Enhancement of Tunneling from a Correlated 2D Electron System by a Many-Electron Mössbauer-Type Recoil in a Magnetic Field

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We consider the effect of electron correlations on tunneling from a 2D electron layer in a magnetic field parallel to the layer. A tunneling electron can exchange its momentum with other electrons, which leads to an exponential increase of the tunneling rate compared to the single-electron approximation. The effect depends on the interrelation between the *dynamics* of tunneling and momentum exchange. The results explain and provide a no-parameter fit to the data on electrons on helium. We also discuss tunneling in semiconductor heterostructures.

Low density two-dimensional electron systems (2DESs) in semiconductor heterostructures and on liquid helium are among the most ideal many-electron systems. Such systems display strong effects of the electron-electron interaction, including those specifically related to electron correlations [1,2]. They show up dramatically in various unusual transport properties. One of the most broadly used techniques for investigating many-electron effects is tunneling [3], a recent example being the observation [4] of the giant increase of interlayer tunneling in double-layer heterostructures, apparently related to the onset of interlayer correlations.

For electrons on helium, an exponentially strong deviation from the single-electron rate of tunneling transverse to a magnetic field has been known experimentally since 1993 [5], but remained unexplained. Such a field couples the tunneling motion away from the 2DES to the in-plane degrees of freedom. The effect of the field and the role of electron correlations cannot be described by a simple phenomenological tunneling Hamiltonian.

In this paper we provide a theory of tunneling from a correlated 2DES in a magnetic field **B** parallel to the electron layer. We show, using the model of a Wigner crystal (WC), that the tunneling is affected by the interelectron momentum exchange and its *dynamics*, which is largely determined by short-range order. We discuss tunneling from 2DESs on helium and in single quantum well heterostructures. The results explain and give a no-parameter fit to the experimental data [5], see Fig. 1. They suggest new types of experiments which involve tunneling through broad barriers and will be sensitive to short-range order in a 2DES.

Electron correlations change the tunneling rate by effectively decreasing the single-electron magnetic barrier. This barrier emerges because, when an electron tunnels from the layer (in the z direction), it acquires an in-plane Hall velocity $v_H = \omega_c z$ in the $\mathbf{B} \times \hat{z}$ direction and the corresponding in-plane kinetic energy $m\omega_c^2 z^2/2$, where $\omega_c = eB/mc$ is the cyclotron frequency. Respectively, the

energy for motion along the z axis is decreased, or the tunneling barrier is increased by $m\omega_c^2 z^2/2$.

In a correlated 2DES, the tunneling electron exchanges its Hall momentum with other electrons, thus decreasing the energy loss [6]. This is somewhat similar to the Mössbauer effect, where the momentum of a gamma quantum is given to the crystal as a whole [7]. In our case, the effect is very sensitive to the electron dynamics. If the rate of the interelectron momentum exchange ω_p exceeds the reciprocal duration of underbarrier motion in imaginary time τ_f^{-1} and ω_c, then in-plane velocities of all electrons are nearly the same, and the Hall velocity is $v_H \propto 1/N \to 0$ (N is the number of electrons). In this adiabatic limit the effect of the magnetic field on tunneling is fully compensated. For $\omega_p \tau_f \sim 1$ a part of the tunneling energy goes
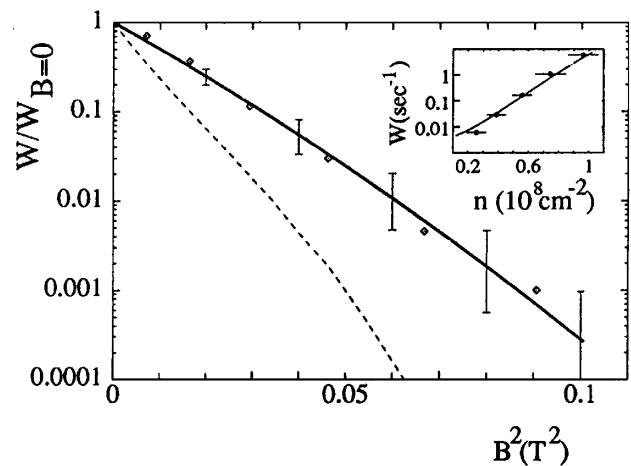

FIG. 1. The rate of electron tunneling from helium surface $W(B)$ as a function of the magnetic field B for the electron density $n = 0.8 \times 10^8$ cm^{-2} and the calculated pulling field $\mathcal{E}_\perp = 24.7$ V/cm (solid curve). Diamonds show the experimental data [5]. The error bars correspond to the uncertainty of the experimental parameters. The dotted curve is the calculation [5] for $T = 0.04$ K without interelectron momentum exchange. Inset: comparison of the present theory for $B = 0$ with the experimentally measured density dependence of the tunneling rate.

to WC phonons, yet the B-induced suppression of tunneling is largely reduced.

In a strongly correlated system, where the electron wave functions overlap only weakly, one can "identify" the tunneling electron. Its out-of-plane motion for $B = 0$ is described by the Hamiltonian

$$H_0 = \frac{p_z^2}{2m} + U(z). \qquad (1)$$

The potential $U(z)$ has a well in which the electron occupies the ground state, with energy E_g. The well is separated by a tunneling barrier from extended states with a quasicontinuous spectrum, cf. Fig. 2(b). We assume that the tunneling length L is much less than the average interelectron distance $\sim n^{-1/2}$, where n is the electron density. Then small-amplitude in-plane electron vibrations about lattice sites are only weakly coupled to tunneling for $B = 0$ [8]. We neglect this coupling.

A magnetic field parallel to the electron layer mixes up the in-plane and out-of-plane motions. The Hamiltonian of the tunneling electron and phonons of the WC is $H = H_0 + H_\nu + H_B$ with

$$H_\nu = \frac{1}{2} \sum_{\mathbf{k},j} [m^{-1} \mathbf{p}_{\mathbf{k}j} \mathbf{p}_{-\mathbf{k}j} + m\omega_{\mathbf{k}j}^2 \mathbf{u}_{\mathbf{k}j} \mathbf{u}_{-\mathbf{k}j}] \qquad (2)$$

and

$$H_B = \frac{1}{2} m\omega_c^2 z^2 - \omega_c z N^{-1/2} \sum_{\mathbf{k},j} [\hat{\mathbf{B}} \times \mathbf{p}_{\mathbf{k}j}]_z. \qquad (3)$$

Here, $\mathbf{p}_{\mathbf{k}j}$, $\mathbf{u}_{\mathbf{k}j}$, and $\omega_{\mathbf{k}j}$ are the momenta, displacements, and frequencies of the normal modes of the 2D Wigner crystal with the wave vector $\mathbf{k}$, respectively ($j = 1, 2$). We assumed that the equilibrium in-plane position of the tunneling electron is at the origin. Then its in-plane momentum is $\mathbf{p} = N^{-1/2} \sum \mathbf{p}_{\mathbf{k}j}$.

The Hamiltonian H_B couples the out-of-plane motion to lattice vibrations. The problem of many-electron tunneling is thus mapped onto a familiar problem of a particle coupled to a bath of harmonic oscillators [9,10], with the coupling strength controlled by the magnetic field. However, there are two distinctions from the standard formulation. First, the coupling mixes together the particle *coordinate* z and the *momenta* of the lattice. These two quantities have different symmetry with respect to time inversion. Because of broken time-reversal symmetry, the general problem of tunneling in a 3D potential in a magnetic field requires a special approach, which was developed earlier for an isolated particle [11]. For the present model, the problem is simplified by the fact that in-plane motion is harmonic vibrations and the coupling is independent of $\mathbf{u}_{\mathbf{k}j}$ [10].

The second distinction arises because, for 2DES, the potential well $U(z)$ is strongly nonparabolic near the minimum (cf. Fig. 2). As a result, the standard instanton technique [12] does not apply [13].

We will evaluate the tunneling rate in the WKB approximation. In the presence of a magnetic field it is convenient to look for the WKB wave function under and behind the barrier in the momentum representation with respect to phonon variables,

$$\psi(z, \{\mathbf{p}_{\mathbf{k}j}\}) = \exp[iS(z, \{\mathbf{p}_{\mathbf{k}j}\})], \qquad \hbar = 1, \qquad (4)$$

and make a canonical transformation so that $\mathbf{p}_{\mathbf{k}j}$ and $-\mathbf{u}_{\mathbf{k}j}$ become new canonical coordinates and momenta.

To the lowest order in $\hbar$, the action S in (4) can be obtained from the Hamiltonian equations for the trajectories of the system,

$$\dot{S} = p_z \dot{z} - \sum_{\mathbf{k}j} \mathbf{u}_{\mathbf{k}j} \dot{\mathbf{p}}_{\mathbf{k}j}, \qquad \dot{z} = \frac{\partial H}{\partial p_z}, \qquad \dot{p}_z = -\frac{\partial H}{\partial z},$$

$$\dot{\mathbf{u}}_{\mathbf{k}j} = \frac{\partial H}{\partial \mathbf{p}_{\mathbf{k}j}}, \qquad \dot{\mathbf{p}}_{\mathbf{k}j} = -\frac{\partial H}{\partial \mathbf{u}_{\mathbf{k}j}}. \qquad (5)$$

In the $(z, \{\mathbf{p}_{\mathbf{k}j}\})$ representation, the Hamiltonian equations (5) have time-reversal symmetry. This allows us to solve them under the barrier in a standard way [12] by keeping the coordinates z, $\mathbf{p}_{\mathbf{k}j}$ real and making the momenta p_z, $-\mathbf{u}_{\mathbf{k}j}$, time $t = -i\tau$, and action $S(z, \{\mathbf{p}_{\mathbf{k}j}\}) = iS_E(z, \{\mathbf{p}_{\mathbf{k}j}\})$ purely imaginary.

The Euclidean action $S_E(\tau)$ as a function of time is evaluated along a multidimensional trajectory (5) that goes under the barrier from the potential well to the boundary of the region which is classically allowed to both the tunneling electron and the WC vibrations. At this boundary, one has to match the underbarrier solution (with imaginary momenta) with the WKB solution behind the barrier (with real momenta). Therefore, for smooth $U(z)$,

$$p_z(\tau_f) = 0, \qquad \mathbf{u}_{\mathbf{k}j}(\tau_f) = \mathbf{0}, \qquad (6)$$

where τ_f is the imaginary time at which the boundary is reached.

We now discuss the initial conditions for the trajectories (5). Typically, the characteristic intrawell localization length $1/\gamma$ in the potential $U(z)$ is small compared to the tunneling length L. For large $\gamma L \gg 1$, the magnetic field may have a strong cumulative effect on the tunneling rate, even where it only weakly perturbs the intrawell motion.
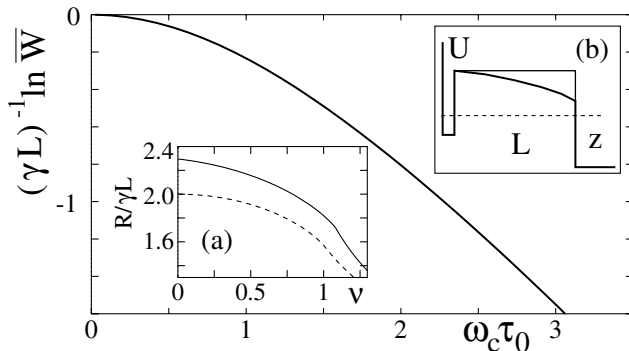

FIG. 2. Relative rate of tunneling $\bar{W} = W(B)/W(0)$ vs magnetic field for a 2D WC in a semiconductor heterostructure, with $\overline{\omega}\tau_0 = 0.5$. Inset (a): the tunneling exponent R vs $\nu = \sqrt{2}\,\overline{\omega}\tau_0$ for $\omega_c\tau_0 = 1.0$ (solid line) and $B = 0$ (dashed line). Inset (b): the tunneling potential with (bold line) and without (thin line) barrier reduction due to static electron correlations.

Inside the well, and close to it, the electron in-plane and out-of-plane motions are then separated. We can set initial conditions at an arbitrary plane $z = z_0$ close to the well, yet deep enough under the barrier so that the wave function $\psi(z, \{\mathbf{p}_{\mathbf{k}j}\})$ is semiclassical. For a harmonic WC, the dependence of ψ on $\mathbf{p}_{\mathbf{k}j}$ is Gaussian. Then from (4)

$$
\begin{aligned}
S_E(0) &= \sum_{\mathbf{k}j} \mathbf{p}_{\mathbf{k}j}(0)\mathbf{p}_{-\mathbf{k}j}(0)/2m\omega_{\mathbf{k}j} \\
\mathbf{u}_{\mathbf{k}j}(0) &= -i\mathbf{p}_{-\mathbf{k}j}(0)/m\omega_{\mathbf{k}j}.
\end{aligned}
\tag{7}
$$

$$
S_E[z] = \frac{1}{2} \int_0^{2\tau_f} d\tau_1 \left[\frac{m}{2}(dz/d\tau_1)^2 + U(z) - E_g + \frac{1}{2}m\omega_c^2 z^2(\tau_1) - (m\omega_c^2/4N)\sum_{\mathbf{k}j}\omega_{\mathbf{k}j} \right.
$$
$$
\left. \times \int_0^{\tau_1} d\tau_2\, z(\tau_1)z(\tau_2)\exp[-\omega_{\mathbf{k}j}(\tau_1 - \tau_2)] \right].
\tag{9}
$$

In (9) we symmetrically continued the trajectory $z(\tau)$ from τ_f to $2\tau_f$, with $z(\tau_f + x) = z(\tau_f - x)$ for $0 \le x \le \tau_f$, and set $z_0 = 0$. The added section of the trajectory corresponds to underbarrier motion from the boundary of the classically accessible range back to the potential well. The tunneling rate $W \propto \exp[-R]$, with $R = 2\min S_E$.

For small magnetic fields, the field-induced correction to the tunneling exponent (9) is quadratic in ω_c. It can be calculated along the zero-field trajectory $dz/d\tau = (2[U(z) - E_g]/m)^{1/2}$. This correction is always positive: magnetic field decreases the tunneling rate. However, the correction is smaller than in the absence of the electron-electron interaction.

Remarkably, although a part of the energy of the tunneling electron goes to WC phonons, the tunneling rate increases with the increasing phonon frequencies. If the characteristic $\omega_{\mathbf{k}j}$ largely exceed the reciprocal tunneling time $1/\tau_f$ and ω_c, then $z(\tau_2) \approx z(\tau_1)$ in the second term in (9). As a result, the B-dependent terms in (9) cancel each other, and tunneling is not affected by the magnetic field at all. This happens because, as the tunneling electron moves under the barrier, its in-plane momentum is adiabatically transferred to the entire WC, similar to the Mössbauer effect. This can be contrasted with the case of an electron confined inside the well but not under the barrier. Here the magnetic barrier is reduced by a factor of 2 compared to the free-electron case, but does not disappear [6].

We now apply the results to electrons on helium and compare them with the experiment [5]. We will use the Einstein model of the WC in which all phonons have the same frequency ω_p, which we set equal to the characteristic plasma frequency $(2\pi e^2 n^{3/2}/m)^{1/2}$. The numerical results change only slightly when this frequency is varied within reasonable limits, e.g., is replaced by the root mean square frequency of the WC $\overline{\omega}$ equal to [14]

$$
\overline{\omega} = \left[\sum_{\mathbf{k}j} \omega_{\mathbf{k}j}^2/2N \right]^{1/2} \approx (4.45 e^2 n^{3/2}/m)^{1/2}.
\tag{10}
$$

For an electron which is pulled away from the helium surface by the field $\mathcal{E}_\perp$, the potential $U(z)$ has the form

In the cases of interest, the dependence of ψ on z is exponential near the well, $\psi \propto \exp(-\gamma z)$. Therefore

$$
z(0) = z_0, \qquad p_z(0) = i\gamma = i\sqrt{2m[U(z_0) - E_g]}.
\tag{8}
$$

Under the barrier, the potential $U(z)$ varies on the scale bigger than $1/\gamma$, and then γ in (8) is independent of the exact position of the plane $z = z_0$.

By solving the linear equations of motion (5) for the phonon variables $\mathbf{u}_{\mathbf{k}j}$, $\mathbf{p}_{\mathbf{k}j}$ with the boundary conditions (6) and (7), we can eliminate them (cf. [9]). Then S_E takes the form of a retarded action for 1D motion,

$$
U(z) = -\Lambda z^{-1} - |e\mathcal{E}_\perp|z - m\overline{\omega}^2 z^2
\tag{11}
$$

for $z > 0$ (outside the helium). On the helium surface (located at $z = 0$), $U(z)$ has a high barrier ~ 1 eV which prevents the electron from penetrating into the helium.

In (11), the term $\propto \Lambda = e^2(\epsilon - 1)/4(\epsilon + 1)$ describes the image potential, and $\epsilon \approx 1.057$ is the dielectric constant. The field $\mathcal{E}_\perp$ is determined by the helium cell geometry and depends on the applied voltage and the electron density n, cf. [15]. The term $\propto m\overline{\omega}^2 \equiv e^2 \sum' |\mathbf{R}_l|^{-3}/2$ describes the Coulomb field created by other electrons at their lattice sites $\mathbf{R}_l$ (the "correlation hole" [8,16]), for the tunneling length $L < n^{-1/2}$. The conditions $1/\gamma \ll L \ll n^{-1/2}$ are typically very well satisfied in the experiment, with $1/\gamma = 1/\Lambda m \approx 0.7 \times 10^{-6}$ cm, $L = \gamma^2/2m|e\mathcal{E}_\perp| \sim 10^{-5}$ cm for typical $\mathcal{E}_\perp \sim 10$ V/cm, and $n^{-1/2} \sim 10^{-4}$ cm.

The magnetic field dependence of the tunneling rate calculated from Eqs. (5)–(8) is shown in Fig. 1. The actual calculation is largely simplified by the fact that, deep under the barrier, the image potential $-\Lambda/z$ in (11) can be neglected. The equations of motion (5) then become linear, and the tunneling exponent $R = 2S_E(\tau_f)$ can be obtained in an explicit (although somewhat cumbersome) form, which was used in Fig. 1. The correction to R from the image potential is $\sim 1/\gamma L$. When this and other corrections $\sim 1/\gamma L$ are taken into account, the theoretical curve in Fig. 1 slightly shifts down (by $\lesssim 20\%$ even for strong B), which is *much less* than the uncertainty in R due to the uncertainties in n and $\mathcal{E}_\perp$ in the experiment [5]. The theory is in excellent agreement with the experiment, with no adjustable parameters.

The dependence of the potential $U(z)$ on n gives rise to the density dependence of the escape rate $W(B)$ even for $B = 0$. We calculated the exponent and the prefactor in $W(0)$ by matching the WKB wave function under the barrier for $1/\gamma \ll z \ll L$ with the intrawell solution. The latter was sought in the form $\psi(z) = z\exp[-A(z)]$. The

function dA/dz satisfies a Riccati equation which can be solved near the well ($z \ll L$) by considering the last two terms in (11) as a perturbation. When calculated to the first order in this perturbation, A allows one to find not only the exponent but also the leading term in the prefactor in the WKB wave function. The resulting tunneling rate is shown in the inset of Fig. 1. It fully agrees with the experiment [17].

For semiconductor heterostructures, tunneling in correlated systems has been investigated mostly for the magnetic field **B** perpendicular or nearly perpendicular to the electron layer (cf. [4]). The data on tunneling in a field parallel to the layer refer to high-density 2DESs [18], where correlation effects are small. We expect that tunneling experiments on low-density 2DESs in parallel fields will reveal electron correlations not imposed by the magnetic field, give insight into electron dynamics, and possibly even reveal a transition from an electron fluid to a pinned Wigner crystal with decreasing n.

The effect of a parallel magnetic field is most pronounced in systems with shallow and broad barriers $U(z)$. For example, in a GaAlAs structure with a square barrier of width $L = 0.1$ μm and height $\gamma^2/2m = 0.02$ eV, for the electron density $n = 1.5 \times 10^{10}$ cm^{-2} and $B = 1.2$ T, we have $\omega_p \tau_0 \approx 0.6$ and $\omega_c \tau_0 \approx 1$ ($\tau_0 = mL/\gamma$ is the tunneling duration for $n = B = 0$).

Electron correlations give rise to a coordinate-dependent lowering of the barrier (see Fig. 2). For $nL^2 \ll 1$, $U(z) = \gamma^2/2m - m\overline{\omega}^2 z^2$, $0 < z < L$ (we count U off from the intrawell energy level E_g). The picture of tunneling depends on the parameter $\nu = \sqrt{2}\,\overline{\omega}\tau_0$. For $\nu < 1$ the electron comes out from the barrier at the point $z = L$, where $U(z)$ is discontinuous (cf. Fig. 2b). In this most important case, the boundary conditions (6) for the tunneling trajectory should be changed to

$$z(\tau_f) = L, \qquad \mathbf{u}_{\mathbf{k}j}(\tau_f) = \mathbf{0}, \qquad (12)$$

but the tunneling exponent is still given by Eq. (9).

For $B = 0$ the tunneling exponent R decreases with n, $R = \gamma L[\nu^{-1}\arcsin\nu + (1 - \nu^2)^{1/2}]$ for $\nu < 1$, and $R = \pi\gamma L/2\nu$ for $\nu > 1$. Magnetic field causes R to increase and the tunneling rate to decrease. The effect is reduced by the interelectron momentum exchange. The results for the Einstein model of the WC with $\omega_{\mathbf{k}j} = \omega_p$ are shown in Fig. 2. The inset of Fig. 2 shows how R is decreased by the electron correlations even for $B = 0$.

We have used the model of a WC to analyze the effect of electron correlations on tunneling in a magnetic field parallel to the electron layer. We showed that the electron-electron interaction gives rise to an exponential increase of the tunneling rate compared to its single-electron value in a strong magnetic field. The effect is determined by the interrelation between the frequencies of in-plane electron vibrations and the reciprocal tunneling time. For long tunneling time, the physics of large changes in the decay rate is closely tied to the physics of the recoilless fraction in the Mössbauer effect. Since the major contribution comes from the short-wavelength vibrations, the results should apply not only to WCs, but also to all 2DESs with short-range order. Our results give a quantitative no-parameter fit to the experimental data [5] on tunneling of strongly correlated electrons on helium.

[1] E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, cond-mat/0006055.

[2] *Two-Dimensional Electron Systems on Helium and other Cryogenic Substrates,* edited by E. Andrei (Kluwer, New York, 1997).

[3] *Perspectives in Quantum Hall Effects,* edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997).

[4] I. B. Spielman *et al.,* Phys. Rev. Lett. **84**, 5808 (2000).

[5] L. Menna, S. Yücel, and E. Y. Andrei, Phys. Rev. Lett. **70**, 2154 (1993); E. Y. Andrei, in Ref. [2], p. 207.

[6] A strong effect on tunneling of the momentum transfer to defects was first discussed by B. I. Shklovskii [JETP Lett. **36**, 51 (1982)]; B. I. Shklovskii and A. L. Efros, Sov. Phys. JETP **57**, 470 (1983).

[7] A physically different problem of tunneling between lattice sites of the WCs at the edges of a quantum Hall system was discussed by M. B. Hastings and L. S. Levitov [Phys. Rev. Lett. **77**, 4422 (1996)]. In contrast to the present case, the tunneling probability was determined by coupling to low-frequency long-wavelength modes, it oscillated with B and went to zero for $T \rightarrow 0$.

[8] M. Ya. Azbel and P. M. Platzman, Phys. Rev. Lett. **65**, 1376 (1990).

[9] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[10] A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).

[11] T. Barabash-Sharpee, M. I. Dykman, and P. M. Platzman, Phys. Rev. Lett. **84**, 2227 (2000).

[12] J. S. Langer, Ann. Phys. (N.Y.) **41**, 108 (1967); S. Coleman, Phys. Rev. D **15**, 2929 (1977).

[13] For a parabolic well and a special form of electron coupling to harmonic oscillators, the instanton technique was applied to tunneling in a magnetic field by P. Ao [Phys. Rev. Lett. **72**, 1898 (1994)]; Phys. Scr. **T69**, 7 (1997).

[14] M. I. Dykman, J. Phys. C **15**, 7397 (1982).

[15] M. J. Lea *et al.,* Phys. Rev. B **55**, 16 280 (1997).

[16] Y. Iye *et al.,* J. Low Temp. Phys. **38**, 293 (1980).

[17] For $B = 0$, a good agreement between measured and numerically evaluated tunneling rates for electrons on helium was obtained by G. F. Saville, J. M. Goodkind, and P. M. Platzman [Phys. Rev. Lett. **70**, 1517 (1993)].

[18] J. Smoliner *et al.,* Phys. Rev. Lett. **63**, 2116 (1989); J. P. Eisenstein *et al.,* Phys. Rev. B **44**, 6511 (1991); S. Q. Murphy *et al.,* Phys. Rev. B **52**, 14 825 (1995).