QCD Prediction for Heavy Boson Transverse Momentum Distributions

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We investigate the predictive power of the Collins, Soper, and Sterman b-space QCD resummation formalism for transverse momentum (Q_T) distributions of heavy boson production in hadronic collisions. We show that the predictive power has a strong dependence on the collision energy $\sqrt{S}$ in addition to its well known Q^2 dependence, and the $\sqrt{S}$ dependence improves the predictive power at collider energies. We demonstrate that, at the Fermilab Tevatron and the CERN LHC energies, the Q_T distributions derived from b-space resummation are not sensitive to the nonperturbative input at large b, and give good descriptions of the Q_T distributions of heavy boson production at all transverse momenta $Q_T \leq Q$.

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With new data from Fermilab Run II on the horizon and the LHC in the near future, we expect to test quantum chromodynamics (QCD) to a new level of accuracy, and also expect that a better understanding of QCD will underpin precision tests of the electroweak interactions and particle searches beyond the standard model [1]. As pointed out in Ref. [1], the description of vector and scalar boson production properties, in particular their transverse momentum (Q_T) distribution, is likely to be one of the most intensively investigated topics at both Fermilab and the CERN LHC, especially in the context of Higgs searches. The main purpose of this Letter is to quantitatively demonstrate the predictive power of QCD resummation formalism for the Q_T distributions of heavy boson production at the Fermilab Tevatron and the CERN LHC. In particular, we concentrate on the small transverse momentum region: $Q_T \leq Q$, where the bulk of the data is. This region is also most relevant to the hadronic Higgs production.

When $Q_T \ll Q$, the Q_T distribution calculated in conventional fixed-order perturbation theory receives large logarithm, $\ln^2(Q^2/Q_T^2)$, at every power of α_s, which is a direct consequence of emissions of soft and collinear gluons by incoming partons. Therefore, at sufficiently small Q_T, the convergence of conventional perturbative expansion in powers of α_s is impaired, and the logarithms must be resummed.

Resummation of the large logarithms can be carried out either in Q_T space directly, or in the impact parameter, b space, which is the Fourier conjugate of Q_T space. It was first shown by Dokshitzer, Diakonov, and Troyan that in the double leading logarithm approximation, the dominant contributions in the small Q_T region can be resummed into a Sudakov form factor [2]. By imposing the transverse momentum conservation without assuming the strong ordering in transverse momenta of radiating gluons, Parisi and Petronzio introduced the b-space resummation method which allows one to resum some subleading logarithms [3]. Using the renormalization group equation technique, Collins and Soper improved the b-space resummation to resum all logarithms as singular as $\ln^m(Q^2/Q_T^2)/Q_T^2$, as $Q_T \to 0$ [4]. Using this renormalization group improved b-space resummation, Collins, Soper, and Sterman (CSS) derived a formalism for the transverse momentum distributions of vector boson production in hadronic collisions [5]. This formalism, which is often called CSS formalism, can be also applied to the hadronic production of Higgs bosons [1].

For Drell-Yan vector boson ($V = \gamma^*, W^\pm, Z$) production in hadronic collisions between hadrons A and B, the CSS formalism has the following generic form [5]:

$$\frac{d\sigma_{A+B\to V+X}}{dQ^2 dy dQ_T^2} = \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{Q}_T\cdot\vec{b}}\, \tilde{W}(b,Q,x_A,x_B) + Y(Q_T,Q,x_A,x_B), \qquad (1)$$

where $\tilde{W}$ gives the dominant contribution when $Q_T \ll Q$, while Y gives corrections that are negligible for small Q_T, but become important when $Q_T \sim Q$. In Eq. (1), $x_A = e^y Q/\sqrt{S}$ and $x_B = e^{-y} Q/\sqrt{S}$ with the rapidity y and collision energy $\sqrt{S}$. The $\tilde{W}$ in Eq. (1) includes all powers of large logarithms from $\ln(1/b^2)$ to $\ln(Q^2)$ and has the following form [5]:

$$\tilde{W}(b,Q,x_A,x_B) = e^{-S(b,Q)}\, \tilde{W}(b,c/b,x_A,x_B), \qquad (2)$$

where c is a constant of the order of 1 [5,6], and $S(b,Q) = \int_{c^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}[\ln(\frac{Q^2}{\mu^2})A(\alpha_s(\mu)) + B(\alpha_s(\mu))]$, with $A(\alpha_s)$ and $B(\alpha_s)$ perturbatively calculable [5]. The $\tilde{W}(b,c/b,x_A,x_B)$ in Eq. (2) depends on only one momentum scale, $1/b$, and is perturbatively calculable as long as $1/b$ is large enough. The large logarithms from $\ln(c^2/b^2)$ to $\ln(Q^2)$ in $\tilde{W}(b,Q,x_A,x_B)$ are completely resummed into the exponential factor $\exp[-S(b,Q)]$.

Since the perturbatively resummed $\tilde{W}(b,Q,x_A,x_B)$ in Eq. (2) is reliable only for the small b region, an extrapolation to the large b region is necessary in order to complete the Fourier transform in Eq. (1). In the CSS formalism, a variable b_* and a nonperturbative function $F^{\rm NP}(b,Q,x_A,x_B)$ were introduced [5],

$$\tilde{W}_{\rm CSS}(b,Q,x_A,x_B) \equiv \tilde{W}(b_*,Q,x_A,x_B)F^{\rm NP}(b,Q,x_A,x_B), \quad (3)$$

where $b_* = b/\sqrt{1+(b/b_{\rm max})^2} < b_{\rm max} = 0.5\ {\rm GeV}^{-1}$, and $F^{\rm NP}$ has a Gaussian-like form, $F^{\rm NP} \sim \exp(-\kappa b^2)$ and the parameter κ has some dependence on Q^2, x_A, and x_B. The data are not inconsistent with such a form [7–10]. However, improvements are definitely needed for the precision tests of the theory [1,11].

Although the b-space resummation formalism has been successful in interpreting existing data, it was argued [1,11] that the formalism has many drawbacks associated with working in impact parameter space. As listed in Ref. [1], the first is the difficulty of matching the resummed and fixed-order predictions; and the second is to know the quantitative difference between the prediction and the fitting because of the introduction of a nonperturbative $F^{\rm NP}$. In the viewing of these difficulties, major efforts have been devoted to resumming the large logarithms directly in Q_T space [1,11].

In the following, we argue and demonstrate that both of these drawbacks can be overcome. We show that b-space formalism works smoothly for all $Q_T \leq Q$. We demonstrate that the Q^2 and $\sqrt{S}$ dependence of the resummed b-space distribution ensure that the Fourier transform is completely dominated by the small b region in high energy collisions, and consequently, the Q_T distribution is insensitive to the details of $F^{\rm NP}$.

It was known [1] that the b-space resummed Q_T distribution from Eq. (1) becomes unphysical or negative when Q_T is large. For example, a matching between the resummed and fixed-order calculations has to take place at $Q_T \sim 50$ GeV for W production when these two predictions cross over [11]. On the other hand, we expect that the predictions given by the b-space resummation in Eq. (1) should work better when Q_T is large, because the perturbatively calculated Y term dominates and the predictions should be less sensitive to $\tilde{W}$ and its nonperturbative input. We find that this puzzle was mainly caused by the lack of numerical accuracy of the Fourier transform from the b space to Q_T space.

Since there is no preferred transverse direction, the $\tilde{W}$ in Eq. (1) is a function of $b = |\vec{b}|$, and the Fourier transform can be written as

$$\frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{Q}_T\cdot\vec{b}}\, \tilde{W}(b,Q,x_A,x_B) = \frac{1}{2\pi}\int_0^\infty db\, bJ_0(Q_Tb)e^{-S(b,Q)}\, \tilde{W}\left(b,\frac{c}{b},x_A,x_B\right), \quad (4)$$

where $J_0(z)$ with $z = Q_Tb$ is the Bessel function. Because of the oscillatory nature of the Bessel function, high accuracy of the numerical integration over b is crucial for a reliable result. The number of oscillations strongly depends on the value of Q_T for the same range of b. For example, when $b \in (0,2)\ {\rm GeV}^{-1}$, $J_0(Q_Tb)$ crosses zero 0, 6, and 63 times for $Q_T = 1$, 10, and 100 GeV, respectively. It is clear that numerical accuracy is extremely important for the large Q_T region. We noticed that most work published in the literature used some kind of asymptotic form to approximate the Bessel function when $z = Q_Tb$ is large. We believe that the asymptotic form is a source of the uncertainties observed for the large Q_T region. Instead of using an asymptotic form, we use an integral form for the Bessel function $J_0(z) = \frac{1}{\pi}\int_0^\pi \cos[z\sin(\theta)]\,d\theta$. The great advantage of using an integral form is that we can control the numerical accuracy of the Bessel function by improving the accuracy of the integration. With the integral form of the Bessel function, we show below that the b-space resummed Q_T distributions are smoothly consistent with data for all transverse momenta up to Q [6].

The predictive power of the CSS resummation formalism relies on the fact that the integration over b in Eq. (4) is dominated by the region where $b \sim 1/Q$, because the exponential factor $\exp[-S(b,Q)]$ in Eq. (4) suppresses the b integral when b is larger than $1/Q$ [5]. Using the saddle point method, it was shown [3,5] that even at $Q_T = 0$, the b integration in Eq. (4) is dominated by an impact parameter of order

$$b_{\rm SP} = \frac{1}{\Lambda_{\rm QCD}}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)^{\lambda}, \quad (5)$$

where $\lambda = 16/(49-2n_f) \approx 0.41$ for quark flavors $n_f = 5$. When $Q \sim M_Z$ or M_W, the momentum scale $c/b_{\rm SP}$ in Eq. (4) is of a few GeV, at which the perturbation theory is expected to work. Consequently, the predictive power of the CSS formalism should not be very sensitive to the $F^{\rm NP}$ at large b as long as Q is large.

The $b_{\rm SP}$ in Eq. (5) was derived with an assumption that the b dependence in $\tilde{W}(b,c/b,x_A,x_B)$ is smooth around $b_{\rm SP}$. We find that the b dependence in $\tilde{W}(b,c/b,x_A,x_B)$ is important. When x_A and x_B are small, this b dependence reduces the numerical value of saddle point, and consequently, increases the predictive power of the b-space resummation formalism [6].

Taking into account the full b dependence of $\tilde{W}(b,c/b,x_A,x_B)$, the saddle point for the b integration in Eq. (4) at $Q_T = 0$ is determined by solving the following equation:

$$\frac{d}{db}\ln[be^{-S(b,Q)}] + \frac{d}{db}\ln\left[\tilde{W}\left(b,\frac{c}{b},x_A,x_B\right)\right] = 0. \quad (6)$$

The $b_{\rm SP}$ in Eq. (5) corresponds to a solution of Eq. (6) without the second term and keeps only the first order term of the $A(\alpha_s)$ in $S(b,Q)$. As shown in Ref. [6], the second term in Eq. (6) is proportional to the evolution of parton distribution $\phi(x,\mu)$: $-\frac{d}{d\ln(1/b)}\phi(x,\mu=\frac{c}{b})$. The evolution $(d/d\ln\mu)\phi(x,\mu)$ is positive (or negative) for $x < x_0 \sim 0.1$ (or $x > x_0$), and is very steep when x is far away from x_0. Therefore, the second term in Eq. (6) *reduces* the numerical value of the saddle point when x_A and x_B are much smaller than x_0. As a demonstration, let $Q = 6$ GeV and $\sqrt{S} = 1.8$ TeV. Using CTEQ4 parton distribution and $\Lambda_{\rm QCD}(n_f = 5) = 0.202$ GeV [12], one derives from Eq. (5) that $b_{\rm SP} \approx 1.2$ GeV^{-1}, and might conclude that perturbatively resummed prediction for Q_T distribution at the given values of Q and $\sqrt{S}$ is not reliable. However, as shown in Fig. 1(a), the integrand of the b integration in Eq. (4) has a nice saddle point in the perturbative region at $b_0 \approx 0.38$ GeV^{-1}. This is due to the fact that $x_A \sim x_B \sim 0.003$ are very small and the second term in Eq. (6) is negative and important. Figure 1(b) shows that the second term in Eq. (6) (dashed line) cancels the first term (solid line) at the saddle point b_0.

In Figs. 1(c) and 1(d), we show the effect of the second term in Eq. (6) on the saddle point of Z production at the LHC energy. At $\sqrt{S} = 14$ TeV, the $\sqrt{S}$ dependence for Z production improves the value of the saddle point from $b_0 = 0.24$ GeV^{-1} at $\sqrt{S} = 1.8$ TeV to 0.13 GeV^{-1}, in comparison to an estimated $b_{\rm SP} \approx 0.40$ GeV^{-1} from Eq. (5). The narrow width of the b distribution shown in Fig. 1(c) also ensures that the b integration is dominated by $b \sim b_0$. Similarly, we find the same improvements on the saddle point and width of the b distribution for Higgs production at LHC energy [6]. In conclusion, the b-space resummation formalism for heavy boson production at collider energies should not be very sensitive to the nonperturbative input $F^{\rm NP}$ at large b.

To quantitatively demonstrate the sensitivities on the $F^{\rm NP}$, we reexamine the extrapolation, $\tilde{W}_{\rm CSS}$, defined in Eq. (3). We find that using the fitting parameters from Refs. [7–10] to fix the $F^{\rm NP}$, the ratio, $\tilde{W}_{\rm CSS}(b,Q,x_A,x_B)/\tilde{W}(b,Q,x_A,x_B)$, differs from one by as much as 20% within the perturbative region: $b < b_{\rm max} \sim 0.5$ GeV^{-1}. That is, the $\tilde{W}_{\rm CSS}(b,Q,x_A,x_B)$ introduces a significant fitting parameter dependence to the resummed b distribution in the perturbative region.

In order to separate the perturbative prediction in small b region from the nonperturbative physics at large b, we

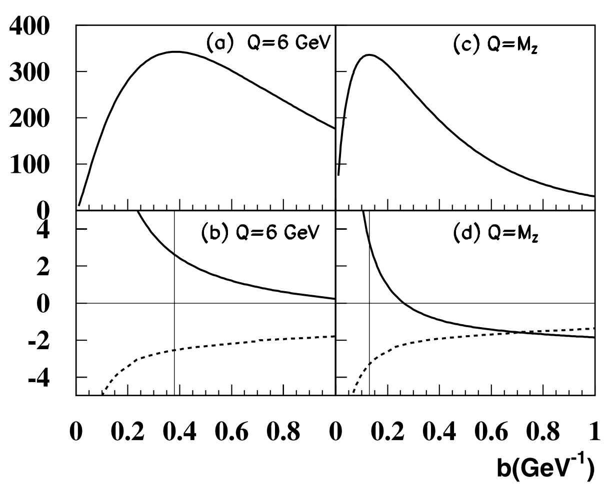

FIG. 1. Integrand of the b integration in Eq. (4) at $Q_T = 0$ as a function of b for $Q = 6$ GeV (a) and $Q = M_Z$ (c) with an arbitrary normalization; and the first (solid line) and second (dashed line) terms in Eq. (6) as a function of b in (b) and (d) at the respective Q.

review the resummation of large logarithms in the CSS formalism, and introduce a new functional form for the extrapolation. In Ref. [5], the large logarithms are resummed by solving the evolution equation

$$\frac{\partial}{\partial \ln Q^2} \tilde{W}(b,Q,x_A,x_B) = [K(b\mu, \alpha_s(\mu)) + G(Q/\mu, \alpha_s(\mu))] \times \tilde{W}(b,Q,x_A,x_B), \quad (7)$$

from $\ln(c^2b^2)$ to $\ln(Q^2)$, and the renormalization group equations for K and G from $\ln(c^2/b^2)$ to $\ln(\mu^2)$ and from $\ln(\mu^2)$ to $\ln(Q^2)$, respectively [5]. Since the evolution equation and the renormalization group equations do not include any power corrections, the solution, $\tilde{W}$ in Eq. (2), is valid only for $b < b_c$ with $\ln(1/b_c^2) \sim b_c^2$ (or $b_c \sim 0.75$ GeV^{-1}). The choice of $b_{\rm max} = 0.5$ GeV^{-1} in Ref. [5] is consistent with the approximation.

Taking advantage of our early conclusion that heavy boson production at collider energies should not be very sensitive to the large b region, we extrapolate $\tilde{W}(b,Q,x_A,x_B)$ to the large b region without introducing the b_* [6]. For $b < b_{\rm max}$, the $\tilde{W}(b,Q,x_A,x_B)$ is the same as the perturbatively calculated one given in Eq. (2). For $b > b_{\rm max}$, we solve the evolution equation in Eq. (7) from $\ln(c^2/b_{\rm max}^2)$ to $\ln(Q^2)$, and solve the renormalization group equations for K and G from $\ln(c^2/b^2)$ to $\ln\mu^2$ and from $\ln(\mu^2)$ to $\ln(Q^2)$, respectively. Because $b > b_{\rm max}$, we add possible power corrections to the renormalization group equations of K and G. We find [6]

$$\tilde{W}_{QZ}(b,Q,x_A,x_B) = \tilde{W}(b_{\rm max},Q,x_A,x_B) \times F^{\rm NP}(b,b_{\rm max},Q,x_A,x_B), \quad (8)$$

for $b > b_{\rm max}$. Including only the first power correction, the $F^{\rm NP}$ has the following functional form [6]

$$\ln(F^{\rm NP}) = -g_1[(b^2)^\alpha - (b_{\rm max}^2)^\alpha] - g_2[b^2 - b_{\rm max}^2], \quad (9)$$

where g_1, g_2, and $\alpha(<1)$ are parameters. Their dependence on Q, x_A, and x_B, which is more relevant for the low Q^2 Drell-Yan data, is explained in Ref. [6]. In Eq. (9), the first term corresponds to a direct extrapolation of the logarithmic contributions to the function K to large b region. The $(b^2)^\alpha$ dependence is a result of replacing a series of logarithmic dependence on μ^2 in the renormalization group equations by $(\mu^2)^\alpha$. The second term is a consequence of the first power correction $(1/\mu^2)$ to the renormalization group equations. Since the saddle point has a small numerical value in b, high power corrections to the renormalization group equations of the K and G, which are sensitive to the very large b region, could be neglected. In addition, we neglect a term, $\ln[\tilde{W}(b,c/b_{\rm max},x_A,x_B)] - \ln[\tilde{W}(b_{\rm max},c/b_{\rm max},x_A,x_B)]$, for the $\ln(F^{\rm NP})$ by assuming that parton distributions are saturated when $b > b_{\rm max}$. More detailed discussions on the functional form of the $F^{\rm NP}$ are given in Ref. [6].

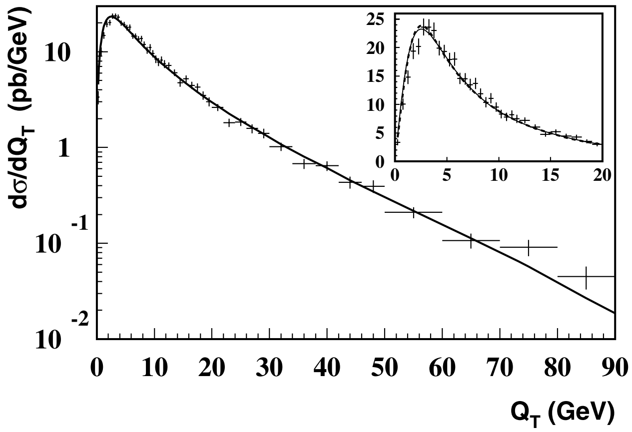

FIG. 2. Comparison between the b-space resummed Q_T distribution and CDF data [13]. The inset shows the $Q_T < 20$ GeV region.

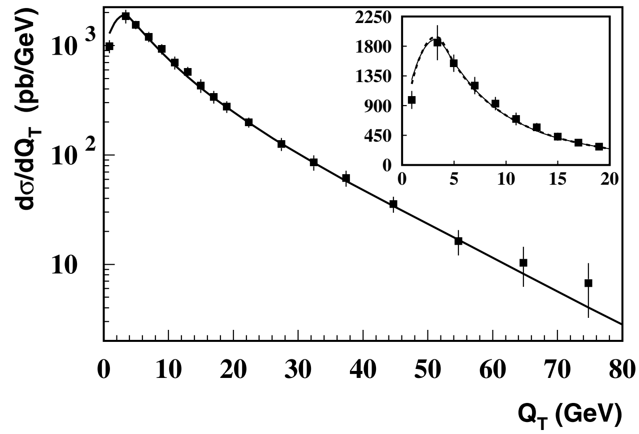

FIG. 3. Comparison between the b-space resummed Q_T distribution and D0 data [14]. The inset shows the $Q_T < 20$ GeV region.

We quantitatively test the sensitivities on the $F^{\rm NP}$ by studying the dependence on $b_{\rm max}$, g_2, and α. We first set $g_2 = 0$ (no power corrections) and fix g_1 and α in Eq. (9) by requiring the first and second derivatives of the $\tilde{W}$ to be continuous at $b = b_{\rm max} = 0.5\ {\rm GeV}^{-1}$. We plot our predictions (solid lines) to the Q_T distributions of Z and W production at Tevatron in Figs. 2 and 3, respectively. In Fig. 2, we plot the $d\sigma/dQ_T$ of e^+e^- pairs as a function of Q_T at $\sqrt{S} = 1.8$ TeV. The data are from CDF Collaboration [13]. Theory curves (Z only) are from Eq. (1) with the $\tilde{W}$ given in Eqs. (2) and (8) for $b < b_{\rm max}$ and $b > b_{\rm max}$, respectively. CTEQ4 parton distribution and an overall normalization 1.09 are used [13]. In Fig. 3, we plot the $d\sigma/dQ_T$ for W production with the same $b_{\rm max}$ and g_2 and without any overall normalization [6]. The data for W production are from D0 Collaboration [14]. The QCD predictions from the b-space resummation formalism are consistent with the data for all $Q_T < Q$. Furthermore, we let g_2 be a fitting parameter for any given value of $b_{\rm max}$. Although the fitting prefers $g_2 \sim 0.4\ {\rm GeV}^2$, the Q_T distributions are extremely insensitive to the choices of $b_{\rm max}$ and g_2. The total χ^2 is very stable for $b_{\rm max} \in (0.25, 0.8)\ {\rm GeV}^{-1}$ and $g_2 \in (0, 1)\ {\rm GeV}^2$. In Figs. 2 and 3, we also plot the theory curves (dashed lines) with $g_2 = 0.8\ {\rm GeV}^2$ (twice of the fitting value). Nonvanish g_2 gives a small improvement to the Q_T distributions at small Q_T. We also vary the value of α in Eq. (9) by requiring only the first derivative to be continuous at $b = b_{\rm max}$, and find equally good theoretical predictions, except very mild oscillations in the curves at very large Q_T due to the Fourier transform of a less-smooth b distribution. The observed insensitivity on $b_{\rm max}$, g_2, and α is clear evidence that the b-space resummation formalism is not sensitive to the nonperturbative input at large b for heavy boson production.

We notice that the theory curve is below the data at large Q_T. We believe that it is because we have only the leading order contribution to the Y term in Eq. (1). At large Q_T, the Y term dominates. Similar to the fixed-order perturbative calculations, the next-to-leading order contribution will enhance the theoretical predictions [15]. In conclusion, the CSS b-space resummation formalism has a good predictive power for heavy boson production at Tevatron energy, and it should provide even better predictions at the LHC energy [6].

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