Environment-Independent Decoherence Rate in Classically Chaotic Systems

Rodolfo A. Jalabert[1] and Horacio M. Pastawski[1,2]
[1]*Université Louis Pasteur, IPCMS-GEMME, 23 rue du Loess, 67037 Strasbourg Cedex, France*
[2]*Facultad de Matemática, Astronomía y Física, Universidad Nacional de Córdoba, Ciudad Universitaria, 5000 Córdoba, Argentina*
(Received 5 October 2000)

We study the decoherence of a one-particle system, whose classical correspondent is chaotic, when it evolves coupled to a weak quenched environment. This is done by analytical evaluation of the Loschmidt echo (i.e., the revival of a localized density excitation upon reversal of its time evolution), in the presence of the perturbation. We predict an exponential decay for the Loschmidt echo with a (decoherence) rate which is asymptotically given by the mean Lyapunov exponent of the classical system, and therefore independent of the perturbation strength, within a given range of strengths. Our results are consistent with recent experiments of polarization echoes in nuclear magnetic resonance and numerical simulations.

DOI: 10.1103/PhysRevLett.86.2490 PACS numbers: 03.65.Yz, 03.65.Sq, 03.67.–a, 05.45.Mt

The coupling of a system to environmental degrees of freedom plays an important role in many areas of physics. Already on a classical level, it leads to fluctuations, damping, and irreversibility. In quantum mechanics, the environmental coupling induces decoherence, destroying quantum superpositions and reducing pure states to a mixture of states [1]. It is then not surprising that the concepts of environment, decoherence, and irreversibility have been the object of scholar discussions for a long time [2]. Renewed interest has been fostered by the crucial role that decoherence plays in the problem of quantum computation [3] and by the technical advances that make it possible to perform experiments envisioned as *gedanken*.

Experiments with Rydberg atoms in a microwave cavity [4] allow one to observe the progressive decoherence in a quantum measurement problem, while analysis of conductances through semiconductor microstructures [5] make it possible to address the "which path" problem in a solid state environment. In addition, nuclear magnetic resonance (NMR) offers unlimited possibilities for the study of decoherence and irreversibility in a tailored environment. The phenomenon of *spin echo* shows how an *individual spin,* in an ensemble, loses its "phase memory" [6] as a consequence of the interaction with other spins that act as an "environment." The failure of recovering the initial ordered state in a time scale T_2 manifests the effect of a many-spin environment on the reversibility of simple systems.

A conceptual breakthrough was enabled by experiments that revert [7] and control [8] the *whole* entangled *state* of the strongly interacting nuclear spins to obtain the NMR *polarization echo.* A local spin excitation $|\psi\rangle$ created at time $t = 0$ spreads out through the crystal under the action of a *many-spin* Hamiltonian $\mathcal{H}_0$ allowing exchange between spins. This complex quantum evolution is macroscopically assimilated to a "spin-diffusion" [9] process (consistently with the usual hypothesis of microscopic chaos describing many particle systems [10]). At time t, a radio-frequency pulse sequence produces a new effective Hamiltonian $-(\mathcal{H}_0 + \Sigma)$. Here Σ, a perturbation containing the pulse imperfections and residual interactions with additional spins, can be made very small [8]. Hence, the pulse at t implements the *gedanken* backwards dynamics proposed by Loschmidt in his argument against the Boltzmann's H theorem. At time $2t$, one measures a maximum in the return probability, that we call Loschmidt echo (LE):

$$M(t) = |\langle\psi|e^{i(\mathcal{H}_0+\Sigma)t/\hbar}e^{-i\mathcal{H}_0 t/\hbar}|\psi\rangle|^2. \tag{1}$$

The buildup of the LE depends on a precise interference between the "diffusive" wave packets $e^{-i\mathcal{H}_0 t/\hbar}|\psi\rangle$ and $e^{-i(\mathcal{H}_0+\Sigma)t/\hbar}|\psi\rangle$, which is degraded by Σ. Clearly, $M(t)$ should be a decreasing function of the elapsed time t before the reversal of $\mathcal{H}_0$, with a decoherence rate $1/\tau_\phi < 1/T_2$. A surprising outcome of the experiment [8] is that, for small Σ's, $1/\tau_\phi$ *depends only on the intrinsic properties of the system* (that is, on $\mathcal{H}_0$).

In this work, we develop a simple analytical model exhibiting the independence of the decoherence rate on the perturbation found in the experiment. The system is represented by a *single-particle Hamiltonian* $\mathcal{H}_0$ whose underlying classical dynamics is strongly chaotic. This is clearly an oversimplification with respect to the many-body Hamiltonian of interest, but still it introduces enough complexity in the intrinsic system (quantified by its mean Lyapunov exponent λ) which is absent in simpler dissipative systems, where $\mathcal{H}_0$ is integrable. Placing ourselves between the limits of a trivial and a many-body $\mathcal{H}_0$ allows us to have a tractable model and explore the influence of classical chaos in quantum dynamics. To account for the "noninverted" part of the Hamiltonian evolution, we consider a Hermitian operator Σ representing the coupling with a quenched environment acting in the backward evolution (from t to $2t$). This approach is not only consistent with the experimental situation but it is also able to provide a new insight into the problem of decoherence because the calculation can be handled within the precise framework of the Schrödinger equation. In contrast, most of the previous studies of decoherence

 0031-9007/01/86(12)/2490(4)$15.00

use extremely simple Hamiltonian systems [1] interacting with a dissipative environment (e.g., stochastic noise), which justifies the use of a master equation for the reduced density matrix. In this context, the entropy growth of a dissipative system hinted [11] at the importance of the chaotic classical dynamics in setting the characteristic time scales for decoherence. This is consistent, under conditions that we specify below, with our results for the time decay of $M(t)$. However, since we use a purely Hamiltonian approach, our conceptual framework is very different.

We start with a localized state in a d-dimensional space,

$$\psi(\overline{\mathbf{r}}; t = 0) = \left(\frac{1}{\pi\sigma^2}\right)^{d/4} \times \exp\left[i\mathbf{p}_0 \cdot (\overline{\mathbf{r}} - \mathbf{r}_0) - \frac{1}{2\sigma^2}(\overline{\mathbf{r}} - \mathbf{r}_0)^2\right], \tag{2}$$

centered at $\mathbf{r}_0$ with dispersion σ. The momentum $\mathbf{p}_0$ selects the energy range of the excitation. This choice also renders the calculations tractable. The time evolution of the initial state is best described using the propagator $K(\mathbf{r}, \overline{\mathbf{r}}; t) = \langle \mathbf{r} | e^{-i\mathcal{H}t/\hbar} | \overline{\mathbf{r}} \rangle$ by

$$\psi(\mathbf{r}; t) = \int d\overline{\mathbf{r}}\, K(\mathbf{r}, \overline{\mathbf{r}}; t)\psi(\overline{\mathbf{r}}; 0). \tag{3}$$

Using the Hamiltonian $\mathcal{H}_0 + \Sigma$ or $\mathcal{H}_0$ in the propagator K yields $\psi_{\mathcal{H}_0+\Sigma}$ or $\psi_{\mathcal{H}_0}$, respectively. We take Σ as a static disordered potential given by N_i impurities with a Gaussian potential with a correlation length ξ:

$$\Sigma = \tilde{V}(\mathbf{r}) = \sum_{\alpha-1}^{N_i} \frac{u_\alpha}{(2\pi\xi^2)^{d/2}} \exp\left[-\frac{1}{2\xi^2}(\mathbf{r} - \mathbf{R}_\alpha)^2\right]. \tag{4}$$

The independent impurities are uniformly distributed with density $n_i = N_i/V$, (V is the sample volume). The strengths u_α obey $\langle u_\alpha u_\beta \rangle = u^2 \delta_{\alpha\beta}$. This assumption simplifies analytical evaluation of the ensemble average of the observable $M(t)$. We stress that we are not simply describing the physics of disordered systems (which is obviously phase coherent), since the potential $\tilde{V}(\mathbf{r})$ acts in the backwards propagation but not in the forward path.

We use the semiclassical approximation for $K(\mathbf{r}, \overline{\mathbf{r}}; t)$, as the sum over all the classical trajectories $s(\mathbf{r}, \overline{\mathbf{r}}; t)$ joining the points $\overline{\mathbf{r}}$ and $\mathbf{r}$ in a time t [12]:

$$K(\mathbf{r}, \overline{\mathbf{r}}; t) = \sum_{s(\mathbf{r}, \overline{\mathbf{r}}; t)} K_s(\mathbf{r}, \overline{\mathbf{r}}; t), \qquad \text{with}$$
$$K_s(\mathbf{r}, \overline{\mathbf{r}}; t) = \left(\frac{1}{2\pi i\hbar}\right)^{d/2} \times C_s^{1/2} \exp\left[\frac{1}{\hbar} S_s(\mathbf{r}, \overline{\mathbf{r}}; t) - \frac{i\pi}{2}\mu_s\right], \tag{5}$$

valid in the limit of large energies for which the de Broglie wavelength ($\lambda_F = 2\pi/k = 2\pi\hbar/p_0$) is the minimal length scale. S is the action specified by the integral of the Lagrangian $S_s(\mathbf{r}, \overline{\mathbf{r}}; t) = \int_0^t d\overline{t}\, \mathcal{L}$ along the classical path, μ is the Maslov index that counts the number of conjugate points, and the Jacobian $C_s = |\det \mathcal{B}_s|$, accounting for the conservation of the classical probability, is expressed in terms of the initial and final position components j and i as $(\mathcal{B}_s)_{ij} = -\partial^2 S_s/\partial r_i \partial \overline{r}_j$. This approximation gives the wave function with great accuracy up to very long times [13]. Besides, it provides the leading-order corrections in $\hbar$ due to $\tilde{V}(\mathbf{r})$, in the limit of $k\xi \gg 1$, from the classical perturbation theory for the actions [14].

Using Eqs. (2) and (5), we can write

$$\psi(\mathbf{r}; t) = (4\pi\sigma^2)^{d/4} \sum_{s(\mathbf{r}, \mathbf{r}_0; t)} K_s(\mathbf{r}, \mathbf{r}_0; t) \times \exp\left[-\frac{\sigma^2}{2\hbar^2}(\overline{\mathbf{p}}_s - \mathbf{p}_0)^2\right], \tag{6}$$

where we used $\partial S/\partial \overline{r}_i|_{\overline{\mathbf{r}} = \mathbf{r}_0} = -\overline{p}_i$, and neglected the second order terms of S in $(\overline{\mathbf{r}} - \mathbf{r}_0)$. This is justified under the assumption that $\xi \gg \sigma \gg \lambda_F$, i.e., an initial wave packet concentrated in a smaller scale than the fluctuations of $\tilde{V}(\mathbf{r})$. In Eq. (6), only trajectories with initial momentum $\overline{\mathbf{p}}_s$ closer than $\hbar/\sigma$ to $\mathbf{p}_0$ are relevant for the propagation of the wave packet.

The semiclassical approximation to the LE is

$$M(t) = \left(\frac{\sigma^2}{\pi\hbar^2}\right)^d \left| \int d\mathbf{r} \sum_{s,\tilde{s}} C_s^{1/2} C_{\tilde{s}}^{1/2} \exp\left[\frac{i}{\hbar}(S_s - S_{\tilde{s}}) - \frac{i\pi}{2}(\mu_s - \mu_{\tilde{s}})\right] \exp\left[-\frac{\sigma^2}{2\hbar^2}[(\overline{\mathbf{p}}_s - \mathbf{p}_0)^2 + (\overline{\mathbf{p}}_{\tilde{s}} - \mathbf{p}_0)^2]\right] \right|^2, \tag{7}$$

and involves two spatial integrations and four trajectories. The integration requires the change from final position variable $\mathbf{r}$ to initial momentum $\overline{\mathbf{p}}$ using the Jacobian C.

The perfect echo of $\Sigma = 0$ is already obtained considering only trajectories $s = \tilde{s}$, which leaves aside terms with a highly oscillating phase:

$$M_{\Sigma=0}(t) = \left(\frac{\sigma^2}{\pi\hbar^2}\right)^d \left| \int d\mathbf{r} \sum_s C_s \times \exp\left[-\frac{\sigma^2}{\hbar^2}(\overline{\mathbf{p}}_s - \mathbf{p}_0)^2\right] \right|^2 = 1. \tag{8}$$

In the coupled case, the square modulus requires a second integration variable $\mathbf{r}'$. We see that only the terms with slightly perturbed trajectories $s = \tilde{s}$ (as well as $s' = \tilde{s}'$) survive the average over impurities. Thus,

$$M(t) \simeq \left(\frac{\sigma^2}{\pi\hbar^2}\right)^d \int d\mathbf{r} \int d\mathbf{r}' \sum_{s,s'} C_s C_{s'} \times \exp\left[\frac{i}{\hbar}(\Delta S_s - \Delta S_{s'})\right] \times \exp\left[-\frac{\sigma^2}{\hbar^2}[(\overline{\mathbf{p}}_s - \mathbf{p}_0)^2 + (\overline{\mathbf{p}}_{s'} - \mathbf{p}_0)^2]\right], \quad (9)$$

where $\Delta S_s = -\int_0^t d\bar{t}\, \tilde{V}[\mathbf{q}_s(t)]$ and $\Delta S_{s'}$ are the phase differences, along the trajectories s and s', resulting from the perturbation $\tilde{V}$. We can decompose M into

$$M(t) = M^{nd}(t) + M^d(t), \quad (10)$$

where the first term (nondiagonal) contains trajectories s and s' exploring different regions of phase space, while in the second (diagonal) s' remains close to s.

In the *nondiagonal* term, the impurity average can be done independently for s and s'. For trajectories longer than ξ, the phase accumulation ΔS_s results from uncorrelated contributions, and therefore can be assumed to be Gaussian distributed [14]. The disorder contribution involved in Eq. (9) is then given by

$$\left\langle \exp\left[\frac{i}{\hbar}\Delta S_s\right]\right\rangle = \exp\left[-\frac{1}{2\hbar^2}\int_0^t d\bar{t}\int_0^t d\bar{t}' \times C_{\tilde{V}}[|q_s(\bar{t}) - q_s(\bar{t}')|]\right], \quad (11)$$

where the correlation function is

$$C_{\tilde{V}}(|\mathbf{q} - \mathbf{q}'|) = \langle \tilde{V}(\mathbf{q})\tilde{V}(\mathbf{q}')\rangle = \frac{u^2 n_i}{(4\pi\xi^2)^{d/2}} \exp\left[-\frac{1}{4\xi^2}(\mathbf{q} - \mathbf{q}')^2\right]. \quad (12)$$

The change of variables $q = v\bar{t}$ and $q' = v\bar{t}'$ yields two integrals along the trajectory s. Since the length L_s of the trajectory is supposed to be much larger than ξ, the integral over $q - q'$ can be taken from $-\infty$ to $+\infty$, while the integral on $(q + q')/2$ gives a factor of L_s. Assuming that the velocity along the trajectory remains almost unchanged with respect to its initial value $v_0 = p_0/m = L_s/t$, one gets

$$M^{nd}(t) \simeq \left(\frac{\sigma^2}{\pi\hbar^2}\right)^d \left|\int d\mathbf{r} \sum_s C_s \exp\left[-\frac{\sigma^2}{\hbar^2}(\overline{\mathbf{p}}_s - \mathbf{p}_0)^2\right] \times \exp\left[-\frac{L_s}{2l}\right]\right|^2 \simeq \exp[-tv_0/\tilde{l}]. \quad (13)$$

In analogy with disordered systems [14], the typical length over which the quantum phase is modified is

$$\tilde{l} = \hbar^2 v_0^2 \left(\int dq\, C(\mathbf{q})\right)^{-1} = \frac{4\sqrt{\pi}\,\hbar^2 v_0^2 \xi}{u^2 n_i}. \quad (14)$$

We then see that $M^{nd}(t)$ has its time scale determined by Σ (through $\tilde{l}$).

In computing the *diagonal* term $M^d(t)$, we use

$$\Delta S_s - \Delta S_{s'} = \int_0^t d\bar{t}\, \nabla\tilde{V}[\mathbf{q}_s(\bar{t})] \cdot [\mathbf{q}_s(\bar{t}) - \mathbf{q}_{s'}(\bar{t})], \quad (15)$$

since the trajectories s and s' remain close to each other. The difference between the intermediate points of both trajectories can be expressed using $\mathcal{B}$:

$$\mathbf{q}_s(\bar{t}) - \mathbf{q}_{s'}(\bar{t}) = \mathcal{B}^{-1}(\bar{t})(\overline{\mathbf{p}}_s - \overline{\mathbf{p}}_{s'}) = \mathcal{B}^{-1}(\bar{t})\mathcal{B}(t)(\mathbf{r} - \mathbf{r}'). \quad (16)$$

In the chaotic case, the behavior of $\mathcal{B}^{-1}(\bar{t})$ is dominated by the largest eigenvalue $e^{\lambda\bar{t}}$. Therefore we make the simplification $\mathcal{B}^{-1}(\bar{t})\mathcal{B}(t) = \exp[\lambda(\bar{t} - t)]I$, where I is the unit matrix, and λ is the mean Lyapunov exponent. Here, we use our hypothesis of strong chaos which excludes marginally stable regions [15] with anomalous time behavior. Assuming a Gaussian distribution for the extra phase,

$$\left\langle \exp\left[\frac{i}{\hbar}(\Delta S_s - \Delta S_{s'})\right]\right\rangle = \exp\left[-\frac{1}{2\hbar^2}\int_0^t d\bar{t}\int_0^t d\bar{t}' \exp[\lambda(\bar{t} + \bar{t}' - 2t)] C_{\nabla\tilde{V}}[|\mathbf{q}_s(\bar{t}) - \mathbf{q}_s(\bar{t}')|](\mathbf{r} - \mathbf{r}')^2\right]. \quad (17)$$

We are now led to consider the "force correlator":

$$C_{\nabla\tilde{V}}(|\mathbf{q} - \mathbf{q}'|) = \langle \nabla\tilde{V}(\mathbf{q}) \cdot \nabla\tilde{V}(\mathbf{q}')\rangle = \frac{u^2 n_i}{(4\pi\xi^2)^{d/2}}\left[\frac{d}{2} - \left(\frac{\mathbf{q} - \mathbf{q}'}{2\xi}\right)^2\right] \exp\left[-\frac{1}{4\xi^2}(\mathbf{q} - \mathbf{q}')^2\right]. \quad (18)$$

We change from the variables $\bar{t}$ and $\bar{t}'$ to the coordinates q and q' along the trajectory s and use the fact that $C_{\nabla\tilde{V}}$ is short-ranged (in the scale of ξ) to write

$$M^d(t) \simeq \left(\frac{\sigma^2}{\pi\hbar^2}\right)^d \int d\mathbf{r} \int d\mathbf{r}' \sum_s C_s^2 \exp\left[-\frac{2\sigma^2}{\hbar^2}(\overline{\mathbf{p}}_s - \mathbf{p}_0)^2\right] \exp\left[-\frac{A}{2\hbar^2}(\mathbf{r} - \mathbf{r}')^2\right] \simeq \left(\frac{\sigma^2}{\pi\hbar^2}\right)^d \int d\mathbf{r} \sum_s C_s^2 \left(\frac{2\pi h^2}{A}\right) \exp\left[-\frac{2\sigma^2}{\hbar^2}(\overline{\mathbf{p}}_s - \mathbf{p}_0)^2\right], \quad (19)$$

where $A = (d - 1)u^2 n_i/[4\lambda v_0 (4\pi\xi^2)^{(d-1)/2}]$ results from the $\bar{t}$ and $\bar{t}'$ integrations of Eq. (11) in the limit $\lambda t \gg 1$. The last line comes from Gaussian integration over $(\mathbf{r} - \mathbf{r}')$. The factor C_s^2 reduces to C_s when we make the change of variables from $\mathbf{r}$ to $\mathbf{p}$. In the long-time limit $C_s^{-1} \propto e^{\lambda t}$, while for short times $C_s^{-1} = t/m$. Using a form that interpolates between these two limits, we have

$$M^d(t) \simeq \left(\frac{\sigma^2}{\pi\hbar^2}\right)^d \int d\overline{\mathbf{p}}\left(\frac{2\pi\hbar^2}{A}\right)^{d/2}\left\{\frac{m}{t}\exp[-\lambda t]\right\} \times \exp\left[-\frac{2\sigma^2}{\hbar^2}(\overline{\mathbf{p}}-\mathbf{p}_0)^2\right] \simeq \overline{A}\exp[-\lambda t], \tag{20}$$

with $\overline{A} = m/(A^{d/2}t)$. Since the integral over $\overline{\mathbf{p}}$ is concentrated around $\mathbf{p}_0$, the exponent λ is considered constant. The coupling Σ appears only in the prefactor (through $\overline{A}$), and therefore its detailed description is not crucial in discussing the time dependence of M^d. The t factor in $\overline{A}$ induces a divergence for small t. However, our calculations are valid only in the limit $\lambda t \gg 1$. Long times (of the order of the Ehrenfest time $t_E = \lambda^{-1}\ln[ka]$, where a is a length characterizing $\mathcal{H}_0$), are also excluded since the diagonal approximation would fail.

Our semiclassical approach made it possible to estimate the two contributions to $M(t)$. The nondiagonal component $M^{nd}(t)$ is the dominant one for small Σ. In particular, it makes $M_{\Sigma=0}(t) = 1$ in Eq. (8). The small values of Σ are not properly treated in the semiclassical calculation of the diagonal term $M^d(t)$. While increasing Σ, the crossover from M^{nd} to M^d is achieved when $\tilde{l}$ becomes smaller than v_0/λ. This condition is compatible with the assumption that, in the limit $k\xi \gg 1$, classical trajectories shorter than the perturbation's "transport mean-free path" $\tilde{l}_{tr} = 4(k\xi)^2\tilde{l}$ are not affected [14] by Σ. For strong Σ, the perturbative treatment of the actions breaks down. We can now establish our main conclusion.

In a system that classically exhibits strong chaos and can be characterized by a mean Lyapunov exponent λ, a small random static perturbation destroys our control of the quantum phase at a rate

$$\frac{1}{\tau_\phi} = -\lim_{t\to\infty}\frac{1}{t}\ln M(t) = \lambda, \tag{21}$$

provided that $\lambda^{-1} \ll t \ll t_E$, and the perturbation presents long-range potential fluctuations ($k\xi \gg 1$) and a strength quantically strong ($\tilde{l} \ll v_0/\lambda$) but classically weak ($v_0/\lambda \ll \tilde{l}_{tr}$). Notice that the thermodynamic limit, $V \to \infty$, is required to take t arbitrarily large.

These various restrictions provide stringent conditions for its numerical verification. Preliminary results [16] in an Anderson model perturbed by a magnetic flux (environment), though subject to finite size limitations, show an environment-independent behavior for the decay of $M(t)$ provided that the perturbation exceeds a critical value [17]. We expect that this generic behavior is robust when considering Hamiltonians with a larger complexity, such as the many-particle case.

The field of quantum chaos deals with signatures of the classical chaos on quantum properties, such as spectral correlations [18], wave function scars [19], and parametric correlations [17]. In contrast, the studies of the temporal domain have been less developed, mainly because of the lack of clear quantities as those of steady state [20]. A partial success was Ref. [21] which distinguished regular and irregular dynamics on the basis of the asymptotic properties of a perturbation dependent overlap. While some form of dynamical sensitivity to perturbations was expected [22], we are not aware of other predictions of a manifestation of the classical Lyapunov exponent in a *Hamiltonian system.* In view of Eq. (21), the issue of decoherence by quantum evolution in classically chaotic systems, both with strong chaos and with marginal stability, deserves a more thorough examination. Studies with other analytical and numerical techniques should clarify, among other aspects, the effects of different specific perturbations, the subtle effects of thermodynamic limits, the corrections due to Anderson localization, and the different temporal laws observed in one-body and many-body systems. This understanding of dynamical manifestations of chaos in the quantum world is decisive in the efforts to limit the experimental effects of decoherence and irreversibility.

We benefited from discussions with O. Bohigas, P. Leboeuf, C. Lewenkopf, J. P. Paz, and K. Richter and the support of ECOS-Sud, ANPCyT and SeCyT-UNC. H. M. P. is affiliated with CONICET.

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