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Universality of Decoherence

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We consider environment induced decoherence of quantum superpositions to mixtures in the limit in which that process is much faster than any competing one generated by the Hamiltonian H_{sys} of the isolated system. While the golden rule then does not apply we can discard H_{sys}. By allowing for couplings to different reservoirs, we reveal decoherence as a universal short-time phenomenon independent of the character of the system as well as the bath and of the basis the superimposed states are taken from. We discuss consequences for the classical behavior of the macroworld and quantum measurement: For decoherence of superpositions of macroscopically distinct states H_{sys} is always negligible.

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Superpositions of quantum states give rise to interference effects which are, however, more and more difficult to observe as the size of the system is increased and the superimposed states are made more distinct. While quantum interferences are ubiquitous in the microworld, none have been seen for macroscopic bodies. A parameter controlling the discernibility of interference fringes is the ratio of the de Broglie wavelength λ of a particle to a typical linear dimension Δ of the spatial structure used to construct superpositions of different partial waves. When that parameter is of order unity, interference is easily measurable; but upon decreasing λ/Δ, either by letting Δ grow or using ever more massive particles and thus ever smaller λ, wave effects become elusive and eventually escape current detection techniques. As an example, we may think of double-slit experiments (with Δ the slit distance) for which the increasing difficulty in question concerns the resolution of angular apertures of diffraction structures of order λ/Δ.

It is nowadays widely accepted that an even more important reason for the notorious absence of quantum superpositions from the macroscopic world lies in environment imposed decoherence [1,2], and that phenomenon is our concern here. Decoherence is, for microscopic bodies, just a facet of dissipation caused by interactions with many-freedom surroundings. Spontaneous emission of light by an atom is such a dissipative process, with the electromagnetic field acting as a weakly coupled environment. Inasmuch as radiatively coupled states of an atom are only "microscopically" distinct, it would appear overly pedantic and even somewhat misleading to attach the fashionable label decoherence to polarization decay while reserving dissipation to the exponential decrease of the population of the excited state. But "larger" systems under the influence of environments do invite such different names for the following reason. If two sufficiently distinct states $|s\rangle, |s'\rangle$ are brought to an initial superposition $|\rangle = c|s\rangle + c'|s'\rangle$, the density operator $\rho(t)$ starts out as the projector $\rho(0) = |\rangle\langle|$ and then, for suitable coupling to the environment (see below), decoheres to the mixture $|c|^2|s\rangle\langle s| + |c'|^2|s'\rangle\langle s'|$, with the weights $|c|^2, |c'|^2$ still as in the initial superposition, on a time scale τ_{dec} while the subsequent relaxation of that mixture towards an eventually stationary $\rho(\infty)$ has a much longer characteristic time τ_{diss}. The time scale ratio $\tau_{\mathrm{dec}}/\tau_{\mathrm{diss}}$ becomes the smaller the more distinct the two component states are. If that distinction can be measured by a length $\Delta \propto |s - s'|$ the time scale ratio is again determined by the ratio of the de Broglie wavelength to Δ, typically as

$$\tau_{\mathrm{dec}}/\tau_{\mathrm{diss}} \sim (\lambda/\Delta)^2. \qquad (1)$$

The acceleration of decoherence against relaxation of probabilities by the (squared) ratio Δ/λ suggests that decoherence entails classicality of the macroworld. Indeed, for macroscopic bodies and macroscopic values of Δ the acceleration factor is so huge that decoherence appears as instantaneous while dissipation for classically meaningful quantities may not at all be noticeable.

The acceleration factor in question has been studied in recent experiments [3–5]. Arndt *et al.* in Vienna [3] observed multiple-slit diffraction of the largest objects thus far, C_{60} molecules. No decohering influence of the environment was effective, simply because τ_{diss} and the relevant thermal de Broglie length were sufficiently large. Experiments at the ENS in Paris [4] involved superpositions of coherent states of a microwave cavity mode. Even though the cavity was of high quality ($\tau_{\text{diss}} = 160\ \mu\text{s}$) the acceleration factor was controlled between, roughly, 1 and 10. Finally, a NIST group [5] worked with superpositions of translational-motion states of single ${}^9\text{Be}^+$ atoms in Paul traps. Here the acceleration factor was steered through the range 1–25. On the theoretical side the experiments mentioned are well understood. In all cases the decoherence time is not smaller than the time scale of dissipation by many orders of magnitude; it is, in fact, still larger than the characteristic times of the free motion of the system in the absence of any environment. In that limit, both dissipation and its companion decoherence can be treated by Fermi's golden rule or fancied-up variants thereof like master equations. The golden rule involves a certain long-time limit: It cannot yield time independent transition rates before the time elapsed since the preparation of the initial state exceeds the basic periods τ_{sys} of the isolated system. Indeed, inasmuch as it requires energy conservation for the exchange of free-bath and free-system quanta, it presupposes such large times for resonance to become effective.

The golden rule cannot be trusted when it predicts a decoherence time τ_{dec} smaller than τ_{sys}. It thus does not explain why the macroworld behaves classically. When holding decoherence responsible here we mostly rely on an exactly solvable model, a harmonic oscillator harmonically coupled to a bath itself consisting of harmonic oscillators [6]. The acceleration factor Δ/λ is there found in effect already for arbitrarily small times. Invaluable as the oscillator model is for rigorously revealing decoherence, it cannot prove that phenomenon to be a universal one.

The clue to progress lies in the fact that the Hamiltonian $H = H_{\text{sys}} + H_{\text{bath}} + H_{\text{int}}$ of the embedding of a system in an environment (alias bath) can be simplified for times much smaller than the characteristic times τ_{sys} of H_{sys}: We can altogether neglect any motion the isolated system would perform, i.e., discard H_{sys}. For the structure of the interaction Hamiltonian we do not have much of a choice. Introducing a coupling agent each for the system, S, and the bath, B, we may write

$$H_{\text{int}} = SB\,. \tag{2}$$

It follows that the coupling agent S becomes conserved and plays only the role of a fixed parameter. Introducing eigenvectors and eigenvalues of S as $S|s\rangle = s|s\rangle$ we consider the matrix element $\langle s|W|s'\rangle$ of the joint density operator of our compound which still is a density operator for the bath. We shall eventually be interested only in the reduced density matrix $\langle s|\rho|s'\rangle = \text{Tr}_{\text{bath}}\langle s|W|s'\rangle$ and intend to rigorously reveal decoherence as a universal short-time phenomenon by showing

$$\langle s|\rho(t)|s'\rangle = e^{-(s-s')^2 f(t) + i(s^2-s'^2)\,\varphi(t)}\,\langle s|\rho(0)|s'\rangle\,, \tag{3}$$

with functions $f(t) \geq 0$ and $\varphi(t)$ to be determined.

For the next step we momentarily model the bath as a collection of oscillators and specify

$$\begin{aligned} H_{\text{bath}} &= \sum_{i=1}^{N}\left(\frac{1}{2m}\,\hat{p}_i^2 + \frac{1}{2}\,m\omega_i^2\hat{q}_i^2\right), \\ H_{\text{int}} &= S\sum_{i=1}^{N} g_i\hat{q}_i\,. \end{aligned} \tag{4}$$

To write the Liouville–von Neumann equation for the joint density operator W it is convenient to stick to the S representation and employ the Wigner function with respect to the bath oscillators. Denoting by $W(s,s',p,q,t)$ that hybrid representative we get the reduced density matrix by integrating over the $(2N)$-dimensional phase space of the bath, $\langle s|\rho(t)|s'\rangle = \int d^Np\,d^Nq\,W(s,s',p,q,t)$. The evolution equation $\dot{W} = LW$ has the generator

$$\begin{aligned} L = \sum_i\bigg\{&\frac{\partial}{\partial p_i}\,m\omega_i^2 q_i - \frac{\partial}{\partial q_i}\,\frac{p_i}{m} \\ &- g_i\left[\frac{i}{\hbar}\,(s - s')q_i - \frac{s + s'}{2}\,\frac{\partial}{\partial p_i}\right]\bigg\}\,. \end{aligned}$$

The purely parametric role of the eigenvalues s, s' of the system coupling agent is manifest in the generator L. We could proceed to solving the foregoing first-order differential equation for $W(s,s',p,q,t)$. It is more convenient to directly go for the time evolution of the reduced density matrix. To that end we may assume initial statistical independence of system and bath, $W(s,s',p,q,0) = \langle s|\rho(0)|s'\rangle \times W_{\text{bath}}(p,q,0)$. Without loss of generality we momentarily assume the initial bath distribution sharp, $W_{\text{bath}}(p,q,0) = \prod_i \delta(p_i - p_{i0})\delta(q_i - q_{i0})$, since we may later average with whatever weight we please. A reduced time evolution operator can be introduced as $U(t) = \int d^Np\,d^Nq\,e^{Lt}W_{\text{bath}}(p,q,0)$. We readily check

$$\begin{aligned} \dot{U} = &-\frac{i}{\hbar}(s - s') \\ &\times \sum_i g_i \int d^Np\,d^Nq\ q_i e^{Lt}\,W_{\text{bath}}(p,q,0) \end{aligned} \tag{5}$$

and then, using the commutator $[\frac{\partial}{\partial q_i}, q_j] = \delta_{ij}$, shift the factor q_i in the integrand to the right of the exponential e^{Lt}, to get a reduced generator $l(t) = \dot{U}(t)U(t)^{-1}$ as

$$\begin{aligned} l(t) = \frac{i}{\hbar}\sum_i\bigg[&(s^2 - s'^2)\,\frac{g_i^2}{2m\omega_i^2}\,(1 - \cos\omega_i t) \\ &- (s - s')g_i\left(q_{i0}\cos\omega_i t + \frac{p_{i0}}{m\omega_i}\sin\omega_i t\right)\bigg]. \end{aligned} \tag{6}$$

Because of the parametric role of the eigenvalues s, s' we here do not confront a differential operator and get the density matrix as $\langle s|\rho(t)|s'\rangle = \exp\{\int_0^t dt'\, l(t')\}\langle s|\rho(0)|s'\rangle$; this contains the initial coordinates q_{i0} and momenta p_{i0} in the exponent. Now we invoke a thermal bath and average as $\overline{e^{iaq_i}} = e^{-a^2\hbar/4m\omega_i \tanh(\beta\hbar\omega_i/2)}$ and $\overline{e^{ibp_i}} = e^{-b^2 m\hbar\omega_i/4\tanh(\beta\hbar\omega_i/2)}$. The result (3) is so reached with

$$\begin{aligned} f(t) &= \sum_i \frac{g_i^2(1 + 2\overline{n}_i)}{2m\hbar\omega_i^3}(1 - \cos\omega_i t) \\ &= \mathrm{Re}\frac{1}{\hbar^2}\int_0^t ds\, sC(t - s), \\ \varphi(t) &= \sum_i \frac{g_i^2}{2m\hbar\omega_i^2}\left(t - \frac{\sin\omega_i t}{\omega_i}\right) \\ &= \mathrm{Im}\frac{1}{\hbar^2}\int_0^t ds\, sC(t - s), \end{aligned} \tag{7}$$

where $\overline{n}_i = (e^{\beta\hbar\omega_i} - 1)^{-1}$ is the thermal number of quanta and $C(t) = \langle B(t)B(0)\rangle$ the thermal autocorrelation function of the bath coupling agent. The function $f(t)$, which determines the decoherence as thermally enhanced by the factor $1 + 2\overline{n}_i$, begins quadratically in t; for larger times it approaches $f(t) \to \gamma t$ with $\gamma = \mathrm{Re}\hbar^{-2}\int_0^\infty dt\, C(t)$, provided $C(t)$ falls off faster than t^{-2}. The proportionality of $f(t)$ to $\hbar^{-1}$ signals a quantum scale λ^2 of reference for $(s - s')^2$. The phase $\varphi(t)$ begins $\propto t^3$ and is temperature independent. We should appreciate the dramatic difference of the decoherence function $f(t)$ from its golden rule counterpart. If the system were itself a harmonic oscillator with frequency Ω, mass M, and displacement S the golden rule would yield $f_{\mathrm{GR}}(t) = \gamma^{\mathrm{GR}}t$ with $\gamma^{\mathrm{GR}} = [1 + 2\overline{n}(\Omega)] \times (2\hbar)^{-2}\mathrm{Re}\int_0^\infty dt\, e^{i\Omega t}\langle[B(t), B(0)]\rangle$. Most importantly, our decoherence function in (3) describes accelerated decoherence for whatever system with whatever coupling agent S, provided only decoherence is fast in the sense $\tau_{\mathrm{dec}} \ll \tau_{\mathrm{sys}}$ which will always be the case for sufficiently distinct s and s'. In that latter case, $\langle s|\rho(t)|s'\rangle$ will decay to negligible magnitude before deviations from the Gaussian falloff due to $f(t) \propto t^2$ occur.

We have not gone more than halfway towards our goal yet. The decoherence shown by the foregoing reasoning is a privilege of superpositions of eigenstates of the system coupling agent; superpositions of eigenstates of other system observables not commuting with S need not lose their relative quantum phases any faster than probabilities change. But, on the other hand, no privileged representations are known in the macroworld.

For fast decoherence of macroscopic superpositions to take place without distinction of special states, a variety of environmental influences would have to be at work. Rather than privileging a single observable S as the one and only coupling agent to contact but a single bath it seems necessary to account for several noncommuting observables as coupling agents toward several reservoirs.

To model the situation just sketched we accompany the single agent S by a canonically conjugate partner R with $[R, S] = \hbar/i$. More general models would involve a larger set of noncommuting system observables but would not lead to conclusions qualitatively different from the ones to be discussed here. Still interested in times smaller than τ_{sys}, we need not worry about a system Hamiltonian H_{sys} and generalize the Hamiltonian (4) to

$$H_{\mathrm{bath}} = H_1 + H_2, \qquad H_{\mathrm{int}} = SB_1 + RB_2, \tag{8}$$

where the indices 1,2 label independent baths. Were we to assume both baths as composed of harmonic oscillators, the neglect of H_{sys} would still guarantee explicit tractability with the strategy as above. However, the resulting expressions are so unwieldy to not warrant full display. Moreover, we want to ascertain universality of decoherence by showing that the following generalization of (3) holds for short times, i.e., up to $\mathcal{O}(t^3)$ corrections:

$$\begin{aligned} \langle s|\,\rho(t)|s'\rangle &= e^{-(s-s')^2 f_1(t)}e^{\hbar^2(\partial/\partial s+\partial/\partial s')^2 f_2(t)}\langle s|\,\rho(0)|s'\rangle, \\ f_i(t) &= \langle B_i^2\rangle t^2/2\hbar^2 + \mathcal{O}(t^3), \end{aligned} \tag{9}$$

for *all* baths providing zero means and Gaussian statistics to the coupling agents B_i in their initial states. Such behavior is typical for coupling agents to many-freedom baths which are sums of very many effectively independent contributions; the central limit theorem predicts Gaussian statistics for such sums. The harmonic-oscillator bath in thermal equilibrium has the virtue of imparting Gaussian behavior even to each summand $g_i\hat{q}_i$ in the coupling agent B given in (4). The coupling agents of more general baths can achieve Gaussian behavior only by obeying the central limit theorem.

To make peace with the absence of a phase factor from (9) it is good to recall from (7) that $\varphi(t) \propto t^3$ for small times. We see that the coupling SB_1 causes decoherence as in (3). On the other hand, RB_2 generates diffusive behavior for the "center of mass" variable $s + s'$, and that diffusion is not in any way accelerated: Note that Planck's constant cancels in the second exponential and that no factor involving $s - s'$ appears there either. However, upon Fourier transforming to the R representation we see that the second exponential in (9) entails accelerated decay for coherences $\langle r|\rho(t)|r'\rangle$ with respect to eigenstates $|r\rangle$ of R, just as the first factor does for the eigenstates of S. It is good to realize that both S and R have continuous spectra. It follows that $\rho(t)$ cannot be diagonal in either the S or the R representation. Our point is not about strict diagonalization but about the rapid disappearance of *both* $\langle r|\rho(t)|r'\rangle$ *and* $\langle s|\,\rho(t)|s'\rangle$ when $|r - r'|$ and $|s - s'|$ are macroscopic, respectively. In particular, we might initially have $\rho(0) = |\,\rangle\langle\,|$ with the state $|\,\rangle$ a superposition of two wave packets whose widths are small compared to their macroscopic separation, with respect to either S or R or both. Our assertion (9) implies rapid relative decoherence of the two wave packets while leaving each of them practically unchanged.

The proof of our assertion (9) is surprisingly simple. In $\langle s|\,\rho(t)|s'\rangle = \mathrm{Tr}_{\mathrm{bath}}\langle s|e^{-iHt/\hbar}W(0)e^{iHt/\hbar}|s'\rangle$ we factor the evolution operator $e^{-iHt/\hbar}$ into four, each of which involves one of the four pieces of the Hamiltonian (8); that factorization holds up to $\mathcal{O}(t^2)$ corrections. Because of the cyclic invariance of the trace the evolution operators of the baths coalesce as $e^{-iH_it/\hbar}e^{iH_it/\hbar} = 1$, and this is how independence of the bath model arises. Now, the coupling agent S can be replaced with its eigenvalues as $\mathrm{Tr}_{\mathrm{bath}}e^{-i(s-s')B_1t/\hbar}\langle s|e^{-iRB_2t/\hbar}W(0)e^{iRB_2t/\hbar}|s'\rangle$. By invoking $R|s\rangle = i\hbar\frac{\partial}{\partial s}|s\rangle$ and assuming the two baths and the system independent initially we get

$$\langle s|\rho(t)|s'\rangle = \langle e^{-i(s-s')B_1t/\hbar}\rangle\langle e^{-(\partial/\partial s+\partial/\partial s')B_2t}\rangle\langle s|\rho(0)|s'\rangle .$$

The central limit theorem then yields the decoherence law (9) as discussed above, provided the $\mathcal{O}(t^2)$ correction arising from the factorization of $e^{-iHt/\hbar}$ turns into the asserted $\mathcal{O}(t^3)$ correction through the average over the initial state. To check on that latter detail [7] we may look at the first factor in the matrix element displayed above which reads, with the $\mathcal{O}(t^2)$ correction from the factorization accounted for, $\langle e^{-i(s-s')(B_1t+\dot{B}_1t^2/2)/\hbar}\rangle$ with $\dot{B}_1 = \frac{i}{\hbar}[H_1, B_1]$. But if B_1 has the structure required for the law of large numbers to apply, so will $\dot{B}_1$, and then indeed $\langle(B_1t + \dot{B}_1t^2/2)^2\rangle = \langle B_1^2\rangle t^2 + \mathcal{O}(t^3)$.

As expected, by introducing different reservoirs coupling to noncommuting observables we break the privilege of a single representation. Noncommuting agents contacting different reservoirs occur, e.g., for a body probing an electric field through its charge or electric dipole moment and a magnetic field with its magnetic moment. That simple example indicates that for a macroscopic body the exclusive action of a single reservoir may be as unrealistic a fiction as complete isolation.

Our discussion is of relevance for the quantum measurement problem. It had long been considered a puzzle how a microscopic object prepared in a superposition of, say, two eigenstates of an observable to be measured can, through unitary evolution of its composition with a macroscopic pointer, cause that pointer to reach one of two distinct positions in each run of the measurement, with those positions uniquely related to the two eigenvalues and repeated runs building up probabilities equal to the weights in the original superposition. Enigmatic was not the entanglement of micro-object and pointer into a superposition associating each eigenstate of the measured micro-observable with a unique pointer state (a superposition of the type often called a Schrödinger cat state); such entanglement is accessible through unitary evolution, as was already explained by von Neumann [8]. The puzzle rather was the "collapse" of that superposition to the mixture with unchanged probabilities. The current understanding is [1,2] that the environment decoheres the superposition. Zurek has pointed out that decoherence of different pointer displacements is most easily understood if the pointer displacement is taken as the pointer's coupling agent towards a reservoir. The price to be paid is the distinction of a "pointer basis," for times up to τ_{sys}. We now see that no such price is due when the pointer displacement is not the only coupling agent but just one of several, each towards a different reservoir. We should emphasize that we here discuss decoherence for a macroscopic pointer rather than a microscopic object subject to measurement; such a pointer does behave effectively classically with respect to *both* displacement *and* momentum.

Can decoherence be reversed? Like any other dissipative phenomenon in (subdynamics of) unitary evolutions, decoherence is not strictly irreversible. Seemingly spontaneous revivals of coherences out of an apparent mixture or time reversed decoherence could arise for a macroscopic system, in the unitary motion of its composition with an environment, given suitable initial states. As regards ordinary damping, such reversals were demonstrated in the historic spin echo experiments [9]; but decoherence *is* just ordinary damping, starting from an extraordinary initial state.

While decoherence makes quantum superpositions alien to the macroworld, it does not forbid quantum behavior to ever reach out there. For instance, once the initial state of the object-pointer compound of a measurement has decohered, the pointer has vanishing probability to jump between the various positions it could have gone to by the previous interaction with the micro-object. The pointer will rather move classically, up to tiny fluctuations. What is left from the entangled state of micro-object and pointer are finite probabilities for distinct pointer readings, and these are due to the very nonclassical nature of the initial state.

We owe a final remark to present efforts towards realizing quantum computing. A quantum computer would incorporate lots of quantum rather than classical two-state elements and would therefore be a mesoscopic or macroscopic complex. The whole complex would seem prone to accelerated decoherence. No computation relying on coherences seems possible during time spans exceeding τ_{dec}. Hope must therefore be set on error correction codes or elements sufficiently protected against detrimental environments. Symmetry may be of help [10,11].

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[1] W. H. Zurek, Phys. Today **44**, No. 10, 36–44 (1991); W. H. Zurek, S. Habib, and J. P. Paz, Phys. Rev. Lett. **70**, 1187 (1993); J. P. Paz and W. H. Zurek, Phys. Rev. Lett. **82**, 5181 (1999).

[2] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I. O. Stamatescu, and H. D. Zeh, *Decoherence and the Appearance of the Classical World* (Springer, Berlin, 1996).

[3] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. van der Zouw, and A. Zeilinger, Nature (London) **401**, 680 (1999).

[4] M. Brune *et al.,* Phys. Rev. Lett. **77**, 4887 (1996); J. M. Raimond, M. Brune, and S. Haroche, *ibid.* **79**, 1964 (1997).

[5] C. J. Myatt, B. E. King, Q. A. Turchette, C. A. Sackett, D. Kielpinski, W. M. Itano, C. Monroe, and D. J. Wineland, Nature (London) **403**, 269 (2000); C. Monroe, D. M. Meekhof, B. E. King, and D. J. Wineland, Science **272**, 1131 (1996).

[6] R. P. Feynman and F. L. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963); F. Schwabl and W. Thirring, Ergeb. Exakten. Naturwiss. **36**, 219 (1964); P. Ullersma, Physica **32**, 27 (1966); A. O. Caldeira and A. J. Leggett, Phys. Rev. A **31**, 1059 (1985); F. Haake and R. Reibold, *ibid.* **32**, 2462 (1985); F. Haake and M. Żukowski, *ibid.* **47**, 2506 (1993).

[7] If willing to settle for thermal baths, we may abbreviate the argument by modifying the factorization as $e^{-iHt/\hbar} = e^{-iH_{\mathrm{bath}}t/2\hbar}e^{-iH_{\mathrm{int}}t/\hbar}e^{-iH_{\mathrm{bath}}t/2\hbar}[1 + \mathcal{O}(t^3)]$.

[8] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer, Berlin, 1932).

[9] E. L. Hahn, Phys. Rev. **80**, 580 (1950); J. Rothstein, Am. J. Phys. **25**, 510 (1957).

[10] D. Braun, P. A. Braun, and F. Haake, Opt. Commun. **179**, 195 (2000); in *Decoherence: Theoretical, Experimental, and Conceptual Problems,* edited by P. Blanchard, D. Giulini, E. Joos, C. Kiefer, and I. O. Stamatescu (Springer, Berlin, 2000).

[11] A. Beige, D. Braun, B. Tregenna, and P. L. Knight, Phys. Rev. Lett. **85**, 1762 (2000).