X-Ray Amplification by Laser Controlled Coherent Bremsstrahlung

K. Z. Hatsagortsyan* and C. H. Keitel†
Theoretische Quantendynamik, Fakultät für Physik, Universität Freiburg, Hermann-Herder-Strasse 3, D-79104 Freiburg, Germany
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A coherent x-ray generation scheme is proposed which involves characteristics of free electron lasers and atomic high harmonic generation schemes. A thin solid layer or any other periodic atomic structure with limited dimensions is exposed to a short, superintense laser pulse. The electrons are extracted from the layer due to the extreme force and penetrate periodically through the ionic structure. Consequently, thousands of harmonics of the laser radiation field are shown to coherently amplify along the interaction length. The small signal gain of the generated x-ray radiation exceeds that arising from the multiphoton Compton process in plasmas and is competitive with that in the leading x-ray free electron lasers.

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One important task of modern laser physics is the achievement of efficient laser action in the x-ray domain. There have been numerous attempts to tackle this challenging problem such as using sophisticated pumping methods of atomic x-ray transitions [1], employing free electron laser schemes [2–5], or generating high harmonics from intense laser driven atomic systems [6–9]. The x-ray free electron laser (X-FEL) has been proposed based on schemes referred to as channeling, transition, diffraction radiation, or Compton-backscattering [3]. The main hopes, however, for an efficient X-FEL remain connected with the so-called undulator scheme [10] with two international projects TESLA [4] and LCLS [5] being currently implemented in this direction. In these projects the x-ray domain becomes achievable due to the large Doppler upshift of the emission frequency with respect to the oscillation frequency of the electron in the undulator field. Technically, though, the demands are extremely challenging and quite expensive. A high quality ultrarelativistic electron beam is required from an accelerator storage ring with GeV energy and a very low emittance (smaller than $10^{-2}\pi$ nm rad) and in addition a very short wavelength undulator with unprecedented length of about 100 m.

A further approach pursued in the last few years has been to generate high frequency light in the x-ray regime via the nonlinear interaction of high power laser light with atomic systems [6–9]. Here small fractions of the atomic electron wave function are oscillating nearly periodically due to the intense laser field, and high multiples of the applied laser field (harmonics) may be generated due to the repeated interaction with the atomic cores. Recently coherent soft x-rays within the water window have been generated by using near optical light pulses of very few cycles length [7]. The advantage of this scheme is the high quality of the coherence of the harmonics. However, a problem is the low intensity of the generated high harmonics. In addition the hard x-ray regime remains to be implemented via this mechanism in spite of justified hopes associated with highly charged ions in intense laser fields [11]. High harmonic generation (HHG) has also been considered within solids [8] and especially successfully in dense plasmas [9]. In this situation at superhigh laser intensities, however, so-called Rayleigh-Taylor-like surface rippling of the plasma during the interaction substantially reduces the spatial coherence of harmonic radiation and consequently restricts the maximal achievable frequency of the generated harmonics.

In this Letter, we propose an alternative approach towards intense coherent x rays deviating clearly from each of the two main streams discussed before—X-FEL and HHG—however, equally involving ideas of both of them. We consider a thin solid layer (or some other thin periodic atomic or ionic structure) to interact simultaneously with a short superintense laser pulse and a weak high-frequency field as probe (see Fig. 1). Under the influence of the high-power laser pulse, the electrons are extracted almost instantaneously from the layer. The periodic brief interactions with the far slower moving and thus still quasiregular ionic structure then lead to high-frequency coherent bremsstrahlung as shown here by the amplification of the probe field. Thus as in a FEL, we consider stimulated radiation from essentially free electrons which,

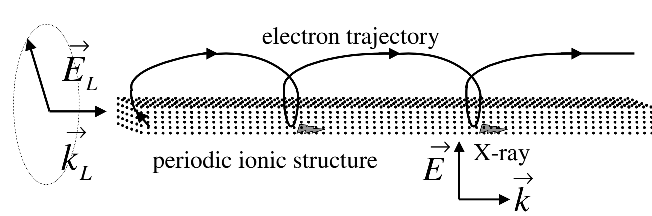

FIG 1. Scheme of interest: Electrons extracted from a thin solid layer propagate along and oscillate in a superstrong laser field ($\vec{k}_L$: wave vector and $\vec{E}_L$: electric field of the laser wave) and interact with a periodic ionic structure. The generation of coherent x rays ($\vec{k}$: wave vector and $\vec{E}$: electric field of the x rays) is depicted at the areas of interaction.

however, are extracted from the laser-driven solid layer rather than being injected into the system. With respect to conventional HHG schemes, the coherent x-ray emission arises from harmonic generation due to laser-induced periodic nonlinear interactions of the electrons with ions. However, in our situation the electron possesses highly relativistic energies and multiphoton scattering dominates over recombinations to bound states. From a more technical point of view we would like to emphasize that we consider the more efficient *stimulated* x-ray emission in our scheme while conventional HHG is known to operate well spontaneously.

In terms of the envisaged crystal and laser parameters we require the laser force to exceed the Coulomb force, i.e., $Zn_i r_0 L_\perp \lambda_L \ll 2\xi$. Here Z is the ion charge, n_i the ion density, r_0 the classical electron radius, $L_\perp$ the transversal width of the solid layer with respect to the laser propagation direction, λ_L the laser wavelength, $\xi = eE_L/mc\omega_L$ the normalized parameter of the laser field strength, and E_L and ω_L are the electric field and frequency of the laser field, respectively; e and m are the electron charge and mass, respectively, and c the speed of light. The electrons thus oscillate most of the time freely with relativistic velocities and a transversal amplitude $R_\perp$ exceeding both much the width of the thin solid layer $L_\perp$ and the characteristic length $\delta = c/\omega_p$ of collective dynamics. Here ω_p denotes the relativistic plasma frequency characterizing the plasma as a collective system and $\delta, L_\perp \ll R_\perp$ induces a decoupling of the electron and ion dynamics. Furthermore, the width of the layer $L_\perp$ (here less than 10^{-5} cm) is assumed substantially smaller than the laser spot size so that the effects of light pressure and ponderomotive forces are thus of no significance. Therefore collective dynamics can be ignored, as also confirmed by the corresponding 2D particle-in-cell (PIC) simulations with mobile ions [12]. As a consequence, typical collective plasma instabilities, such as ion shock wave formation and turbulence which reduce the coherence for HHG in plasmas [9], do not apply in our situation.

The main problem facing the realization of the proposed scheme is the destruction of the periodic ionic structure in the superstrong laser field. We show that the destruction process can be slow enough as compared to the time necessary for the x-ray radiation. The ion regularities are reduced mainly due to Coulomb explosion after the electron extraction by the laser field. The transversal and longitudinal ionic displacements $l_i^\perp$ and $l_i^\parallel$ during the interaction time τ_L can be estimated to be at most $l_i^\perp \sim Z^2 n_i r_0 \times L_\perp c^2 \tau_L^2 m/M$ and $l_i^\parallel \sim Z^2 n_i r_0 L_\perp^2 c^2 \tau_L^2 m/ML$ due to Coulomb repulsion. Here M is the ion mass and L the longitudinal length of the solid layer. Our calculation shows that the coherence of the radiation from the periodic structure is maintained if $l_i^\perp \ll R_\perp$ and $l_i^\parallel \ll a$, with a being the ionic separation length prior the laser pulse. In the case of a Pb sample with density $n_i \sim 10^{22}$ cm^{-3}, $a \sim 5 \times 10^{-8}$ cm, $M \sim 3.8 \times 10^5 \cdot m$, a layer width and length of $L_\perp \sim 1.6 \times 10^{-6}$ cm and $L \sim 4 \times 10^{-4}$ cm, ion charge $Z \sim 10$, laser pulse intensity $I \sim 10^{21}$ W/cm^2, KrF wavelength $\lambda_L = 248$ nm and pulse duration $\tau_L \sim 10$ fs, the above mentioned conditions hold. Thus, during the short times of laser-electron interaction of interest here, the ionic structure can be assumed to remain regular with small deviations which will be included in our calculation. This assumption has been confirmed as well by PIC simulations [12], where no substantial ion dynamics was notable during the short interaction with the laser pulse.

Free electrons in a very strong radiation field are known to emit harmonics of the incident radiation frequency from early works on spontaneous multiphoton Compton radiation [13] and numerous recent works including experimental implementations [14]. The maximal harmonic number corresponding to the fields that can be generated by means of the multiphoton Compton effect is of the order $\tilde{n} \sim \xi^3$ [13]. In order to enhance the generated harmonic number n above the rather small output due to spontaneous multiphoton Compton-radiation it is, loosely speaking, necessary to arrange electron trajectories with sharp edges via additional forces. We propose to implement this task by imposing the laser-accelerated electrons to penetrate periodically through a transient regular ionic structure. We define the multiphoton parameter $\bar{n} = \lambda_L \xi^2/2a$ which turns out to characterize the efficiency of harmonic generation for a given set of parameters. Up to $\bar{n}$ we will show significant harmonic generation, while for higher harmonic numbers the efficiency decreases substantially. To have an idea about the orders of magnitude, for a KrF high-power laser pulse with intensity $I \sim 10^{21}$ W/cm^2 ($\xi = 6$) and $a \sim 5 \cdot 10^{-8}$ cm, the multiphoton parameter in the Compton process is $\tilde{n} \sim 200$, while for our proposed scheme $\bar{n} \sim 9 \cdot 10^3$, due to the periodicity of the ionic structure. The expression for $\bar{n}$ and those estimations thus indicates that relativistic laser intensities, small ion separations, as well as high ion charges and densities are most favorable.

We calculate the small signal gain G of x-ray radiation for our scheme of interest in the framework of classical radiation theory [15]. The high power pump laser field is considered exactly while the scattering potential as well as the probe field of x-ray radiation are treated perturbatively to second order. The interaction of the electrons with the intense laser and probe field and the ions is governed by a resonance condition which stems from an energy-momentum conservation law:

$$\omega + 2\pi\nu s/a + l\omega_L = 0. \quad (1)$$

Here ω is the angular frequency of the probe wave, $\vec{\nu} = -c\overline{\vec{p}}_o/\Lambda$ and $\overline{\vec{p}}_o = \vec{p}_o - \hat{x}(\mathrm{mc}\xi)^2/2\Lambda$ is the renormalized momentum of the electron in the laser field. Further we denote $\Lambda = p_{ox} - \varepsilon_o/c$, ε_o and $\vec{p}_o$ are the initial electron energy and momentum, respectively; $\hat{x}$ is the unit vector along the laser wave propagation direction; l and s are integers, indicating the number of photons absorbed from the laser wave and the number of quanta of momentum transferred to the scattering field, respectively. For the sake of periodicity, parameters are chosen such that $N = d/a$ happens to be an integer number. Then coherence of the generated radiation from each period of the trajectory would be secured. We show that the constructive interference among the various scattering events secures high coherence of the harmonics and an enhancement of the gain as compared with bremsstrahlung in a plasma [16].

There are various distinct regimes of amplification depending on the characteristics of the width of the resonance of the process. The most favorable regime for the probe amplification is what we call the exponential instability (EI) regime. This is defined by the situation when the gain G is large enough, such that the spectral width of the probe wave, due to its fast increase in intensity, prevails the resonance width due to the limited interaction time, as well as that imposed by the momentum spread Δ of the electrons $G \succeq \max\{1/L, N\Delta/mc\lambda_L\}$, where Δ is the electron momentum spread. In this regime the gain is found on the basis of a self-consistent set of Maxwell equations for the probe wave and the equation of motion for the electrons. The scattering potential of each ion is specified as Coulomb potential. The ions are assumed to be centered along a regular lattice with a Gaussian distribution for the random variation from the crystal knot centers. In the case of a strong laser field ($\xi \gg 1$) and concentrating on the high harmonic regime ($n \gg 1$), the x-ray small signal gain is then given by the expression

$$G = \sqrt{3}\left(\frac{\pi}{4}\right)^{1/3} \exp\left\{-\frac{4\pi^2 u^2}{3a^2}\right\} \times \left[\frac{Z^2 n_e n_i r_0^3 \xi^6 \lambda_L^2 \gamma_o^3 m^4 c^4}{n^2 a^4 \Lambda^4 R_\perp^2} L_\perp^4 (1 + \beta_o)^4 I_{n-N}(\zeta) \times K_{n-N}(\zeta)\right]^{1/3} \quad (2)$$

where u is the mean deviation of ions from the crystal knot points; n_e, n_i are the electron and ion densities, respectively; γ_o is the initial Lorentz-factor of the electron, $\beta_o = v_o/c$, v_o is the electron initial velocity; $\overline{\varepsilon_o} = \varepsilon_o - c(mc\xi)^2/2\Lambda$ is the renormalized energy of the electron in the laser field, $n = \omega/\omega_L$ is the harmonic number, $\zeta = \lambda_L \xi \gamma_o (1 + \beta_o)/a$ and $I_n(\zeta)$, $K_n(\zeta)$ are the modified Bessel functions.

The gain dependence of the generated radiation frequency is shown in Fig. 2. The parameters employed involve $\xi = 6$, $\lambda_L = 3.5 \times 10^{-6}$ cm, $\tau_L = 6$ fs, $R_\perp = 2.1 \times 10^{-5}$ cm, $L_\perp = 7 \times 10^{-6}$ cm, $L = 3.8 \times 10^{-4}$ cm, $Z = 10$, $n_e = 10n_i = 10^{23}$ cm^{-3}, and $\Delta/mc = 10^{-4}$. Harmonic generation of order 10^3 is shown with wavelength $\lambda = 0.35$ Å and gain $G \simeq 2.1 \times 10^4$ cm^{-1}, i.e., $GL \simeq 7.6$. This gain clearly dominates that due to the multiphoton Compton process in a plasma (see Fig. 2a). The multiphoton parameter $\bar{n}$ in this process can be deduced from Eq. (2). It corresponds to the number of harmonics at the peak of the function $I_{n-N}(\zeta) \cdot K_{n-N}(\zeta)$ (see Fig. 2a, dotted line). In Fig. 2b, we display the harmonic structure, showing reasonable coherence of the harmonics and no restriction to odd harmonics as in atomic high harmonic generation in the nonrelativistic regime. This deviation is not surprising as opposed to our situation in bound atomic systems inversion symmetry is given because the electron is accelerated towards the nucleus alternatively along the positive and negative polarization direction. Since Eq. (2) is restricted to the peak values of the harmonics only, we needed to evaluate this part numerically.

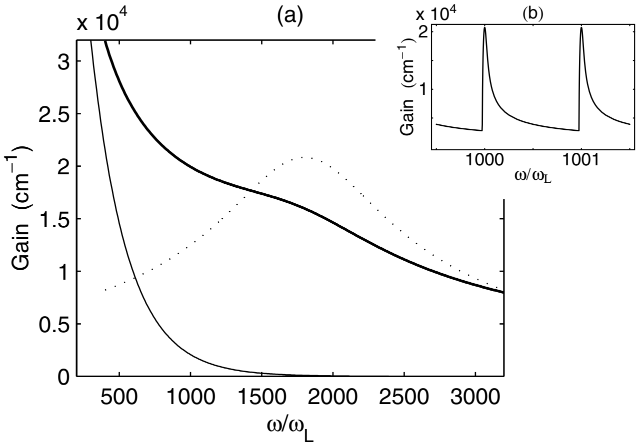

FIG. 2. (a) The envelope function of the gain G according to Eq. (2) as a function of the probe frequency in the EI regime (thick line). The electron and ion densities are $n_e = 10n_i = 10^{23}$ cm^{-3}, the laser wavelength is $\lambda_L = 3.5 \cdot 10^{-6}$ cm, the laser intensity parameter is $\xi = 6$, $R_\perp = 2.1 \cdot 10^{-5}$ cm, the ion charge $Z = 10$, the ion structure period $a = 3.5 \cdot 10^{-8}$ cm, the initial velocity $v_0 = 0$, the transversal size of the ionic structure is $L_\perp = 7 \cdot 10^{-6}$ cm, $u/a = l_i^{\parallel}/a = 0.087$, and the momentum spread of the electron beam is $\Delta/mc = 10^{-4}$. The thin line describes the corresponding gain for the multiphoton Compton radiation amplification in a plasma [17] for the same parameters. The dotted line depicts the function $I_{n-N}(\zeta) \cdot K_{n-N}(\zeta)$ multiplied by $2.5 \cdot 10^7$ which peaks at the multiphoton parameter $\bar{n}$. (b) A window of the fine structure of the gain corresponding to the thick line in (a).

We proceed by estimating the gain efficiency and the conditions for the EI regime for a different set of parameters. Employing the KrF laser wavelength with $\lambda_L = 248$ nm and an intensity of $I \simeq 10^{21}$ W/cm^2 ($\xi = 6$) and a Pb crystal with $n_e = 10n_i = 10^{23}$ cm^{-3}, $a = 4.95 \cdot 10^{-8}$ cm, $L_\perp = 5.2 \cdot 10^{-6}$ cm, $R_\perp = 2.5 \cdot 10^{-5}$ cm, $L = 4 \cdot 10^{-4}$ cm and further $Z = 10$, $\Delta/\text{mc} = 3 \cdot 10^{-5}$, then the EI regime operates yielding very high gain, e.g., for the $n = 1000$ harmonic $G \simeq 1.8 \cdot 10^4$ cm^{-1}. Compared to this at the same parameters the multiphoton Compton effect gives rise to merely $\tilde{n} \simeq 200$ harmonics [17]. Furthermore for the chosen parameters the gain via incoherent x-ray bremsstrahlung in plasmas is very small as well, around 10^{-6} cm^{-1} for the 1000th harmonic [16]. In our scheme, even for the very short interaction time of $\tau_L = 10$ fs, the enhancement of radiation is substantial as the gain-length product, characterizing the lasing possibility at $\lambda = \lambda_L/n = 2.5$ Å, is $GL = 7$. This gain-length product is somewhat less, but competitive with the gain of leading present day and near future x-ray lasers. Those include $GL \simeq 15$ at $\lambda = 13.9$ nm [1] and especially the ultimate of $GL \simeq 41$ at $\lambda = 1$ Å due to unprecedented long interaction lengths of $L = 95m$ currently in construction [4].

We now turn to the coherence of the emitted radiation. The temporal coherence is determined by the time scale which characterizes the growth of radiation $1/cG$ being here of order 2 fs. Then $\delta\omega/\omega \sim cG/n\omega_L \sim 10^{-4}$. The spatial coherence of the generated radiation is determined by the transversal width of the layer $L_\perp$: $\delta k_\perp/k \sim \lambda_L/2\pi n L_\perp \sim 8 \cdot 10^{-4}$. The discussed scheme has thus quite reasonable coherence properties but cannot compete with the leading schemes [1] where $\delta\omega/\omega \sim 10^{-5}$, $\delta k_\perp/k \sim 2.5 \cdot 10^{-5}$. However, we emphasize that our scheme is not a large-scale facility and especially has proven superior for its very high gain on short distances.

Lowering the electron and ion density to $n_i = n_e \sim 10^{21}$ cm^{-3} we pass to the so-called warm beam regime with less substantial gain. This is also the case when keeping the densities but reducing the laser intensities. One problem with present day terawatt femtosecond laser pulses at an intensity of 10^{21} W/cm^2 is still the pedestal in the ns regime. Recent work by Tournois *et al.* on pulse compression and shaping [18], however, has shown means of reducing the pedestal. In addition, nonionizing pedestals in the picosecond regime with a pulse/pedestal contrast ratio of 10^8 are already now available at an intensity of 10^{18} W/cm^2. Then for $\lambda_L = 780$ nm, $Z = 1$, $L_\perp = 4 \cdot 10^{-6}$ cm and $n_e = n_i = 10^{22}$ cm^{-3} the gain-length product for the 100th harmonic can reach 1.4 considering an interaction length of $L = 10$ μm. This is still substantial, especially taking into account that harmonic generation due to the Compton effect is still negligible for those parameters. Other regimes of the present scheme as well as related schemes shall be the topic of a subsequent more detailed work.

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*Permanent address: Department of Theoretical Physics, Yerevan State University, A. Manoukian Street 1, 375049 Yerevan, Armenia.

†Email address: keitel@uni-freiburg.de

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