mechanism first postulated by Faraday, in which the surrounding fluid is drawn towards the center of a convecting domain, dragging particles along with it, as a vibrating plate accelerates downwards [1,10–12]. In our experiments with thin layers of wet granular material, surface tension at the air-water interface precludes some aspects of this mechanism. Our experimental observations, which highlight novel nonlinear responses of slurrylike materials, should be useful for developing an improved understanding of natural situations and industrial applications involving the transport, mixing, and rheology of wet granular media [13].

The slurry we study consists of a combination of measured quantities of bronze microspheres (ACU Powder International; diameter equal to 143 ± 21 μm) and tap water in a circular dish of diameter 7.5 cm (Fig. 1a). The layer thickness ranges from 50 to 150 particle diameters and, within this range, does not influence the qualitative features of the response. A number of experiments were also conducted with 150 ± 9 μm diameter soda lime glass particles (MO-SCI Corporation). We selected a water volume fraction of $\phi = 0.44$, which fills the pore space in a loosely compacted granular material and provides a minimal excess that facilitates leveling of the surface; the material is just saturated. The container is sealed to prevent evaporation.

An electromagnetic shaker supplies vertical vibrations driving the periodic motion $z(t) = A\sin(\omega t)$. The motion is parametrized by the frequency $f = \omega/2\pi$ and the dimensionless acceleration $\Gamma = A\omega^2/g$, where g is the gravitational acceleration. We explore frequencies f from 10–150 Hz and accelerations Γ from 0–10. Both of these ranges correspond in magnitude with those previously used in studies of vibrated liquids and dry granular material.

In a typical experiment we maintain constant frequency and adjust the acceleration amplitude. From the state of rest the system progresses through a number of distinct dynamical states, the first of which is the convective heaping shown schematically in Fig. 1b. Photographs representative of the entire set of observations are shown in Fig. 2; note the very regular patterns that can be observed (e.g., Figs. 2a and 2c). Whereas thin layers of dry granular material respond with patterning that oscillates on the time scale of the driving force (e.g., [3]), the result here is convection cycles of hundreds of periods of the forcing, or longer. The formation of multiple convection domains appears similar to that observed with a thick granular layer in a completely aqueous environment [11] and with fine powders in air [12]. Our study, instead, examines relatively thin liquid-saturated granular layers with air above. As a result we document *isolated* convecting structures (e.g., Figs. 2b and 2e) and a "cracking" surface relief at higher

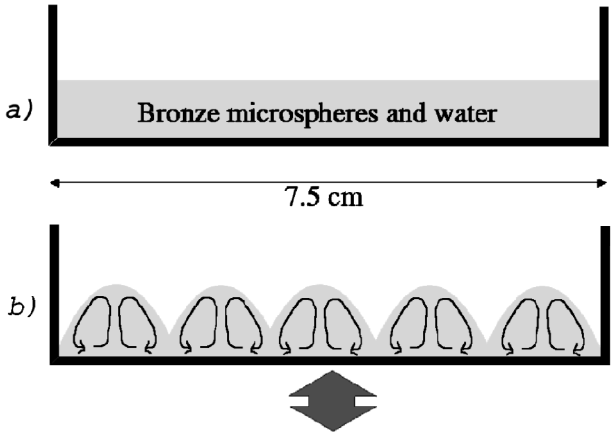

FIG. 1. Schematic profile of the vibrated granular slurry experiment. The circular sample dish is filled by combining measured quantities of dry granules and water. In contrast to the response of dry granular systems, the saturated granular material remains in contact with the plate throughout the motion, even when the downward acceleration exceeds that of gravity.

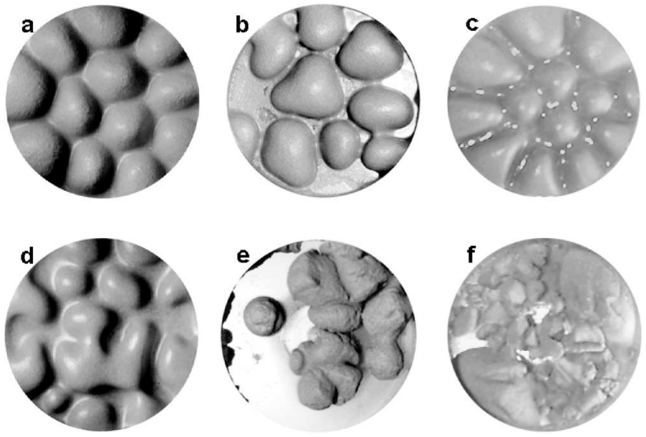

FIG. 2. Top views illustrating the variety of surface reliefs observed with bronze particles; liquid volume fraction $\phi = 0.44$. Images correspond to different drive frequencies f and accelerations Γ (layer depth is 0.5 cm unless otherwise noted): (a) $f = 50$ Hz, $\Gamma \approx 3$ (particle diameter here is about 200 μm, layer depth approximately 1 cm); (b) $f = 80$ Hz, $\Gamma = 2.4$; (c) $f = 65$ Hz, $\Gamma = 3.0$; (d) $f = 50$ Hz, $\Gamma \approx 3$ (layer depth approximately 1 cm); (e) $f = 48$ Hz, $\Gamma = 5.8$; (f) $f = 55$ Hz, $\Gamma = 12$. Though the details (e.g., irregularities) of the surface responses are transient, their characteristic large-scale features are stable and reproducible.

drive amplitudes, both results which are summarized in a phase diagram.

Whereas at low accelerations the long-lived heaps appear as regular (Fig. 2c) or irregular (Fig. 2d) height modulations on a layer that covers the substrate completely, at higher accelerations heaps increase in height and decrease in lateral dimension, exposing the floor of the experimental cell and leading to domains that are isolated from one another (Fig. 2b). Note that some of these structures have nontrivial topology, including those with certain symmetries (cases of sixfold and threefold symmetry are shown in Figs. 2a and 2b) and those that appear to lack low-order symmetry. As acceleration is further increased, the heaps convect with increasing speed and instabilities begin to manifest themselves in the form of surface fissures (Figs. 2e and 2f). This cracking transition is followed by various states of violent convection and breakup of irregularly shaped aggregates of material.

Experiments spanning a wide range of drive frequency and amplitude allow us to construct the phase portrait for bronze particles, as shown in Fig. 3. Initially, a flat state is achieved by striking the test vessel from below, delivering an impulsive forcing. We developed a reproducible standard for defining and measuring transitions in the dynamical state of the surface as follows. After sample preparation, the frequency and acceleration are brought quickly to some constant value and the experiment is allowed to run for up to 5 min, at which point we observe whether there remain regions where the flat layer is stable, and whether fissures are visible on the surfaces of any heaps that may have formed. At each frequency the symbols in Fig. 3 indicate the highest acceleration at which the transition is not observed and the lowest acceleration at which it is observed.

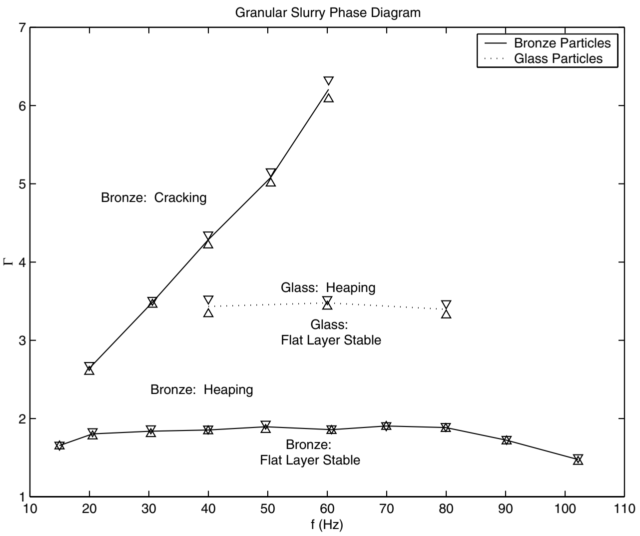

FIG. 3. Composite phase portrait in acceleration versus frequency for layers of either bronze or glass particles; 0.5 cm thick layer of 150 μm spheres. Further experiments, with a 0.75 cm thick layer, produce similar results, shifting the cracking transition, but without a measurable effect on the transition to heaping; $\phi = 0.44$. For accelerations below $\Gamma_c = 1.9$, no motion is observed except at the highest frequencies, where the heaping response is somewhat ambiguous, forming only one pile. This is the best characterization of the high-frequency limit our experimental apparatus was able to provide. The data shown for glass particles were taken to verify the scaling of Eq. (1).

A significant feature in Fig. 3 is that the onset of heaping occurs at nearly constant critical acceleration, $\Gamma_c = 1.9$ for bronze particles, across the range of frequencies where the response can be identified readily. We note that the cracking transition appears nonhysteretic, while the heaping transition exhibits hysteresis; when decreasing the forcing amplitude below Γ_c, convection stops and the surface relief cannot relax to the flat state. This latter observation is a consequence of the nonlinear constitutive response of the wet granular medium, which is a material with a yield stress. Further experiments demonstrate that varying layer thickness affects the amplitude of the cracking transition but not its heaping instability. It is noteworthy that experiments with 150 μm and 1 mm glass beads reproduce all of the observed phenomena, despite the significantly different size and density of the particles. In particular, our data with 150 μm glass particles for frequencies between 40–80 Hz show a transition to heaping at $\Gamma_c = 3.4$. We note that, for all of the above, surface tension is clearly important for maintaining a distinct interface separating the slurry from air. Also, across the menisci that span the particles at the air-water interface, there is a pressure drop that gives the material a cohesiveness (referred to as the apparent cohesion in the soil mechanics literature) that limits dilation induced by vibration.

A force balance hints at the origin of the critical acceleration for the transition to heaping. The vibrated system has an effective vertical acceleration $a(t) = -g(1 + \Gamma \sin\omega t)$. We note that the slurry remains in contact with the moving substrate even when the downward acceleration of the plate exceeds that of gravity. During the upswing the force on the grain network is accounted for by normal forces, $N(t)$, transmitted from the plate via interparticle contacts, but during the downswing a combination of liquid mediated viscous and hydrostatic forces, $F(t)$, serve to accelerate the grain network in correspondence with the substrate motion. Equal and opposite forces, $-F(t)$, act on the fluid, in effect as a body force. The force balance for the grains is $-\rho_g a(t) = N(t) + F(t)$, where ρ_g is the granular density. It is clear that the normal force on the grain network can only be positive, thus we set $N(t) = \rho_g[|a(t)| - a(t)]/2$. For accelerations greater than $\Gamma = 1$, the time-averaged body force on the fluid $\langle \rho_\ell a(t) - F(t) \rangle$ will be less than $\rho_\ell g$ (ρ_ℓ denotes the density of the liquid and $\langle\cdot\rangle$ denotes the time average). Since the period of the motion is much shorter than the time scale on which surface instabilities form, it is reasonable to consider the instability of the flat layer by analogy to the case of acceleration of a fluid of high density into a fluid of low density, i.e., the Rayleigh-Taylor instability. A calculation then gives the criterion for sign reversal of the body force on the liquid:

$$2\pi \frac{\rho_\ell}{\rho_g} = -\pi + 2\sqrt{\Gamma_c^2 - 1} + 2\arcsin\left(\frac{1}{\Gamma_c}\right), \tag{1}$$

which is independent of the frequency. For the bronze particles used in our experiments, the ratio $\rho_\ell/\rho_g = 1/5.4$ and the value $\Gamma_c = 1.9$ is obtained numerically. Also, for the glass particles $\rho_\ell/\rho_g = 1/1.6$ and Eq. (1) predicts $\Gamma_c = 3.4$. The agreement of both of these values with the results reported in the phase diagram (Fig. 3) suggests that the mechanism of the instability is similar to that of the Rayleigh-Taylor instability. The arguments that lead to (1) produce an instability threshold that is independent of viscosity; the viscosity is expected to influence the rate at which the instability develops, as is known in other familiar viscous flow problems (e.g., the Rayleigh-Taylor or Rayleigh-Plateau instability).

The significance of Γ_c is corroborated by direct measurement of the diameter of isolated convecting structures as a function of drive parameters. We prepared spherical heaps of granular slurry on a flat surface with moisture content $\phi = 0.46$. We collected digital images from above of the convecting heaps of bronze particles at various Γ at each of three fixed frequencies. The drop diameter depends upon the level of acceleration above $\Gamma_c = 1.9$, and in Fig. 4 we show that the data of drop radius versus driving frequency may be collapsed when the diameter is plotted against $(\Gamma - \Gamma_c)/\omega$. These experiments further demonstrate the significance of Γ_c when characterizing dynamics of wet granular systems.

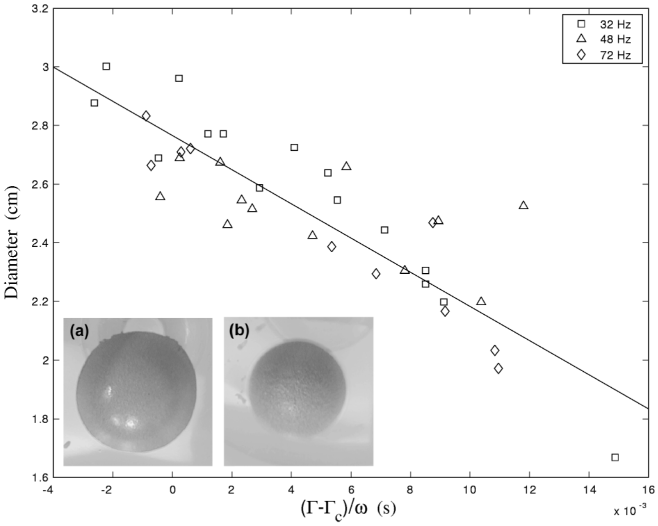

FIG. 4. Data for the diameter of isolated heaps (volume = 2.2 cm^3) of a slurry of bronze particles plotted versus $(\Gamma - \Gamma_c)/\omega$; $\phi = 0.46$. Scaling, using the onset acceleration for flat layers (Γ_c), collapses all of the data. Insets show examples at (a) low and (b) high values of $(\Gamma - \Gamma_c)/\omega$.

Our experiments with isolated convecting heaps led to the discovery of a unique response that again illustrates the breadth of dynamical phenomena observed with granular slurries. When the liquid volume fraction of an isolated heap was increased slightly to $\phi = 0.50$, a crown of jets was observed as shown in Fig. 5. The fingers

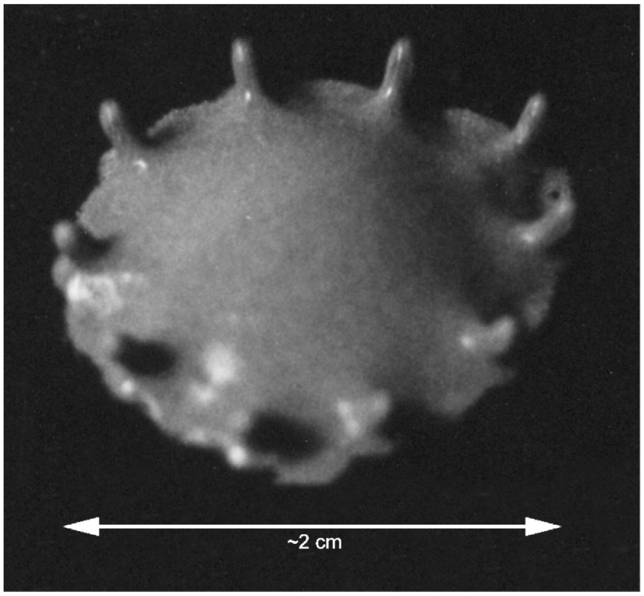

FIG. 5. A single drop of a slurry of bronze particles on a vibrating plate forms a standing crown of jets that oscillates subharmonically. The total drop volume is 1.6 cm^3 and $\phi = 0.50$. Drive parameters are $f = 40$ Hz and $\Gamma = 7.5$.

consist of two interlaced sets that oscillate subharmonically. Particle rarefaction (increased ϕ) is observed at the peaks of the fingers. This standing wave resembles the well-known Edgerton Crown seen when a drop impacts a liquid.

In our experiments with a granular slurry, we have obtained new convecting surface reliefs, including isolated structures of varied topology, and documented their dependence on driving frequency and amplitude. We further show how the critical vibration amplitude Γ_c results from a Rayleigh-Taylor instability. The fact that this parameter is useful in characterizing the diameter of a vibrated heap implies that it may have a more general role in the dynamics of granular slurries. Wet granular systems such as studied here are present in many industrial and natural processes [14], e.g., food handling, liquefaction of soils during vibratory motions such as earthquakes, mixing of building materials, processing and handling of coal, etc. Our experimental results should be useful as dynamical investigations of granular matter come to address wet systems.

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