Entanglement, Interference, and Measurement in a Degenerate Parametric Oscillator

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Quantum dynamical equations of motion for homodyne detection of the degenerate optical parametric oscillator are solved exactly. Nonclassical photon statistics are shown to be a consequence of interference of probability amplitudes, entanglement of photon pairs from such an oscillator, and the role of measurement in quantum evolution.

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Fluctuations of photon beams reflect the quantum dynamics of photoemissive sources. In quantum mechanics, probabilities for observed events are derived from an underlying wave function that can interfere and collapse as it evolves. A consequence of this is that quantum mechanics can lead to correlations between observed events which a classical stochastic theory may not. Examples of these nonclassical correlations include squeezing, antibunching, and violations of Bell's inequalities [1,2].

The subthreshold degenerate parametric oscillator (DPO) has played a central role in the study of nonclassical photon correlations, particularly, squeezing [1,2]. The DPO radiates a highly bunched light beam that exhibits a large degree of squeezing. Interestingly, the squeezed and highly bunched light from the DPO when combined with a coherent light field, as in homodyne detection, is expected to display a rich variety of nonclassical photon correlations including antibunching [3]. It is intriguing that a highly bunched entangled photon beam from the DPO when mixed with a coherent field will exhibit correlations similar to those exhibited by a single-atom resonance fluorescence in free space or in cavity quantum electrodynamics (QED) [4,5]. Antibunching of light emitted by a single two-level atomic system can be eventually traced to the atomic dead time that a two-level atom cannot emit a second photon immediately after the emission of a first photon. The situation is not so simple for homodyne detection of the light from the DPO because there is no obvious mechanism for a dead time. By solving the equations of motion for homodyne detection exactly, we show that nonclassical photon correlations in homodyne detection of the DPO are a consequence of the interference of probability amplitudes, entangled nature of photon pairs generated by the DPO, and measurement. These are the features that most distinguish quantum mechanics from classical mechanics.

An outline of the experimental setup for homodyne detection of the DPO light is shown in Fig. 1. The DPO and local oscillator (LO) fields are combined by a beam splitter to produce the source field at the detector. The field from the DPO is governed by the interaction Hamiltonian for a phase matched DPO driven by a classical injected signal of amplitude ε [6]:

$$\hat{H} = \frac{i\hbar\kappa\varepsilon}{2}(\hat{a}_d^{\dagger 2} - \hat{a}_d^2) + \hat{H}_{\text{loss}}. \tag{1}$$

Here κ is the mode-coupling constant and $\hat{a}_d$ and $\hat{a}_d^\dagger$ are the annihilation and creation operators, respectively, for the DPO. $\hat{H}_{\text{loss}}$ describes the loss suffered by the DPO field. The combination $\kappa\varepsilon$ can be chosen to be real by a suitable definition of phases.

The equation of motion for the density matrix $\hat{\rho}_d$ of the DPO field is then

$$\dot{\hat{\rho}}_d = \frac{\kappa\varepsilon}{2}[\hat{a}_d^{\dagger 2} - \hat{a}_d^2, \hat{\rho}_d] + \gamma(2\hat{a}_d\hat{\rho}_d\hat{a}_d^\dagger - \hat{a}_d^\dagger\hat{a}_d\hat{\rho}_d - \hat{\rho}_d\hat{a}_d^\dagger\hat{a}_d), \tag{2}$$

where 2γ is the cavity decay rate. The steady-state solution to this equation in positive-P representation is given by [6,7]

$$(\hat{\rho}_d)_{ss} = \frac{1}{\sqrt{2\bar{n}_d}\,\pi}\iint dx\,dy\,\frac{|x\rangle\langle y|}{\langle y\,|\,x\rangle} \times \exp\left[2xy - \frac{\gamma}{\kappa\varepsilon}(x^2+y^2)\right], \tag{3}$$

where $-\infty < x, y < \infty$ are both real variables and $|x\rangle$ is a coherent state of $\hat{a}_d$ with $\hat{a}_d|x\rangle = x|x\rangle$. From this expression for the density matrix, the steady-state expectation value of an operator $\hat{O}$ can be calculated as $\langle\hat{O}\rangle_{ss} = \text{Tr}(\hat{O}\hat{\rho}_{ss})$. This leads to the following expectation values

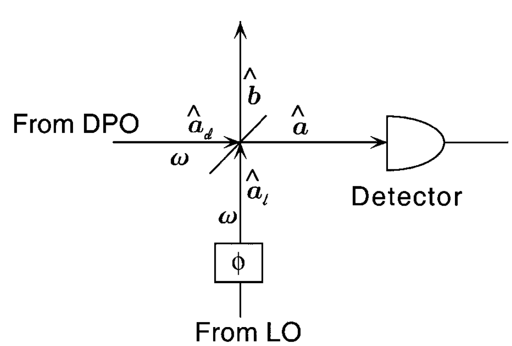

FIG. 1. Schematic experimental setup for the homodyne detection of the light from a degenerate parametric oscillator.

for the DPO field:

$$\langle \hat{a}_d^\dagger \hat{a}_d \rangle_{ss} = \frac{1}{2}\left[\frac{(\kappa\varepsilon/\gamma)^2}{1-(\kappa\varepsilon/\gamma)^2}\right] \equiv \bar{n}_d\,, \tag{4a}$$

$$\langle \hat{a}_d \rangle_{ss} = \langle \hat{a}_d^\dagger \rangle_{ss} = 0\,, \tag{4b}$$

$$\langle \hat{a}_d^2 \rangle_{ss} = \langle \hat{a}_d^{\dagger 2} \rangle_{ss} = \frac{\gamma}{\kappa\varepsilon}\,\bar{n}_d\,. \tag{4c}$$

The mean photon number $\bar{n}_d$ for the DPO depends only on the ratio $\kappa\varepsilon/\gamma$ and the steady-state mean field amplitude $\langle \hat{a}_d \rangle_{ss}$ for the DPO given by Eq. (4b) is zero.

The light emitted by the DPO is combined with the light from a coherent local oscillator (annihilation operator $\hat{a}_\ell$) by the beam splitter in Fig. 1 to produce the superposed field at the detector [3]

$$\hat{a} = \sqrt{\mathcal{R}}\,\hat{a}_\ell + \sqrt{\mathcal{T}}\,\hat{a}_d\,, \tag{5}$$

where $\mathcal{T}$ and $\mathcal{R}$ denote the power transmittivity and reflectivity of the beam splitter, respectively. Here we have assumed the local oscillator field is radiated by a cavity prepared in a coherent state. All measurements in this paper refer to the superposed field given by Eq. (5). We will call this field the homodyne degenerate parametric oscillator (HMDPO) field. The steady-state density matrix for the HMDPO field is

$$\hat{\rho}_{ss} = (|\alpha_\ell\rangle\langle\alpha_\ell|)\,\frac{1}{\sqrt{2\bar{n}_d}\,\pi}\iint dx\,dy\,\frac{|x\rangle\langle y|}{\langle y\,|\,x\rangle} \times e^{2xy-(\gamma/\kappa\varepsilon)(x^2+y^2)}, \tag{6}$$

where $|\alpha_\ell\rangle$ is the LO coherent state. The average field amplitude and photon number for the HMDPO are calculated with the help of Eq. (6) to be

$$\langle \hat{a} \rangle_{ss} = \sqrt{\mathcal{R}}\,\alpha_\ell\,, \tag{7a}$$

$$\langle \hat{a}^\dagger \hat{a} \rangle_{ss} = \mathrm{Tr}(\hat{\rho}_{ss}\hat{a}^\dagger\hat{a}) = \mathcal{R}\bar{n}_\ell + \mathcal{T}\bar{n}_d \equiv \bar{n}_{ss}\,. \tag{7b}$$

Thus the average field radiated by the DPO in the steady state is zero and the average photon number is the sum of the contributions from the DPO and the LO (no phase coherence between the DPO and LO fields). Let us now look at the second-order intensity correlation function $g^{(2)}(\tau) = \langle \hat{a}^\dagger(0)\hat{a}^\dagger(\tau)\hat{a}(\tau)\hat{a}(0)\rangle_{ss}/\langle \hat{a}^\dagger\hat{a}\rangle_{ss}^2$ for the HMDPO field. This correlation function is proportional to the probability of detecting a photon at time $t = \tau$ conditioned on the detection of a first photon at time $t = 0$ starting in the steady state. We can therefore write $g^{(2)}(\tau)$ as

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(\tau)\hat{a}(\tau)\rangle_c \langle \hat{a}^\dagger(0)\hat{a}(0)\rangle_{ss}}{\langle \hat{a}^\dagger\hat{a}\rangle_{ss}^2} = \frac{\langle \hat{a}^\dagger(\tau)\hat{a}(\tau)\rangle_c}{\bar{n}_{ss}}\,, \tag{8}$$

where we have used the fact that the probability of detecting the first photon is proportional to $\langle \hat{a}^\dagger(0)\hat{a}(0)\rangle_{ss} = \langle \hat{a}^\dagger\hat{a}\rangle_{ss} \equiv \bar{n}_{ss}$. The probability of detecting the second photon is thus proportional to the conditional mean photon number $\langle \hat{a}^\dagger(\tau)\hat{a}(\tau)\rangle_c$ evaluated with respect to the reduced density matrix after the first detection.

After the first detection at $\tau = 0$ the state of the system reduces to [8]

$$\hat{\rho}_c(0) = \frac{\hat{a}\hat{\rho}_{ss}\hat{a}^\dagger}{\mathrm{Tr}(\hat{\rho}_{ss}\hat{a}^\dagger\hat{a})} = \frac{\hat{a}\hat{\rho}_{ss}\hat{a}^\dagger}{\bar{n}_{ss}}\,. \tag{9}$$

With the help of the reduced density matrix (6) and (9), the mean values immediately after the first detection are found to be

$$\begin{aligned}\langle \hat{a}(0)\rangle_c &= \sqrt{\mathcal{R}}\,\alpha_\ell + \frac{\bar{n}_d\mathcal{T}\sqrt{\mathcal{R}}}{\bar{n}_{ss}}\left(\alpha_\ell + \frac{\gamma}{\kappa\varepsilon}\,\alpha_\ell^*\right)\\ &= \sqrt{\mathcal{R}}\,\alpha_\ell + \sqrt{\mathcal{T}}\,\langle \hat{a}_d(0)\rangle_c\,,\end{aligned} \tag{10a}$$

$$\begin{aligned}\langle \hat{a}^\dagger(0)\hat{a}(0)\rangle_c = \frac{1}{\bar{n}_{ss}}\Big[&(\bar{n}_\ell\mathcal{R} + 2\bar{n}_d\mathcal{T})^2\\ &+ \frac{\gamma}{\kappa\varepsilon}\,\bar{n}_d\mathcal{T}\mathcal{R}(\alpha_\ell^2 + \alpha_\ell^{*2})\\ &+ \frac{\bar{n}_d\mathcal{T}^2}{2}(1 - 2\bar{n}_d)\Big],\end{aligned} \tag{10b}$$

$$\langle \hat{a}^2(0)\rangle_c = \frac{3\gamma}{\kappa\varepsilon}\,\bar{n}_d\mathcal{T} + \frac{\alpha_\ell^2\mathcal{R}}{\bar{n}_{ss}}[\bar{n}_\ell\mathcal{R} + 3\bar{n}_d\mathcal{T}], \tag{10c}$$

$$\langle \hat{a}^{\dagger 2}(0)\rangle_c = \frac{3\gamma}{\kappa\varepsilon}\,\bar{n}_d\mathcal{T} + \frac{\alpha_\ell^{*2}\mathcal{R}}{\bar{n}_{ss}}[\bar{n}_\ell\mathcal{R} + 3\bar{n}_d\mathcal{T}]. \tag{10d}$$

Note that after the detection of the first photon, the mean field amplitude for the HMDPO jumps from $\sqrt{\mathcal{R}}\,\alpha_\ell$ to that given by (10a). This jump is caused by the discontinuity in the mean field emitted by the DPO which acquires a nonzero expectation value. The expectation values in Eqs. (10) serve as the initial values for the evolution of the system toward steady state.

The equations of motion for the time evolution of the HMDPO field variables can be derived using Eqs. (2) and (5). We recall that the LO state $|\alpha_\ell\rangle$ reduces to itself after the detection of each photon so that $\langle \dot{\hat{a}}_\ell\rangle = 0$. We then obtain the following dynamical equations for the HMDPO field:

$$\frac{d}{d\tau}\langle \hat{a}\rangle = \kappa\varepsilon\langle \hat{a}^\dagger\rangle - \gamma\langle \hat{a}\rangle + \sqrt{\mathcal{R}}\,(\gamma\alpha_\ell - \kappa\varepsilon\alpha_\ell^*), \tag{11a}$$

$$\frac{d}{d\tau}\langle \hat{a}^\dagger\rangle = \kappa\varepsilon\langle \hat{a}\rangle - \gamma\langle \hat{a}^\dagger\rangle + \sqrt{\mathcal{R}}\,(\gamma\alpha_\ell^* - \kappa\varepsilon\alpha_\ell), \tag{11b}$$

$$\begin{aligned}\frac{d}{d\tau}\langle \hat{a}^\dagger\hat{a}\rangle = [&\kappa\varepsilon\langle \hat{a}^2\rangle - \gamma\langle \hat{a}^\dagger\hat{a}\rangle\\ &+ \sqrt{\mathcal{R}}\,(\gamma\alpha_\ell^* - \kappa\varepsilon\alpha_\ell)\langle \hat{a}\rangle] + \text{c.c.},\end{aligned} \tag{11c}$$

$$\begin{aligned}\frac{d}{d\tau}\langle \hat{a}^2\rangle = &\kappa\varepsilon\mathcal{T} + 2\kappa\varepsilon\langle \hat{a}^\dagger\hat{a}\rangle - 2\gamma\langle \hat{a}^2\rangle\\ &- 2\kappa\varepsilon\sqrt{\mathcal{R}}\,\alpha_\ell^*\langle \hat{a}\rangle + 2\gamma\sqrt{\mathcal{R}}\,\alpha_\ell\langle \hat{a}\rangle,\end{aligned} \tag{11d}$$

$$\begin{aligned}\frac{d}{d\tau}\langle \hat{a}^{\dagger 2}\rangle = &\kappa\varepsilon\mathcal{T} + 2\kappa\varepsilon\langle \hat{a}^\dagger\hat{a}\rangle - 2\gamma\langle \hat{a}^{\dagger 2}\rangle\\ &- 2\kappa\varepsilon\sqrt{\mathcal{R}}\,\alpha_\ell\langle \hat{a}^\dagger\rangle + 2\gamma\sqrt{\mathcal{R}}\,\alpha_\ell^*\langle \hat{a}^\dagger\rangle.\end{aligned} \tag{11e}$$

These equations form a closed set and can be solved exactly for the time dependence of expectation values as the reduced state evolves toward the steady state. With initial values given by Eqs. (10) the results for the relevant quantities are

$$\langle \hat{a}(\tau)\rangle_c = \sqrt{\mathcal{R}}\,\alpha_\ell + \sqrt{\mathcal{T}}\,\langle \hat{a}_d(\tau)\rangle_c\,, \tag{12a}$$

$$\langle \hat{a}_d(\tau)\rangle_c = \frac{\kappa\varepsilon\sqrt{\mathcal{R}\mathcal{T}}}{4\bar{n}_{ss}}\left[\left(\frac{e^{-\lambda_1\tau}}{\lambda_1} - \frac{e^{-\lambda_2\tau}}{\lambda_2}\right)\alpha_\ell + \left(\frac{e^{-\lambda_1\tau}}{\lambda_1} + \frac{e^{-\lambda_2\tau}}{\lambda_2}\right)\alpha_\ell^*\right], \tag{12b}$$

$$\langle \hat{a}^\dagger(\tau)\hat{a}(\tau)\rangle_c = \mathcal{R}\bar{n}_\ell + \mathcal{T}\langle \hat{a}_d^\dagger(\tau)\hat{a}_d(\tau)\rangle_c + \sqrt{\mathcal{R}\mathcal{T}}\,[\alpha_\ell\langle \hat{a}_d^\dagger(\tau)\rangle_c + \alpha_\ell^*\langle \hat{a}_d(\tau)\rangle_c], \tag{12c}$$

$$\langle \hat{a}_d^\dagger(\tau)\hat{a}_d(\tau)\rangle_c = \bar{n}_d + \frac{\mathcal{T}\bar{n}_d}{4\bar{n}_{ss}}\left[\frac{\lambda_2}{\lambda_1}e^{-2\lambda_1\tau} + \frac{\lambda_1}{\lambda_2}e^{-2\lambda_2\tau}\right]. \tag{12d}$$

Here the decay constants $\lambda_1 = \gamma - \kappa\varepsilon$ and $\lambda_2 = \gamma + \kappa\varepsilon$. With the help of Eqs. (8) and (12) we find that the intensity correlation function $g^{(2)}(\tau)$ for the HMDPO field is given by

$$g^{(2)}(\tau) = 1 + \frac{\bar{n}_d\mathcal{T}^2}{4\bar{n}_{ss}^2}\left(\frac{\lambda_2}{\lambda_1}e^{-2\lambda_1\tau} + \frac{\lambda_1}{\lambda_2}e^{-2\lambda_2\tau}\right) + \left(\frac{\bar{n}_\ell\kappa\varepsilon\mathcal{R}\mathcal{T}}{\bar{n}_{ss}^2}\right)\left(\frac{e^{-\lambda_1\tau}}{\lambda_1}\cos^2\phi - \frac{e^{-\lambda_2\tau}}{\lambda_2}\sin^2\phi\right), \tag{13}$$

where $\phi = \arg(\alpha_\ell)$ is the LO phase and $\bar{n}_{ss}$ is given by Eq. (7b). The local oscillator phase refers to the pump field. This result can also be obtained directly from the correlation functions of the noise sources [3]. The second term of $g^{(2)}(\tau)$ represents excess fluctuations contributed by the DPO field. The third term in $g^{(2)}(\tau)$ arises due to the interference between the LO and the DPO *after* the detection of a first photon. Figure 2 shows several plots of $g^{(2)}(\tau)$ for $\phi = \pi/2$, all exhibiting violations of one or more of the classical Schwartz inequalities $g^{(2)}(0) \geq 1$, $g^{(2)}(0) \geq g^{(2)}(\tau)$, and $|g^{(2)}(0) - 1| \geq |g^{(2)}(\tau) - 1|$ [9]. These nonclassical correlations illustrate the interference of probability amplitudes, measurement induced coherence, and the entangled nature of photons from the DPO. In the following we will focus on the case $\phi = \pi/2$ because nonclassical effects are most pronounced in this case.

Figure 3 shows the time evolution of the HMDPO field amplitude over an interval during which a first photodetection occurs. Before detection, in the steady state, the HMDPO field amplitude has value $\sqrt{R}\,\alpha_\ell$, which for $\phi = \pi/2$ is a positive imaginary quantity. Note that the DPO contributes nothing to the steady-state average HMDPO field. The field amplitude has been normalized so that its steady-state value is unity. When a photon is detected the mean field amplitude changes discontinuously from the steady-state value $\langle \hat{a}\rangle_{ss} = \sqrt{\mathcal{R}}\,\alpha_\ell$ to the value $\langle \hat{a}(0)\rangle_c$ given by Eq. (10a). This mean the HMDPO amplitude may vanish or even become negative imaginary for $\phi = \pi/2$. After detection the HMDPO amplitude relaxes along the imaginary axis back to the steady-state value. During its passage to the steady state it can vanish at some nonzero delay if initially it starts out negative imaginary. If the reduced state of the HMDPO were a coherent state, the normalized intensity correlation function would be

$$\begin{aligned} g_{\mathrm{coh}}^{(2)}(\tau) &= \frac{|\langle \hat{a}(\tau)\rangle_c|^2}{|\langle \hat{a}\rangle_{ss}|^2} = \cos^2\phi\left(1 + \frac{\kappa\varepsilon\mathcal{T}}{\bar{n}_{ss}}\frac{e^{-\lambda_1\tau}}{2\lambda_1}\right)^2 + \sin^2\phi\left(1 - \frac{\kappa\varepsilon\mathcal{T}}{\bar{n}_{ss}}\frac{e^{-\lambda_2\tau}}{2\lambda_2}\right)^2, \\ &= 1 + \frac{\bar{n}_d\mathcal{T}^2}{2\bar{n}_{ss}^2}\left(\frac{\lambda_2}{\lambda_1}e^{-2\lambda_1\tau}\cos^2\phi + \frac{\lambda_1}{\lambda_2}e^{-2\lambda_2\tau}\sin^2\phi\right) + \frac{\kappa\varepsilon\mathcal{T}}{\bar{n}_{ss}}\left(\frac{e^{-\lambda_1\tau}}{\lambda_1}\cos^2\phi - \frac{e^{-\lambda_2\tau}}{\lambda_2}\sin^2\phi\right). \end{aligned} \tag{14}$$

This function must vanish whenever $\langle \hat{a}(\tau)\rangle_c$ vanishes. It is shown by the dashed curves in Fig. 2 for $\phi = \pi/2$ and is seen to be a good approximation to the exact correlation function (continuous curves) for the parameters used in the figure. The minima of $g^{(2)}(\tau)$ reflect the delays where $\langle \hat{a}(\tau)\rangle_c$ vanishes. The differences in $g^{(2)}(\tau)$ and $g_{\mathrm{coh}}^{(2)}(\tau)$ near the minima are due to the second term in Eq. (13) representing the incoherent fluctuations of the DPO. The importance of this term decreases as $\bar{n}_d$ gets smaller and for $\bar{n}_d \ll 1$ the coherent result $g_{\mathrm{coh}}^{(2)}(\tau)$ is a good approximation to the exact correlation function.

The discontinuity of the HMDPO field amplitude which leads to a vanishing field is actually due to the discontinuous change in the field radiated by the DPO when the first photon is detected. The mean field radiated by the DPO is zero before measurement [Eq. (4b)] and changes discontinuously to $\langle \hat{a}_d(0)\rangle_c = \sqrt{\mathcal{R}\mathcal{T}}\,(\bar{n}_d/\bar{n}_\ell)[\alpha_\ell + (\gamma/\kappa\varepsilon)\alpha_\ell^*]$. For small $\bar{n}_d$ (which implies $\gamma/\kappa\varepsilon \gg 1$) and $\phi = \pi/2$, the conditioned DPO field amplitude is $\langle \hat{a}_d(0)\rangle_c \approx \sqrt{\mathcal{R}\mathcal{T}}\,(\gamma\bar{n}_d/\kappa\varepsilon\bar{n}_\ell)\alpha_\ell^* = -\sqrt{\mathcal{R}\mathcal{T}}\,(\gamma\bar{n}_d/\kappa\varepsilon\bar{n}_\ell)\alpha_\ell$. Thus after the detection of the first photon, the field radiated by the DPO is out of phase with the LO field. It can cancel and even exceed the LO field as seen in Fig. 3. This phase coherence is an example of measurement induced coherence. The first detected photon could have come from the LO (with probability $\mathcal{R}\bar{n}_\ell/\bar{n}_{ss}$) or the DPO (with probability $\mathcal{T}\bar{n}_d/\bar{n}_{ss}$). These possibilities exist as a superposition of probability amplitudes, not as classical choices. The interference of probability amplitudes therefore plays an important role in giving rise to the postdetection coherence between the DPO and the LO fields. It is remarkable that although the DPO field is much weaker than the LO field ($\mathcal{T}\bar{n}_d \ll \mathcal{R}\bar{n}_\ell$),

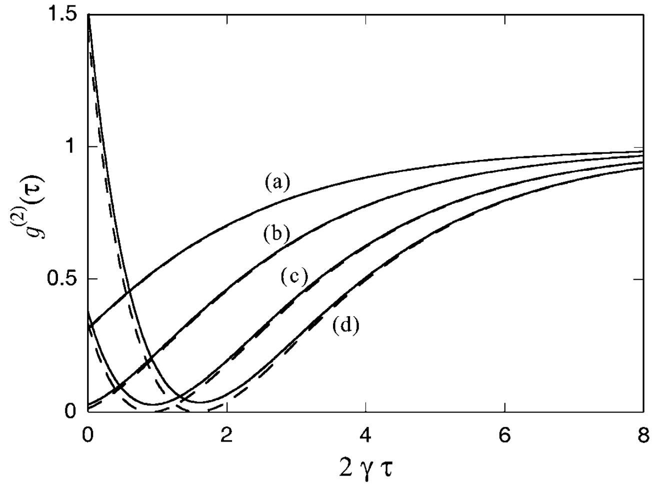

FIG. 2. Two-time intensity correlation function $g^{(2)}(\tau)$ as a function of scaled time $2\gamma\tau$ for $\bar{n}_d = 10^{-5}$, $\phi = \pi/2$ and several different values of the ratio $(\bar{n}_\ell \mathcal{R}/\bar{n}_d \mathcal{T}) = 500$ (a); 250 (b); 140 (c); and 100 (d). The dashed curves show $g^{(2)}_{\text{coh}}(\tau)$ for the same parameters.

the postdetection field radiated by the DPO can cancel or even exceed the LO field. The reason for the enhanced postdetection emission from the DPO is that photons are created as entangled pairs inside the cavity. When the first photon of a pair is detected the conditioned mean photon number for the DPO increases and it emits at an enhanced rate to ensure that the second photon is emitted within $(2\gamma)^{-1}$ of the first.

Nonclassical behavior of $g^{(2)}(\tau)$ is a direct consequence of postdetection coherence induced by the measurement and the entangled nature of photon pairs from the DPO. As a result of the induced phase coherence, the interference term $\sqrt{\mathcal{R}\mathcal{T}}\,[\alpha_\ell \langle \hat{a}_d^\dagger(\tau)\rangle_c + \alpha_\ell^* \langle \hat{a}_d(\tau)\rangle_c]$ in Eq. (12c) gives a nonzero contribution leading to the third term in Eq. (13). In the steady state (before detection) such an interference term contributes nothing. Enhanced postdetection emission, out of phase with the LO, allows the interference term to cancel the first two terms in Eq. (12c). This postdetection enhanced out-of-phase emission is a result of the entangled nature of photon pairs from the DPO. It would, for example, be absent for any value of ϕ if the DPO were replaced by a thermal source.

Induced coherence is reflected also in the conditioned mean photon number of mode $\hat{b}$ (Fig. 1). A calculation similar to the one outlined here shows that after the first detection from mode $\hat{a}$, the DPO radiates in phase (for $\phi = \pi/2$) into mode $\hat{b}$. Thus for $\phi = \pi/2$ the field from the DPO after a first detection interferes destructively with the LO field at the port where the first detection occurs and constructively at the other port.

When ϕ deviates from $\pm\pi/2$, the reduced HMDPO field amplitude does not vanish because α_ℓ^* and α_ℓ are not π out of phase and their real parts never vanish. Consequently, as ϕ deviates from $\pm\pi/2$ and approaches 0, the nonclassical features are gradually washed out. The nonclassical correlations described here are not directly related to squeezing. This can be seen by noting that by adjusting the beam splitter transmittivity we can degrade squeezing but these correlations persist so long as the ratio $\mathcal{R}\bar{n}_\ell/\mathcal{T}\bar{n}_d$ stays the same. Also note that in the limit $\bar{n}_d \ll 1$ the field from the DPO shows negligible squeezing, whereas the correlations described here are the strongest. Likewise, as $\bar{n}_d$ increases, squeezing increases (until the threshold is reached), whereas the nonclassical correlations weaken.

The features of quantum mechanics that most distinguish it from classical mechanics are the interference of probability amplitudes, entanglement, and the role of measurement in the quantum mechanical evolution of a system. The nonclassical behavior of photons in homodyne detection of the light from a degenerate parametric oscillator illustrates all of these within the context of a simple exactly solvable model.

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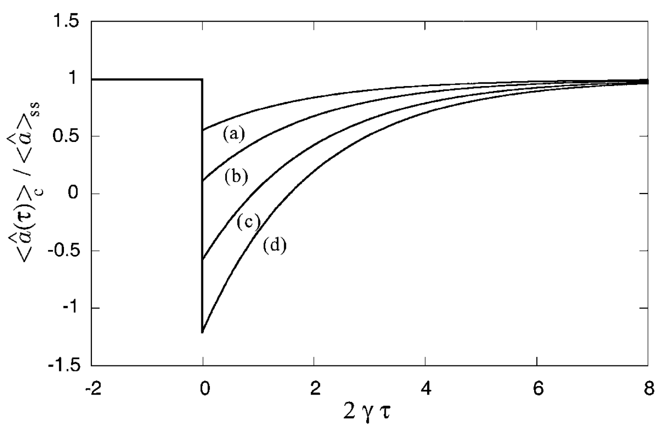

FIG. 3. Evolution of the reduced state field amplitude normalized to the steady-state field amplitude $\langle \hat{a}(\tau)\rangle_c/\langle \hat{a}\rangle_{ss}$ as a function of scaled time $2\gamma\tau$ for $\bar{n}_d = 10^{-5}$, $\phi = \pi/2$ and several different values of the ratio $(\bar{n}_\ell \mathcal{R}/\bar{n}_d \mathcal{T}) = 500$ (a); 250 (b); 140 (c); and 100 (d). At $\tau = 0$ a first photon is detected.

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