

Dark Solitons in a Two-Component Bose-Einstein Condensate

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The creation and interaction of dark solitons in a two-component Bose-Einstein condensate is investigated. For a miscible case, the interaction of dark solitons in different components is studied. Various possible scenarios are presented, including the formation of a soliton-soliton bound pair. We also analyze the soliton propagation in the presence of domains, and show that a dark soliton can be transferred from one component to the other at the domain wall when it exceeds a critical velocity. For lower velocities multiple reflections within the domain are observed, where the soliton is evaporated and accelerated after each reflection until it finally escapes from the domain.

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The realization of Bose-Einstein condensation (BEC) in weakly interacting atomic gases [1] has opened the possibility to investigate nonlinear properties of atomic matter waves. Several remarkable phenomena, which strongly resemble well-known effects in nonlinear optics (NLO), have been observed in BEC, such as four-wave mixing [2], vortices [3,4], and dark solitons [5,6].

A dark soliton in BEC is a macroscopic excitation of the condensate with a corresponding positive scattering length, which is characterized by a local density minimum and a phase gradient of the wave function at the position of the minimum [7]. The shape of the dip does not change due to the balance between kinetic energy and repulsive atom-atom collisions. The recent creation of dark solitons in BEC by means of phase imprinting (PI) [5,6] has posed several fundamental questions concerning the dynamics, stability, and dissipation in such systems [8–11]. Also, the interaction of two solitons in a BEC has been experimentally addressed [12].

In the recent years, the development of trapping techniques has allowed the creation of multicomponent condensates, which are formed by trapping atoms in different internal (electronic) states [13,14]. The multicomponent BEC, far from being a trivial extension of the single-component one, presents novel and fundamentally different scenarios for its ground state [15] and excitations [16]. In particular, it has been observed that the BEC can reach an equilibrium state characterized by the separation of the species in different domains [14].

In the present Letter, we analyze the creation, propagation, and interaction of dark solitons in a two-component condensate. In this more complex scenario, novel phenomena can be expected, as has already been reported in the context of NLO [7]. We show that the dynamics of the soliton interaction is completely different from the single-component case. In particular, two dark solitons in a single-component BEC always repel each other [17], whereas the opposite is true for two solitons interacting in the two-component case [18]. We consider two situations. For the first one, the two components are miscible, and one soliton is created in each component.

We show, both analytically and numerically, that this system presents nontrivial dynamics, which includes the formation of a soliton-soliton bound state. The properties of such a binary bound state, in particular the period of its oscillations, should be sensible to dissipation effects and therefore constitutes an excellent tool to analyze these effects. In addition, the dissipation can be studied in a much clearer and controllable way than in current experiments with single-component BEC, since the soliton-soliton interaction keeps the solitons within a central region of the trap of several healing lengths. In the second scenario, the propagation of a soliton in a two-component BEC with domains is considered. In particular, we demonstrate that the soliton can be transferred from one component to the other at the domain wall. Below a critical velocity the soliton is reflected, performing multiple oscillations inside the corresponding domain. At each reflection the soliton is partially evaporated by emission of phonons, and accelerates until it eventually escapes through the wall.

In the following we consider a trapped BEC with two components, where the dynamics takes place only in one dimension due to the strong trap confinement in the transverse direction. This approximation is valid if the mean-field interaction is smaller than the typical energy separation E_{\perp} in the other directions [19], and has been successfully employed in the analysis of dark solitons in single-component condensates [6]. For sufficiently low temperatures the dynamics is well described by two coupled Gross-Pitaevskii equations

$$i\hbar \frac{\partial}{\partial t} \psi_j(x, t) = \left\{ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g_{jj} |\psi_j(x, t)|^2 + g_{jl} |\psi_l(x, t)|^2 - \mu_j \right\} \psi_j(x, t), \quad (1)$$

where $V(x)$ is the trap potential, μ_j is the chemical potential of component j , $g_{jl} = 4\pi\hbar^2 a_{jl}/mS$ is the coupling constant (averaged over the frozen-out transversal directions) between the components j and l with the transversal

area S , m is the atomic mass, and a_{jl} the scattering length between j and l ($j, l = 1, 2$).

We consider the situation in which a dark soliton is created in one or in both components. To generate dark solitons we use the well established PI technique [20], although other methods [21] could in principle be employed. This method consists of applying a homogeneous potential U generated by the dipole potential of a far detuned laser beam to one part of the condensate. The potential is pulsed on for a time t_p , such that the wave function locally acquires an additional phase factor $\exp(iUt_p/\hbar)$. The pulse duration is chosen to be short compared to the minimal correlation time of the system \hbar/μ , with $\mu = \max(\mu_1, \mu_2)$. This ensures that the effect of the light pulse is mainly a change in the phase of the condensate, whereas changes of the density during this time can be neglected. Note, however, that due to the imprinted phase at larger times an adjustment of the density in the condensate appears, leading to the formation of the dark soliton and also additional structures. In principle the described method can be employed to selectively create a soliton in only one of the components, or to create two different solitons in each one. The creation of a soliton in one component modifies the density in the other one, due to the coupling in Eq. (1). But as we show below, under appropriate conditions, solitons can also be created in a two-component condensate. In the following, given a particular PI, we define the imprinted velocity as the one corresponding to a soliton created by the same PI in a single-component BEC.

We consider first the case in which one soliton is created in each component. This situation can be analytically studied by employing a variational approach [22,23]. We assume for simplicity the solitons are moving in a homogeneous condensate of densities $n_1 = n_2 = n_0$. This situation corresponds to a very elongated trap with equal concentrations of both components. Additionally we assume that the coupling constants are $g_{11} = g_{12} = g_{22} = g$. The latter assumption matches well the experimental conditions [3,14]. As variational wave functions the single-component soliton solutions [17] are considered,

$$\frac{\psi_1(x, t)}{\sqrt{n_0}} = \frac{i\dot{q}}{c_s} + \sqrt{1 - \frac{\dot{q}^2}{c_s^2}} \tanh \sqrt{1 - \frac{\dot{q}^2}{c_s^2}} \frac{(x - q)}{l_0}, \quad (2a)$$

$$\frac{\psi_2(x, t)}{\sqrt{n_0}} = \frac{-i\dot{q}}{c_s} + \sqrt{1 - \frac{\dot{q}^2}{c_s^2}} \tanh \sqrt{1 - \frac{\dot{q}^2}{c_s^2}} \frac{(x + q)}{l_0}, \quad (2b)$$

where $\mu_1 = \mu_2 = \mu$, $\dot{q} = dq/dt$, $2q(t)$ denotes the relative distance between the solitons, $c_s = \sqrt{gn_0/m}$ the sound velocity, and $l_0 = \hbar/\sqrt{gn_0m}$ is the healing length for a single component. Equations (2a) and (2b) represent a kink-antikink situation, i.e., when the phase fronts of the solitons are facing each other. The case kink-kink, when both phase fronts are in the same direction, is also discussed below. The previous expressions describe a

symmetric situation around $x = 0$, or equivalently they describe the system in the center of mass frame.

The problem of solving Eq. (1) can be restated as a variational problem [22,23], corresponding to the stationary point of the action related to the Lagrangian density,

$$\mathcal{L} = \sum_{j=1,2} \left\{ \frac{i\hbar}{2} \left(\psi_j \frac{\partial \psi_j^*}{\partial t} - \psi_j^* \frac{\partial \psi_j}{\partial t} \right) + \frac{\hbar^2}{2m} \left| \frac{\partial \psi_j}{\partial x} \right|^2 + \frac{g}{2} |\psi_j|^4 \right\} + g |\psi_1|^2 |\psi_2|^2. \quad (3)$$

Our goal is to find the equation which governs the evolution of $q(t)$. In order to do that, we insert the variational Ansatz (2a) and (2b) into Eq. (3), and calculate an effective Lagrangian $L = \int dx \mathcal{L}$ which becomes

$$L = -2gn_0^2 l_0 \left(1 - \frac{\dot{q}^2}{c_s^2} \right)^{3/2} \frac{\cosh(2b)}{\sinh^3(2b)} [\sinh(4b) - 4b] - \frac{4}{3} gn_0^2 l_0 \left(1 + 2 \frac{\dot{q}^2}{c_s^2} \right) \sqrt{1 + \frac{\dot{q}^2}{c_s^2}}, \quad (4)$$

where $b(q, \dot{q}) = \sqrt{1 - \dot{q}^2/c_s^2} (q/l_0)$. The equation for $q(t)$ is given by the Euler-Lagrange equation $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0$, which we have solved for different initial conditions, obtaining the trajectories in the phase space (q, \dot{q}) (Fig. 1). For sufficiently small deviations from $(0, 0)$, the system behaves periodically, i.e., the solitons form a bound pair (soliton molecule). For bound trajectories with $\dot{q}(0)/c_s < 0.4$ at $q = 0$, the frequency of the oscillation is $4/\sqrt{15} c_s/l_0$. Contrary to single component solitons [17], the free trajectories are characterized by the acceleration of the approaching solitons, and the deceleration of the outgoing ones. For $q \gtrsim 2l_0$, L becomes the sum of two single-soliton Lagrangians and \dot{q} vanishes. For practical purposes the trajectories can be considered periodic if they cross $\dot{q} = 0$ at $q \lesssim 2l_0$, and free otherwise. The free trajectories close to the periodic ones are squeezed together at $q = 0$ and $\dot{q} = \dot{q}_c = 0.67c_s$, which constitutes the critical escape velocity.

The kink-kink situation is, however, different, as recently shown in the context of multimode NLO [24]. In

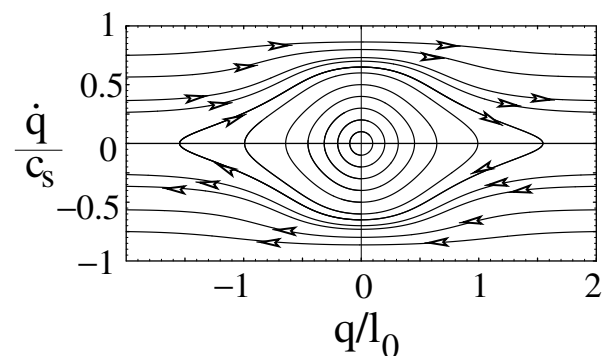


FIG. 1. Phase map of the kink-antikink relative motion.

this case the solitons move in the same direction with velocities \dot{q}_1 and \dot{q}_2 , such that in our formalism $2\dot{q} = \dot{q}_2 - \dot{q}_1$. Contrary to the kink-antikink case, the solitons form a molecule which can never be broken. The special kink-kink case ($q = 0, \dot{q} = 0$) coincides with the optical vector dark soliton solution [18,25].

We have numerically analyzed the dynamics of the system by solving Eq. (1) using standard split operator techniques. We have studied the creation of a soliton in each component using the PI method. We consider the case of a BEC with equal linear density in both components, $n = 2.0 \times 10^8 \text{ cm}^{-1}$, in a box trap and with $g_{11} = g_{22} = 1.05g_{12}$. This choice allows a homogeneous region of equal densities for both components, and therefore provides a better quantitative comparison with the analytical model. However, for more general nonhomogeneous situations a similar qualitative picture has been observed in simulations.

We restrict ourselves here to the case kink-antikink. The creation of a soliton in one component perturbs the density of the other one. We observe that if the PI is applied in order to create two solitons which are initially separated by distances larger than approximately $4l_0$, the fluctuations in the densities prevent the formation of the solitons. Therefore, the solitons have to be created initially with $q(0) < 2l_0$. The soliton in each component induces a local density increase centered at the position of the soliton in the other component. In other words, the solitons are filled by the other component. Therefore, they become wider and slower than a corresponding single-component soliton. This fact introduces some quantitative corrections to the analytical estimates, although such corrections are, in fact, small.

Figure 2 shows the case of two solitons with an imprinted velocity $\dot{q} = 0.81c_s$ (corresponding to an initial PI razor of $8l_0$ [5,6]). For such velocity, the solitons form a molecule, as described above. The soliton separation, which can reach $4l_0$, depends on the imprinted velocity. We have numerically found a critical imprinted velocity $\dot{q}_c = 0.83c_s$ at which the solitons become free, in good agreement with the variational approach.

Figure 3 shows the evolution of the solitons corresponding to an imprinted velocity $\dot{q} = 0.89c_s$ (dashed line in Fig. 3). The solitons indeed move apart much slower with

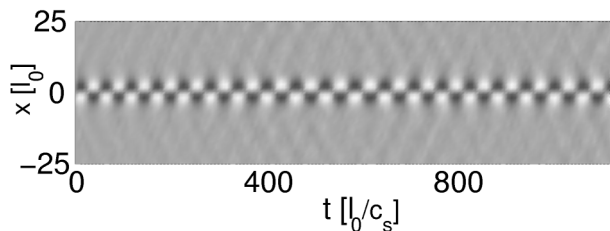


FIG. 2. Density of component 1 for the kink-antikink case with $q(0) = 0$ and $\dot{q}(0) = 0.81c_s$. Darker regions are those with less density. Component 2 is the mirror image of component 1 around $x = 0$.

$\dot{q} = 0.095c_s$. The closer the imprinted velocity is to \dot{q}_c the larger the soliton deceleration is, in agreement with our analytical results. Therefore, the deceleration is an effect of the soliton-soliton interaction and not a consequence of the filling of the soliton by the other component [26]. We have also depicted in Fig. 3 the trajectory after the reflection from the box boundaries, in order to illustrate the behavior when both solitons collide. As predicted from our variational approach, it can be observed that the solitons are accelerated when approaching each other, and decelerated after crossing. The maximal velocity at $q = 0$ is comparable to the critical velocity.

In the last part of this Letter we analyze the soliton propagation in separate domains. If the relation between the coupling constants and densities is appropriately chosen, separate domains of each component can be produced [14]. We consider the case in which a soliton is created in one of the components and move towards the domain wall. In order to illustrate the different possible scenarios, we study the situation in which $g_{12}/g_{11} = 1.7$, $g_{22}/g_{11} = 0.96$ for different initially imprinted velocities. Both components have equal linear density $n = 4.0 \times 10^8 \text{ cm}^{-1}$. A sufficiently fast soliton will be transferred through the domain wall into the other component. However, if the velocity is sufficiently low, the soliton is reflected at the domain wall, as shown in Fig. 4. This figure shows the case of a box trap with the initial soliton velocity $\dot{q} = 0.15c_s$. At each reflection the soliton is partially evaporated in the form of phonons in the second component. The latter induces an acceleration until the soliton eventually escapes the domain. The critical escape velocity can be estimated from simple energetic considerations, assuming that the soliton must overcome a potential barrier induced by the second component at the domain wall. This gives a critical velocity $\dot{q}_t = \sqrt{(g_{12} - g_{11})/4Sl_0}$, where S is the transversal area. In the considered example, the analytical value $\dot{q}_t = 0.19c_s$ is in excellent agreement with the numerical one $\dot{q}_t = 0.16c_s$. When the soliton is transferred, a back action of the soliton on the domain is observed. This introduces density fluctuations and perturbations in the domain

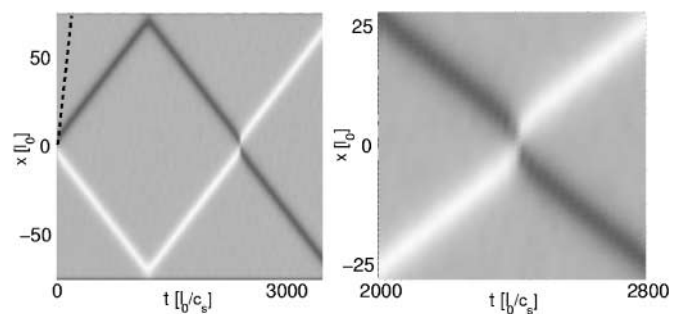


FIG. 3. (Left) Density of component 1 for the kink-antikink case with $q(0) = 0$ and $\dot{q}(0) = 0.89c_s$. Darker regions are those with less density. Component 2 is the mirror image of component 1 around $x = 0$. The dashed line is the soliton trajectory in a single-component condensate. (Right) Detail of the collision region.

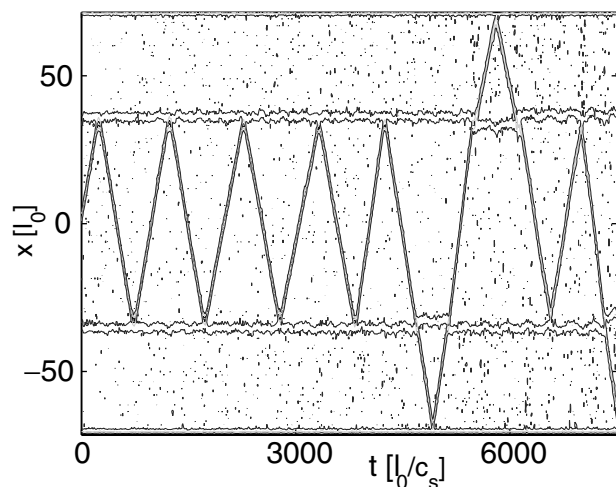


FIG. 4. Interaction with a domain wall of a soliton initially created in component 1 with $\dot{q} = 0.15c_s$.

walls, which slightly modify the critical velocity. The latter can produce a retrapping of the soliton in the original domain, as observed in Fig. 4.

In this Letter we have shown the rich behavior of solitons in two-component BEC. The two components provide solutions such as bound solitons and the possibility to create extremely slowly moving ones. We have analytically studied the dynamics of the system with a variational approach, and determined the possible scenarios. We have finally analyzed a two-component BEC which contains domains, and showed that depending on the physical parameters a dark soliton can be either transferred or reflected at the domain wall.

Several interesting problems, however, remain open. Among them, we stress especially two. In the present Letter we have analyzed a 1D system. If the 1D conditions are not strictly fulfilled, dynamical instability is expected [9,10]. In the new scenario with two-component condensates the properties of such instability should be altered. A second interesting problem is given by the dissipation of the oscillatory motion. The two solitons radiate phonons when oscillating. Contrary to the case of other binary systems, the radiation will increase the elongation of the oscillations, until eventually breaking the soliton molecule, and therefore these systems could be an excellent probe for the dissipation effects.

We should finally stress that the effects considered here appear for realistic situations and can be experimentally analyzed with the state of the art technology. The creation of dark solitons constitutes a well established technique for the case of a single-component BEC. We have numerically simulated the PI mechanism in a two-component BEC and demonstrated that this technique can also be applied in that situation [27]. Since the solitons are indeed wider due to the presence of the second component, some of the predicted effects, as, for example, the appearance of a critical escape velocity, could be experimentally ob-

served in a nondestructive way. Others, however, as, for example, the soliton oscillations, could require the opening of the trap, and subsequent condensate expansion. The dynamics of such expansion will be the subject of a separate investigation.

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- [1] M.H. Anderson *et al.*, *Science* **269**, 198 (1995); K.B. Davis *et al.*, *Phys. Rev. Lett.* **75**, 3969 (1995); C. C. Bradley *et al.*, *Phys. Rev. Lett.* **78**, 985 (1997).
 - [2] L. Deng *et al.*, *Nature (London)* **398**, 218 (1999).
 - [3] M.R. Matthews *et al.*, *Phys. Rev. Lett.* **83**, 2498 (1999).
 - [4] K.W. Madison *et al.*, *Phys. Rev. Lett.* **84**, 806 (2000).
 - [5] J. Denschlag *et al.*, *Science* **287**, 97 (2000).
 - [6] S. Burger *et al.*, *Phys. Rev. Lett.* **83**, 5198 (1999).
 - [7] See, e.g., Y.S. Kivshar and B. Luther-Davies, *Phys. Rep.* **298**, 81 (1998).
 - [8] Th. Busch and J.R. Anglin, *Phys. Rev. Lett.* **84**, 2298 (2000).
 - [9] A.E. Muryshev *et al.*, *Phys. Rev. A* **60**, R2665 (1999).
 - [10] P.O. Fedichev *et al.*, *Phys. Rev. A* **60**, 3220 (1999).
 - [11] D.L. Feder *et al.*, *Phys. Rev. A* **62**, 053606 (2000).
 - [12] K. Sengstock and W. Ertmer (private communication).
 - [13] C.J. Myatt *et al.*, *Phys. Rev. Lett.* **78**, 586 (1997).
 - [14] J. Stenger *et al.*, *Nature (London)* **396**, 345 (1998).
 - [15] B.D. Esry *et al.*, *Phys. Rev. Lett.* **78**, 3594 (1997); T. Ho, *Phys. Rev. Lett.* **81**, 742 (1998); P. Öhberg and S. Stenholm, *Phys. Rev. A* **57**, 1272 (1998).
 - [16] B.D. Esry and Chris H. Greene, *Phys. Rev. A* **57**, 1265 (1998); Th. Busch *et al.*, *Phys. Rev. A* **56**, 2978 (1997); P. Öhberg and S. Stenholm, *J. Phys. B* **32**, 1959 (1999).
 - [17] V.E. Zakharov and A.B. Shabat, *Zh. Eksp. Teor. Fiz.* **64**, 1627 (1973) [*Sov. Phys. JETP* **37**, 823 (1973)].
 - [18] This situation should not be confused with the interaction of two vector dark solitons [7], which are formed by a kink in each component and are centered at the same position moving as a whole. See, e.g., Ref. [25].
 - [19] The stability of a dark soliton in an elongated trap requires a chemical potential $\mu \leq 2.5E_{\perp}$ [9].
 - [20] Ł. Dobrek *et al.*, *Phys. Rev. A* **60**, R3381 (1999).
 - [21] R. Dum *et al.*, *Phys. Rev. Lett.* **80**, 2972 (1998).
 - [22] V.M. Pérez-García *et al.*, *Phys. Rev. A* **56**, 1424 (1997).
 - [23] Y.S. Kivshar and W. Królikowski, *Opt. Commun.* **114**, 353 (1995).
 - [24] Q. Park and H.J. Shin, *Phys. Rev. E* **61**, 3093 (2000).
 - [25] Y.S. Kivshar and S.K. Turitsyn, *Opt. Lett.* **18**, 337 (1993).
 - [26] Such a filling effect has been recently reported in two-component vortices by B.P. Anderson *et al.*, *Phys. Rev. Lett.* **85**, 2857 (2000); see also B.P. Anderson *et al.*, *cond-mat/0012444*.
 - [27] The same PI for both components, together with a displacement of one of the components, leads also to the same effects.