Magnetic Dissipation and Fluctuations in Individual Nanomagnets Measured by Ultrasensitive Cantilever Magnetometry

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Cantilever magnetometry with moment resolution better than $10^4\mu_B$ was used to study individual nanomagnets. By using the fluctuation-dissipation theorem to interpret measurements of field-induced cantilever damping, the low frequency spectral density of magnetic fluctuations could be determined with resolution better than $1\mu_B$ $\mathrm{Hz}^{-1/2}$. Cobalt nanowires exhibited significant magnetic dissipation and the associated magnetic fluctuations were found to have $1/f$ frequency dependence. In individual submicron rare-earth alloy magnets, the dissipation/fluctuation was very small and not distinguishable from that of a bare silicon cantilever.

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Dynamical effects in nanometer-size ferromagnets, including dissipation and thermal-magnetic fluctuations, are of fundamental interest and can have important consequences for magnetic recording and some proposed spin-based devices [1–3]. SQUID magnetometry has been successful in measuring magnetic fluctuations in ensembles of nanomagnets [4], thermal-magnetic noise in macroscopic ferromagnets [5,6], and nuclear spin noise in bulk samples of $NaClO_3$ [7]. Ideally, one would like to be able to investigate individual nanomagnets so that their properties can be studied without averaging over an inhomogeneous ensemble. Although a number of techniques are capable of measuring magnetic reversals in single submicron magnetic particles [8–11], sensitivity limitations have previously precluded observations of small-angle fluctuations in individual particles.

Cantilever magnetometry has previously been shown to be a sensitive technique for the study of single particles [12] and thin magnetic layers [13], especially at high magnetic fields and over a broad temperature range. In this paper we use cantilever magnetometry with resolution better than $10^4\mu_B$ to study the moment, anisotropy, and switching behavior of single particles and nanowires. More importantly, we demonstrate a method to measure small-angle magnetic fluctuations and spin noise with a resolution better than $1\mu_B$ $\mathrm{Hz}^{-1/2}$ based on measurements of cantilever damping. Our technique makes use of the fluctuation-dissipation theorem to determine the spectral density of transverse magnetic fluctuations at the cantilever resonance frequency. We find striking differences in the magnitude of fluctuations in cobalt nanowires relative to individual rare-earth alloy particles, in agreement with recent computer simulations [14]. Since thermal-magnetic noise can adversely affect the relaxation times and quantum coherence of nearby spins, these results may have implications for single-spin magnetic resonance force microscopy [1] and some spintronic [2] and quantum computing devices [3].

Three specific nanomagnets (tips) will be considered here (see Table I and Fig. 1). Two of the tips were ~20-μm-long cobalt nanowires fabricated by evaporating cobalt onto the sidewall of a custom-fabricated silicon cantilever to create an in-plane tip while the rest of the cantilever was shadow masked with a knife edge [15]. The narrowest tip (nanowire 2) had a cross section of 35 nm × 35 nm. In addition, a high anisotropy $Pr_2Fe_{14}B$ tip was fabricated by gluing a micron-size particle near the end of the cantilever in the presence of an orienting magnetic field. A focused ion beam (FIB) was then used to precisely mill the particle to submicron dimensions (Fig. 1b). The particle's easy axis was verified to be aligned within 10° of the cantilever axis by measuring cantilever displacement in an applied field to determine the field induced torque. Tips were capped with 3 nm of Pt or Au to inhibit oxidation. The cantilevers used in this study have small spring constants (33 to 240 μN/m) with low intrinsic dissipation and are therefore ideal for detecting small (attonewton) forces [15,16].

TABLE I. Cantilever and magnet parameters at 4.2 K.

Cantilevers	Nanowire 1	Nanowire 2	PrFeB	Bare
L (μm)	255	114	210	135
Width (μm)	3	7	3	7
Thickness (nm)	250	60	250	60
$\omega_0/2\pi$ (Hz)	3678	5415	5293	4361
k_0 (dyn/cm)	0.134	0.056	0.240	0.033
Q_0	95 000	29 000	75 000	31 000
F_{min} (10^{-13} dyn)	3.8	3.7	4.8	3.0
Magnets				
Length (nm)	20 000	20 000	600	
Width (nm)	250	35	300	
Thickness (nm)	35	35	100	
m (10^{-12} emu)	290	3	27	
H_k (kOe)	10.5	11.6	162	
ϵ	0.12	0.13	···	

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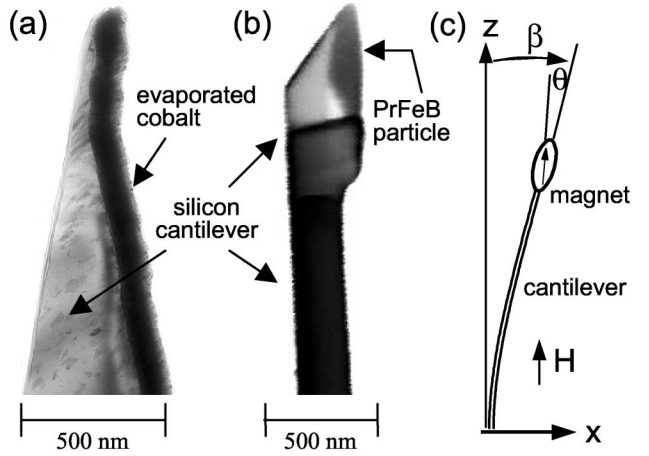

FIG. 1. (a) Transmission electron micrograph (top view) of a cobalt nanowire tip formed on the sidewall of a single crystal silicon cantilever. (b) Scanning electron micrograph (side view) of the $Pr_2Fe_{14}B$ particle tip glued to the end of a cantilever and shaped by focused ion beam milling to an approximate size of 100 nm × 300 nm × 600 nm. (c) Model for a single domain magnetic tip on a vibrating cantilever.

Dynamic-mode cantilever magnetometry was performed in vacuum at low temperatures within a He^4 cryostat equipped with a 6 T superconducting magnet. The basic experiment was to measure cantilever resonance frequency and damping as a function of the applied field, H, which was directed parallel to the cantilever axis (z direction) as shown in Fig 1c. Cantilever motion was detected with a fiber optic interferometer [17] operating at a 1310 nm wavelength with less than 500 nW of optical power incident on the cantilever. The cantilever was oscillated at its natural resonance frequency using a piezoelectric disk and a gain-controlled positive feedback loop [18]. Cantilever frequency was measured with a frequency counter, while damping was typically measured by the cantilever ring-down time after abruptly turning off the piezoelectric drive signal. The noise level of all measurements was limited only by the cantilever's intrinsic thermal vibrations [15,18].

To interpret the magnetometry data, we model the magnetic tip as a single domain ferromagnet of volume V with saturation magnetization M_s and uniaxial anisotropy K_u [19]. As the cantilever vibrates with displacement x, the tip tilts by an angle β and the tip moment cants away from the particle's easy axis by an angle θ due to the z-directed applied field (Fig. 1c). For small vibration amplitudes, $\beta = x/L_e$, where L_e is an effective cantilever length that differs from the actual length L and depends on the vibrational mode shape. For the first two flexural modes of a rectangular cantilever, L/L_e equals 1.38 and 4.79, respectively [1,20].

The canting of the magnetization can be determined by considering the magnetic energy of the particle, which can be written as the sum of anisotropy and Zeeman energy terms: $E_m = K_u V \sin^2\theta - HM_s V \cos(\beta - \theta)$ (in cgs units). Minimizing E_m with respect to θ in the small-angle approximation yields $\theta = \beta[H/(H + H_k)]$, or

$$\theta = \frac{H}{(H + H_k)} \frac{x}{L_e}, \tag{1}$$

where we have defined $H_k = 2K_u/M_s$. The x component of magnetic moment generates a restoring torque given by $\tau = M_s VH(\beta - \theta) = M_s VH\beta[H_k/(H + H_k)]$ which acts to effectively stiffen the cantilever spring constant by $\Delta k = \tau/\beta L_e^2 = (M_s V/L_e^2)[HH_k/(H + H_k)]$. For small $\Delta k/k_0$, the resulting frequency shift is $\Delta\omega/\omega_0 = (1/2)(\Delta k/k_0)$, or

$$\frac{\Delta\omega}{\omega_0} = \frac{mHH_k}{2k_0 L_e^2(H + H_k)}, \tag{2}$$

where $m = M_s V$ is the tip moment, k_0 is the cantilever spring constant, and ω_0 is the frequency in zero field.

Figure 2a shows a hysteresis loop of cantilever frequency vs field for a typical cobalt nanowire (nanowire 1). The portion of the hysteresis loop from 60 kOe down to zero was reversible and fits Eq. (2) with high accuracy. Using the known values of ω_0, k_0, and L_e listed in Table I, a least-squares fit for m and H_k gives $m = 2.9 \times 10^{-10}$ emu and $H_k = 10.5$ kOe. Despite the fact that the wire is polycrystalline, large H_k is obtained because of the strong shape anisotropy. The sudden small

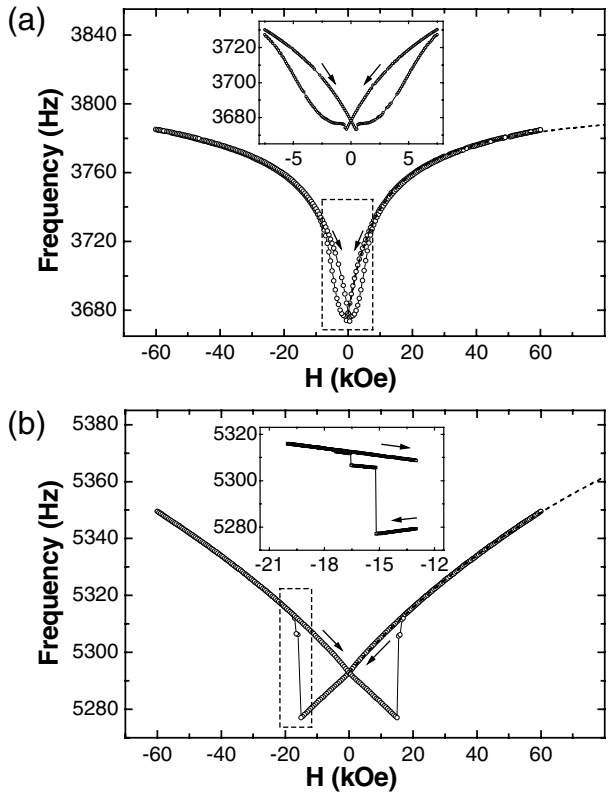

FIG. 2. Hysteresis loops of cantilever frequency vs applied field at 4.2 K for (a) nanowire 1 and (b) the $Pr_2Fe_{14}B$ particle tip. Fits to the data based on Eq. (2) are shown for both tips in the range 0 to 60 kOe (dashed lines). The insets in (a) and (b) zoom in on the switching behavior for each tip.

switching at ± 500 Oe (Fig. 2a inset) may be due to a small amount of cobalt inadvertently deposited onto the face of the cantilever, while the slow switching at larger reverse field is probably due to the gradual propagation of domain walls within the wire. Measurements were also taken for the narrower cobalt nanowire (nanowire 2 with 35 nm $\times$ 35 nm cross section) for both the 5.4 kHz first flexural mode and the 36.1 kHz second mode. Again, excellent fits with Eq. (2) were obtained. From the fit to the first mode data, we determined $m = 3.75 \times 10^{-11}$ emu and $H_k = 11.6$ kOe, while for the second mode data we obtained $m = 3.84 \times 10^{-11}$ emu and $H_k = 13.5$ kOe. The fact that the two different vibration modes gave consistent results for both m and H_k gives us further confidence in the validity of the model.

The $Pr_2Fe_{14}B$ particle exhibited much larger anisotropy and higher switching field, as can be seen from the data in Fig. 2b and Table I. Since the bulk starting material for the $Pr_2Fe_{14}B$ tip had a known grain size of 10 μm, it is likely that the final submicron-size particle was composed of a single crystal. The value we determine for H_k (162 kOe) is more than 10 times larger than for the cobalt nanowires but is smaller than the bulk value of 300 kOe for $Pr_2Fe_{14}B$ single crystals at low temperature [21]. The discrepancy is possibly because of imperfections either in the starting material or on the surface of the shaped particle (FIB damage, oxidation, etc.). This particular particle had a switching field of approximately 15 kOe and was found to always switch in three discrete jumps (see Fig. 2b inset).

Energy dissipation in dynamic-mode cantilever magnetometry has received relatively little attention but is important because it sets the limit for the minimum detectable magnetic moment and provides insight into the spectral density of magnetic fluctuations via the fluctuation-dissipation theorem. The dissipation can be modeled as a velocity dependent drag force given by $F_d = -\Gamma v$, where v is the tip velocity and Γ is a friction coefficient. For a cantilever oscillating with peak displacement $x_{\rm pk}$, the drag force results in an energy loss per cycle of

$$\Delta E = \pi \omega x_{\rm pk}^2 \Gamma . \tag{3}$$

In general, we can write $\Gamma = \Gamma_0 + \Gamma_m(H)$, where Γ_0 is the intrinsic (zero field) cantilever friction and Γ_m is the magnetic friction induced by the applied field. Γ is determined experimentally by measuring the cantilever quality factor Q and using the relation $\Gamma = (k_0/\omega_0^2)\omega/Q$, where k_0/ω_0^2 is the cantilever effective mass. This effective mass is independent of field and flexural mode [1].

For linear dissipative systems, of which the cantilever is a good example, the fluctuation-dissipation theorem relates the spectral density of thermal excitations to the friction coefficient [22]. In the case of a cantilever operating in the high temperature limit (i.e., $k_BT \gg \hbar\omega$), the spectral density of the force fluctuations S_F in thermal equilibrium is given by $S_F = 4\Gamma k_BT$, where we use the convention that S_F is a single-sided spectrum [15]. This relationship, which is the thermomechanical analog of Johnson noise, reflects the fact that as the friction coefficient increases, the thermal excitations driving the cantilever must necessarily increase if the thermal energy of the cantilever oscillation is to remain equal to the value required by the equipartition theorem: $\frac{1}{2}k\langle x^2\rangle = \frac{1}{2}k_BT$.

The thermal force fluctuations determine the noise floor for various cantilever measurements. For example, the smallest detectable force signal (assuming unity signal-to-noise ratio) is given by $F_{\rm min} = S_F^{1/2}B^{1/2} = \sqrt{4\Gamma k_BTB}$, where B is the detection bandwidth. The minimum detectable cantilever frequency shift is $\Delta\omega_{\rm min} = \omega_0 F_{\rm min}/\sqrt{2}\,k_0 x_{\rm pk}$ [18] which, in turn, limits the detectable magnetic moment to $m_{\rm min} = \sqrt{2}\,F_{\rm min}L_e^2/x_{\rm pk}H$, where we have utilized Eq. (2) in the limit of $H_k \gg H$. Using the cantilever parameters for nanowire 2 (Table I), the corresponding $m_{\rm min}$ is $6.5 \times 10^3 \mu_B$ in a 1 Hz bandwidth for $H = 60$ kOe and $x_{\rm pk} = 100$ nm [23].

When the field H is applied, cantilever Q is observed to drop (i.e., Γ_m increases) due to magnetic "friction" associated with the oscillatory canting of the moment within the magnetic particle. Figure 3a shows Γ_m for nanowire 1 and the $Pr_2Fe_{14}B$ particle. The cobalt nanowire exhibited large magnetic friction, while the friction for the rare earth magnet was so small that it was not distinguishable from the magnetic friction found in bare silicon cantilevers. In all cases, it was verified that the magnetic friction was

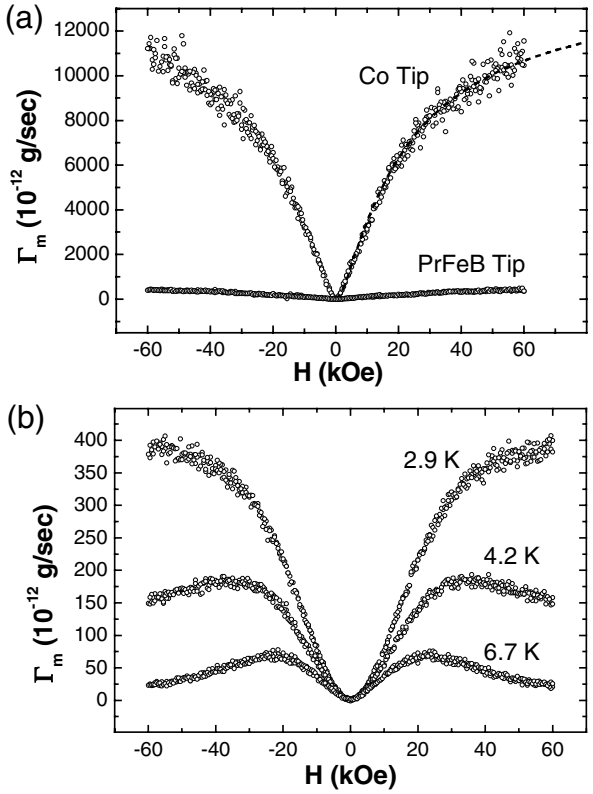

FIG. 3. (a) Cantilever damping vs applied field at 4.2 K for nanowire 1 and the $Pr_2Fe_{14}B$ particle. A fit to Eq. (4) is shown for the cobalt nanowire (dashed line). (b) Cantilever damping vs field for a bare silicon cantilever.

well behaved in the sense that Q and Γ_m were independent of the cantilever oscillation amplitude. It was also verified that the cantilever remains in thermal equilibrium. In other words, the thermal vibration amplitude is independent of magnetic field and maintains the value expected from the equipartition theorem.

We have found that the field dependence of Γ_m is well explained by assuming that the energy lost per cycle is proportional to $\theta_{\rm pk}^2$, where $\theta_{\rm pk}$ is the maximum angle that the magnetization cants as the cantilever oscillates. It is convenient to write the proportionality in terms of the peak anisotropy energy according to $\Delta E_m = \epsilon K_u V \theta_{\rm pk}^2$, where ϵ is a dimensionless quantity that represents the fraction of the peak anisotropy energy that is lost per cycle. Using (1), we obtain $\Delta E_m = (\epsilon K_u V / L_e^2)[H/(H + H_k)]^2 x_{\rm pk}^2$. Combining this result with (3), we find

$$\Gamma_m = \epsilon(\omega)\left(\frac{mH_k}{\pi\omega L_e^2}\right)\left(\frac{H}{H + H_k}\right)^2, \qquad (4)$$

where $\epsilon(\omega)$ indicates a possible frequency dependence.

The dashed line in Fig. 3a shows a fit of (4) to the dissipation data for nanowire 1. The single fitted parameter was for the value of ϵ since all other parameters were determined previously from the frequency vs field results. In this case, we find $\epsilon(3.7\ \text{kHz}) = 0.12$. For nanowire 2, we obtain $\epsilon(5.4\ \text{kHz}) = 0.13$ and $\epsilon(36.1\ \text{kHz}) = 0.15$ for the first and second flexural modes, respectively. Thus, ϵ is similar in magnitude for both nanowires and appears to be nearly frequency independent.

The magnetic dissipation is necessarily accompanied by cantilever thermal excitation with spectral density $S_{F_m} = 4\Gamma_m k_B T$, in accordance with the fluctuation-dissipation theorem. The excitation is not a true force, however, since a uniform field acting on a magnetic moment produces no force. Rather, the excitation is from the torque that the external field exerts on the transverse magnetic fluctuations within the ferromagnet. The force spectral density can be converted into an equivalent transverse moment spectral density according to $S_{m_x} = (L_e/H)^2 S_{F_m} = (L_e/H)^2(4\Gamma_m k_B T)$. Using (4) in the limit of small H, we find

$$S_{m_x} = \frac{4\epsilon(\omega) m k_B T}{\pi\omega H_k}. \qquad (5)$$

Applying (5) for the case of nanowire 2, we find that $S_{m_x}^{1/2}$ is $330\mu_B\ \text{Hz}^{-1/2}$ at 5.4 kHz and $135\mu_B\ \text{Hz}^{-1/2}$ at 36 kHz. Thus the fluctuations for this wire exhibit a strong $1/f$ frequency dependence in the power spectrum, a behavior that has also been observed in SQUID measurements of macroscopic ferromagnetic cores [5,6].

The ability to measure small thermal-magnetic fluctuations is determined by the ability to resolve changes in dissipation. The thermal limit for detecting dissipation is given by [24] $\Gamma_{m,\rm min} = \sqrt{2}\, F_{\rm min}/\omega x_{\rm pk}$. The corresponding minimum detectable spectral density is $S_{m_x,\rm min} = (L_e/H)^2(4\Gamma_{m,\rm min} k_B T) = 4\sqrt{2}\,(L_e/H)^2 k_B T F_{\rm min}/\omega x_{\rm pk}$. For the first flexural mode of the cantilever from nanowire 2, we find the smallest detectable moment fluctuations $S_{m_x,\rm min}^{1/2} = 0.9\mu_B\ \text{Hz}^{-1/2}$ for a 60-kOe field, a 100-nm vibration amplitude, and a 1-Hz bandwidth [23].

Magnetic dissipation for the high anisotropy $Pr_2Fe_{14}B$ tip was very small (Fig. 3a). In fact for this tip, other types of rare-earth magnet tips, nonmagnetic (platinum) nanowire tips, and bare silicon cantilevers, we find virtually identical magnetic dissipation behavior which appears to originate from the silicon cantilever itself. Interestingly, the silicon dissipation increases with decreasing temperature in the range 3 to 7 K (Fig. 3b) in a manner characteristic of spin noise [7] (unlike cobalt tips which showed no temperature dependence below 10 K). Further studies will be necessary to fully elucidate the nature of the magnetic dissipation in bare silicon cantilevers.

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[1] J. A. Sidles *et al.,* Rev. Mod. Phys. **67**, 249 (1995).
[2] P. Ball, Nature (London) **404**, 918 (2000).
[3] G. P. Berman, G. D. Doolen, P. C. Hammel, and V. I. Tsifrinovich, Phys. Rev. B **61**, 14 694 (2000).
[4] D. D. Awschalom, D. P. DiVincenzo, and J. F. Smyth, Science **258**, 414 (1992).
[5] G. A. Prodi *et al.,* J. Appl. Phys. **66**, 5984 (1989).
[6] O. V. Snigirev *et al.,* J. Appl. Phys. **79**, 960 (1996).
[7] T. Sleator *et al.,* Phys. Rev. Lett. **55**, 1742 (1985).
[8] W. Wernsdorfer, D. Mailly, and A. Benoit, J. Appl. Phys. **87**, 5094 (2000); W. Wernsdorfer *et al.,* J. Magn. Magn. Mater. **145**, 33 (1995).
[9] M. Lederman, S. Schultz, and M. Ozaki, Phys. Rev. Lett. **73**, 1986 (1994).
[10] S. J. Hefferman, J. N. Chapman, and S. McVitie, J. Magn. Magn. Mater. **95**, 76 (1991).
[11] R. P. Cowburn *et al.,* Appl. Phys. Lett. **73**, 3947 (1998).
[12] C. Rossel *et al.,* J. Appl. Phys. **79**, 8166 (1996).
[13] J. G. E. Harris *et al.,* Appl. Phys. Lett. **75**, 1140 (1999).
[14] J. D. Hannay, R. W. Chantrell, and D. Rugar, J. Appl. Phys. **87**, 6827 (2000).
[15] T. D. Stowe *et al.,* Appl. Phys. Lett. **71**, 288 (1997).
[16] K. Y. Yasumura *et al.,* J. Microelectromech. Syst. **9**, 117 (2000).
[17] D. Rugar, H. J. Mamin, and P. Guethner, Appl. Phys. Lett. **55**, 2588 (1989).
[18] T. R. Albrecht *et al.,* J. Appl. Phys. **69**, 668 (1991).
[19] J. A. Marohn, R. Fainchtein, and D. D. Smith, Appl. Phys. Lett. **73**, 3778 (1998).
[20] P. M. Morse and K. U. Ingard, *Theoretical Acoustics* (McGraw-Hill, New York, 1968), pp. 181–185.
[21] S. Hirosawa *et al.,* J. Appl. Phys. **59**, 873 (1986).
[22] H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951).
[23] If additional magnetic damping occurs in an applied field, $F_{\rm min}$ will increase and the resolution of the measurement will be somewhat degraded.
[24] W. Denk and D. W. Pohl, Appl. Phys. Lett. **59**, 2171 (1991).