Measuring the CP Violating Phase γ Using $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ Decays

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A new and simple procedure to measure the angle γ from $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ decays using SU(3) symmetry is presented. It is based on a full Dalitz plot analysis of these decays. All diagrams, including strong and electroweak penguins, are considered in the procedure. The method is also free from final state interaction problems. The theoretical error in the extraction of γ within the method should be of the order of 10^0 or even less. Taking into account the B-meson production in the first generation of B factories and recent measurements from CLEO, this method could bring the best measurement of γ in the next years.

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For the next years, large accelerator facilities known as B factories are going to be running. The goal is to produce a large amount of B mesons because their decay should be sensitive to CP violation [1,2]. As a consequence, one hopes to measure the three Cabibbo-Kobayashi-Maskawa (CKM) angles $\alpha \equiv \arg(-V_{td}V_{tb}^{*}/V_{ud}V_{ub}^{*})$, $\beta \equiv \arg(-V_{cd}V_{cb}^{*}/V_{td}V_{tb}^{*})$, and $\gamma \equiv \arg(-V_{ud}V_{ub}^{*}/V_{cd}V_{cb}^{*})$ to check the standard model predictions. It is not yet clear which precise measurements will allow a clean extraction of α and γ. Concerning the last angle, many interesting methods have been proposed so far but all of them suffer from either experimental or theoretical problems. On the one side, theoretically clean procedures based in $B \to DK$ decays [3] suffer from important experimental difficulties and would demand about 10 years of data taking in order to extract γ with an error, at least, of the order of 15^0 [1]. On the other side, other methods based in $B \to \pi\pi$, $B \to \pi K$, or $B \to KK$ decays [4] have theoretical uncertainties which would imply systematic errors in the extraction of γ as large as 20^0 [5]. It is then worth looking for new experimental procedures to extract γ with smaller uncertainties.

Methods proposed so far to measure γ are based in the study of branching ratio asymmetries in two body decays. In a previous Letter [6] we showed that three body decays could be a more interesting tool to extract CP violating phases [7]. The idea is to make use of the fact that Dalitz plot analysis of a three body decay gives a direct measurement of *amplitudes* instead of branching ratios. In other words, one has a direct access to the *phase* of a given process. In Ref. [6] we have illustrated this general remark applying it to the extraction of γ studying the decay $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$. We showed that, due to this direct experimental access to phases, γ could be extracted with a smaller statistical error than the usual methods based in two body decays existing in the literature. Unfortunately, the method suffers from one difficulty also existing in two body decays, i.e., penguin pollution: the bigger the unmeasured penguin contribution compared to the tree one, the larger the systematic error of the method.

In this Letter, making a full use of Dalitz plot analysis features, we present a first procedure to extract γ using three body decays which takes penguin contributions explicitly into account. The method is based in a combined study of two pairs of CP conjugated decays $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ related by SU(3) symmetry. It is free from final state interaction and electroweak penguin problems. The theoretical error of the method is due to SU(3) breaking and the uncertainty about charm penguin diagrams. Considering present expectation values for γ and β, we estimate this error to be of the order of 10^0 or even less. This value is smaller than that of other methods experimentally accessible within a few years.

Let us start the description of the method by presenting the main features of Dalitz plot analysis in our context. Three body decays of heavy mesons seem to be dominated by intermediate resonant decays [8]. All these partial intermediate channels, together with the direct nonresonant channel interfere to give the same—detected—three body final state. The Dalitz plot analysis is a powerful tool that yields a clear separation of all these intermediate channels; moreover, it brings a direct measurement of the *amplitude,* i.e., magnitude and phase, of the contribution of all the intermediate processes. As phases are always measured with respect to a given one, in order to extract a weak phase with this method, one needs at least two distinct channels with different weak phase [6].

This is the case for the two pairs of decays $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ considered here, thanks to the presence of χ_{c0} as a possible intermediate resonance. Indeed, the contribution $\mathcal{A}(B^{\pm} \to \chi_{c0}\mathcal{P}^{\pm}) \times \mathcal{A}(\chi_{c0} \to \pi^{+}\pi^{-})$ (where $\mathcal{P} = \pi$ or K) has no weak phase while for the other possible channels—as, for example, those mediated by a ρ^0 or f_0 resonances—the tree contribution has weak phase γ.

Final state interaction problems do not affect this method. One can easily get rid of them by choosing a contribution mediated by an isospin 0 resonance [6,9]—such as an f_0 or a possible σ resonance. Indeed, a $B^{\pm} \to f_0\mathcal{P}^{\pm}$ decay, for example, proceeds through a unique isospin amplitude. Thus, the method presented in this Letter is completely free from final state interaction difficulties. In the following, we will present the method using a $B^\pm \to f_0 \mathcal{P}^\pm$ decay.

Besides tree contributions driven by the weak phase γ, $\mathcal{A}(B^\pm \to f_0 \mathcal{P}^\pm)$ has also penguin contributions with different weak phases. For $\mathcal{P} = \pi$, top penguin diagrams have weak phase β and charm penguin ones have no weak phase; on the other side, for $\mathcal{P} = K$, both top and charm penguins have no weak phase. Thus, for the four decays, the f_0 amplitudes are written as

$$A_1 = Te^{i(\delta_T + \gamma)} + P_t e^{i(\delta_{P_t} - \beta)} + P_c e^{i\delta_{P_c}}, \tag{1}$$

$$A_2 = Te^{i(\delta_T - \gamma)} + P_t e^{i(\delta_{P_t} + \beta)} + P_c e^{i\delta_{P_c}}, \tag{2}$$

$$A_3 = T' e^{i(\delta_{T'} + \gamma)} + P' e^{i\delta_{P'}}, \tag{3}$$

$$A_4 = T' e^{i(\delta_{T'} - \gamma)} + P' e^{i\delta_{P'}}, \tag{4}$$

where $A_1 = \mathcal{A}(B^+ \to f_0 \pi^+)$, $A_2 = \mathcal{A}(B^- \to f_0 \pi^-)$, $A_3 = \mathcal{A}(B^+ \to f_0 K^+)$, and $A_4 = \mathcal{A}(B^- \to f_0 K^-)$. In the expressions above, δ_T, $\delta_{T'}$, δ_{P_t}, δ_{P_c}, and $\delta_{P'}$ are strong (CP conserving) phases, T and T' contain both tree and color suppressed contributions, P_t includes all strong and electroweak penguins diagrams with weak phase β, P_c includes all those with no weak phase, and P' includes all penguins. In other words, Eqs. (1)–(4) include *all* kinds of diagrams contributing to these decays [10]. Some of these decays can be related by SU(3) symmetry, as we will see below.

The left-hand sides in Eqs. (1)–(4), i.e., A_i, ($i = 1, \ldots, 4$), are *directly measured* complex numbers, that is, 8 real independent quantities. This is one of the main claims of this Letter: Dalitz plot analysis brings more independent measurements than usual two body branching ratios; one thus has more information available for each decay that can be used to treat penguin and other pollutions.

As a first step, we will exclude from our analysis charm penguin contributions which appear only in the first two equations. Note that this is not a strong assumption. Indeed, in $B^\pm \to \pi^\pm \pi^+ \pi^-$ decays, as in $B \to \pi\pi$, one expects penguin contributions to be of the order of 20% of tree ones [2]. Moreover, charm penguin amplitudes are expected to be not larger than half of the top ones [11]. Thus, in order to resolve in γ in Eqs. (1)–(4), the influence of charm penguin is expected to be small. In spite of this, in the last part of this Letter, we will explicitly study their influence in our results.

Now, we make use of SU(3) flavor symmetry. All contributions included in T' are identical to those in T except for an s quark replacing a light one. (Note that one cannot make the same assumption for the penguin sector because in P_t there are two different topologies while in P' there is only one.) One can then write

$$T' = \lambda T, \tag{5}$$

$$\delta_T' = \delta_T, \tag{6}$$

where $\lambda = V_{us}/V_{ud} \approx 0.2$.

The validity of these assumptions will be discussed later in this Letter.

One then has a system with 8 real equations and 8 unknown quantities. After a simple algebra to eliminate unwanted quantities, one can reduce the set of 8 equations to a simpler system containing the desired variable γ:

$$\frac{e^{i(\delta_T - \gamma)} - e^{i(\delta_T + \gamma + 2\beta)}}{|e^{i(\delta_T + \gamma)} - e^{i(\delta_T - \gamma)}|} = \frac{\lambda(A_2 - A_1 e^{i2\beta})}{|A_3 - A_4|} \tag{7}$$

$$\arg(A_3 - A_4) = \arg[e^{i(\delta_T + \gamma)} - e^{i(\delta_T - \gamma)}]. \tag{8}$$

This system, containing 3 real equations, allows us to obtain γ, β, and δ_T. At this stage, one could expect this method to provide not only a measurement of γ but also an independent measurement of β. Unfortunately, this is true only if Eqs. (7) and (8) were exact. This is not the case due to the theoretical assumptions made above. The question is how does any error in these equations propagate to the actual extraction of γ and β.

We found that β is too sensitive to small uncertainties in these equations. This is simply due to the fact that β is present only in Eqs. (1) and (2), in a term which is small. Thus, small uncertainties in coefficients $A_{1,2}$ imply large uncertainties in β.

As a consequence, the method is not well suited to providing an independent measurement of β. Fortunately, in the next years, the phase β will be known within an error of a few degrees. Our strategy is thus to consider β as a known parameter. We then have two independent variables, γ and δ_T, with only two equations, e.g., real and imaginary parts of Eq. (7) [12].

Let us now study the theoretical errors of the method and their influence in the extraction of γ. The sources of systematic errors are (1) the validity of SU(3) symmetry assumptions and (2) charm penguins.

SU(3) symmetry is assumed only for those terms carrying the weak phase γ, i.e., diagrams included in T. This point represents another important issue of this method: one does not need to make SU(3) assumptions about penguin contributions. The two main contributions to T are tree and color suppressed diagrams. Factorization corrections to exact SU(3) symmetry in both diagrams transform Eq. (5) in

$$T' = \frac{f_K}{f_\pi} \lambda T, \tag{9}$$

where f_K and f_π are the kaon and pion decay constants, respectively. As $f_K/f_\pi \approx 1.2$, this represents a 20% correction with respect to the assumption of exact SU(3) symmetry of Eq. (5). The importance of nonfactorizable corrections to Eq. (9) is not yet clear but they are usually assumed to be small. Both theoretical [13] and experimental [14] educated studies seem to favor nonfactorizable corrections not larger than 10%.

We have studied numerically the influence of this uncertainty when solving Eqs. (7). For this, after changing (5) to (9), we have assumed an extra 10% theoretical uncertainty in the relation between T' and T and studied the propagation of this error in the extraction of γ. More precisely, we have first assumed a given set of values for the various parameters entering the right-hand sides of Eqs. (1)–(4)—including γ. With them, we have *calculated* the quantities A_i, $(i = 1, \ldots, 4)$, using Eqs. (1)–(4) considering explicitly a 10% correction to (9). Then we have solved the system of Eqs. (7)—which assumes that there is no such correction—to find γ. We have finally compared the latter with the originally assumed value for γ.

We found that the amount of this error in extracting γ depends only on the actual value of γ and β, as shown in Fig. 1. This error was found to be independent of the actual values of T, P_t, P', δ_T, δ_{P_t}, and $\delta_{P'}$. Figure 1 shows that the smaller β, the larger the errors. When $\beta \to 0$, the error diverges. This is simply because, when β is exactly zero, Eqs. (1)–(4) with $P_c = 0$, are no longer independent; thus Eqs. (7) admit solutions only when SU(3) symmetry assumption is exact.

The other assumption related to the SU(3) symmetry, i.e., Eq. (6), has no influence in the result. To find this, we proceeded in the same way as described above—this time assuming $\delta'_T \neq \delta_T$ when calculating A_i. We found that the error in finding γ due to this assumption is always negligible, independently of the values of T, P_t, P', δ_T, δ_{P_t}, $\delta_{P'}$, β, and γ.

Let us now consider the error due to charm penguins. They would affect only Eqs. (1) and (2) since in the $B^\pm \to K^\pm\pi^+\pi^-$ decays charm penguins have the same weak phase as top ones and are thus already included in P'. This is a crucial remark, because in the $B^\pm \to \pi^\pm\pi^+\pi^-$ decay P_t/T is expected to be small [2]; thus, as γ is present in the T term, any error in the penguin sector is less important. The actual importance of these neglected terms is in fact unknown at present, but a crude estimate gives [11] $0.2 \leq P_c/P_t \leq 0.5$.

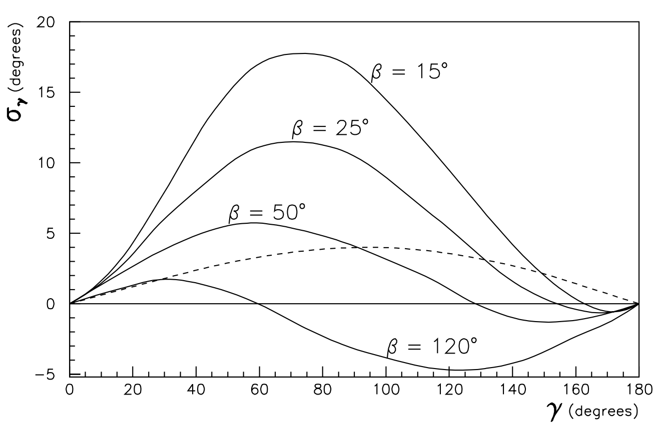

FIG. 1. The error in the extraction of γ as a function of γ. Solid curves are the error due to the uncertainty in the SU(3) assumption, for different values of β. The dashed curve is the error due to charm penguins.

The theoretical uncertainty in the value of this ratio propagates to the actual extraction of γ. To estimate the amount of this error, we made the same kind of numerical analysis as described above to the study of SU(3) symmetry assumptions. In other words, we have calculated the quantities A_i assuming that P_c is not zero, and then solved Eqs. (7)—where P_c is not present. We found that in this case, the error in the extraction of γ does not depend on the actual value of β, but it does depend on the value of γ, as shown in the dashed curve of Fig. 1. As in the SU(3) case, these results are independent of the actual values of δ_T, δ_P, δ'_P, and P'. Nevertheless, the dashed curve in Fig. 1 does depend on the actual value of P_t/T. More precisely, the error is proportional to the value of this ratio. Results presented in Fig. 1 have been obtained using the expected values of $P_t/T = 0.2$ [15], $P_c/P_t = 0.35$ [11], and $\delta_{P_c} = \delta_{P_t}$ [16]. These results do not change if the last assumption is released, and change little if P_c/P_t takes another value within the range 0.2–0.5.

Finally, we have studied how an uncertainty in β propagates to the extraction of γ. Assuming β is known with an error of 3^0—which is the expected error after 4 years of running of BaBar and Belle—this contributes with less than 1^0 in the error to extract γ.

Let us now analyze the total theoretical error of the method. Adding in quadrature all the errors we have discussed above, we conclude the following. If β is 50^0 or more, the total error of the method is always small: 5^0 or less. If β turns out to have the central value estimated by standard model predictions [17], i.e., 25^0 (which is also in reasonable agreement with recent reports from Belle and BaBar [18]), then the error may be larger: if γ is in the range 50^0–80^0, as dictated by an overall analysis of the unitary triangle [17], then the error is of the order of 11^0; if γ is in the range 100^0–130^0, as suggested by experimental constraints based in non-leptonic B decays [19], then the error in extracting γ is of the order of 5^0 to 8^0.

All these values for the theoretical error are smaller than what one expects from other methods to extract γ, proposed so far. This is valid as far as the value of β is larger than 10^0.

It is important to stress that the method is also self-contained for the determination of its systematic error. Indeed, once the actual values of γ and P_t/T are determined from the experiment, the systematic error can easily be read from the numerical study made above.

Our last comment deals with the experimental feasibility of this procedure. The channels involved in the analysis require only the measurement of charged pions and kaons and do not require the necessity of tagging (since the method deals only with charged B meson decays). In Ref. [6] we made a simulation of $B^\pm \to \pi^\pm\pi^+\pi^-$ decays; we showed that the extraction of partial amplitudes—i.e., what we call A_i, $(i = 1, \ldots, 4)$ in this Letter—could be done, with relatively small statistical errors, with about 1000 reconstructed events in each Dalitz plot. Recent CLEO data [20] found that $B^{\pm} \rightarrow \mathcal{P}^{\pm}\rho^0$ (with $\mathcal{P} = K, \pi$) have branching ratios of the order of 10^{-5}. Thus, assuming a total branching ratio of the decays $B^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ and $B^{\pm} \rightarrow K^{\pm}\pi^{+}\pi^{-}$ of the order of 2×10^{-5} and a 60% reconstruction efficiency [1], this method may provide a good measurement of γ after about 4 years of running of the first generation of B machines.

In summary, we have presented in this Letter a method to measure the weak CKM angle γ which deals with all strong and electroweak penguin amplitudes. It exploits a general procedure to extract weak angles using Dalitz plot analysis in three body B meson decays. Here, we discuss a combined study of two pairs of CP conjugated decays, $B^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ and $B^{\pm} \rightarrow K^{\pm}\pi^{+}\pi^{-}$ which allows the treatment of penguin amplitudes. Moreover, tree and penguin magnitudes and strong phases are obtained from the experimental procedure. The method brings a measurement of γ free from final state interaction problems.

The procedure has two theoretical sources of uncertainty. First, it is based in SU(3) symmetry assumptions, even though only for the tree sector, not in the penguin one. Second, charm penguins can be included only in the method by considering a model to estimate P_c/P_t. According to the present knowledge, charm penguins would introduce a small systematic uncertainty. Combining both sources of error, the total theoretical error of the method presented in this Letter is not larger than 5^0 if β is larger than 50^0; if β has the expected value of 25^0, then the error is of the order of 5^0–8^0 if γ turns out to be in the second quadrant and of the order of 11^0 if it is in the first one. Thus, one expects the theoretical error of this method to be smaller than that of other procedures proposed so far to extract γ.

The method presented here should be experimentally feasible at BaBar and KEK. It could then provide the best knowledge of γ in the next years. In any case, this method is based on a new idea and a different experimental procedure, if compared to usual methods based in two body decays. It will then bring an independent measurement of γ.

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