QCD Phase Transition in the Inhomogeneous Universe

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We investigate a new mechanism for the cosmological QCD phase transition: inhomogeneous nucleation. The primordial temperature fluctuations, measured to be $\delta T/T \sim 10^{-5}$, are larger than the tiny temperature interval in which bubbles would form in the standard picture of homogeneous nucleation. Thus the bubbles nucleate at cold spots. We find the typical distance between bubble centers to be a few meters. This exceeds the estimates from homogeneous nucleation by 2 orders of magnitude. The resulting baryon inhomogeneities may affect primordial nucleosynthesis.

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A separation of cosmic phases during a first-order QCD transition [1] could give rise to inhomogeneous nucleosynthesis [2–5]. During a thermal first-order phase transition in a homogeneous medium bubbles nucleate due to statistical fluctuations (homogeneous nucleation). Their mean separation at nucleation introduces a scale for isothermal inhomogeneities in the early Universe, which may influence the local neutron-to-proton ratio, providing inhomogeneous initial conditions for nucleosynthesis. The baryon inhomogeneities may survive until the time of neutron freeze-out, if the mean bubble nucleation distance, $d_{\rm nuc}$, exceeds the diffusion length of the proton. Comparing those scales at the time of the QCD transition, assuming a thermodynamic transition temperature $T_c = 150$ MeV, gives $d_{\rm nuc} > 2$ m [4]. The causal scale is set by the Hubble distance at the QCD transition, $d_H \equiv c/H \sim 10$ km.

The order of the QCD transition and the values of its parameters are still under debate. Nevertheless, there are indications from lattice QCD calculations [6–10]. For the physical masses of the quarks the order of the transition is still unclear [6,7]. Quenched QCD (no dynamical quarks) shows a first-order phase transition with a small latent heat, compared to the bag model, and a small surface tension, compared to dimensional arguments [8]. We assume that the QCD transition is of first order and that the values from quenched lattice QCD (scaled appropriately by the number of degrees of freedom) are typical for the physical QCD transition. Based on these values and homogeneous bubble nucleation a small supercooling, $\Delta_{\rm sc} \equiv 1 - T_f/T_c \sim 10^{-4}$, and a tiny bubble nucleation distance, $d_{\rm nuc} \sim 1$ cm, follow [11]. The actual nucleation temperature is denoted by T_f.

We argue that the assumption of homogeneous nucleation is violated in the early Universe by the inevitable density perturbations from inflation or from other seeds for structure formation. Those fluctuations in density and temperature have been measured by COBE [12] to have an amplitude of $\delta T/T \sim 10^{-5}$. The effect of the QCD transition on density perturbations [13,14] and gravitational waves [15] has been studied previously, while we investigate the effect of the density perturbations on the QCD phase transition here. We conclude that a first-order QCD transition induces an inhomogeneity scale of a few meters. In comparison with heterogeneous nucleation via *ad hoc* dirt [16], we do not introduce any new, unknown objects. Our findings might have interesting implications for precision measurements of primordial abundances [4,5].

First-order phase transitions normally proceed via nucleation of bubbles of the new phase. When the temperature is spatially uniform and no significant impurities are present, the mechanism is homogeneous nucleation. The probability to nucleate a bubble of the new phase per time and volume is approximated by $\Gamma \approx T_c^4 \exp[-S(T)]$. The nucleation action S is the free energy difference of the system with and without the nucleating bubble, divided by the temperature.

Nucleation is a very rapid process, compared with the extremely slow cooling of the Universe. The duration of the nucleation period, $\Delta t_{\rm nuc}$, is found to be [3,17]

$$\Delta t_{\rm nuc} = -\frac{\pi^{1/3}}{dS/dt|_{t_f}} . \qquad (1)$$

The time t_f is defined as the moment when the fraction of space where nucleations still continue equals $1/e$. The heat flow preceding the deflagration fronts reheats the rest of the Universe. We denote by $v_{\rm heat}$ the effective speed by which released latent heat propagates in sufficient amounts to shut down nucleations. In practice, $v_{\rm def} < v_{\rm heat} < c_s$, where $v_{\rm def}$ is the velocity of the deflagration front and c_s is the sound speed [18]. In the unlikely case of detonations $v_{\rm heat}$ should be replaced by the velocity of the phase boundary in all expressions that follow.

The mean distance between nucleation centers, measured immediately after the transition completed, is

$$d_{\rm nuc,hom} = 2 v_{\rm heat} \Delta t_{\rm nuc} . \qquad (2)$$

This nucleation distance sets the spatial scale for baryon number inhomogeneities.

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Lattice simulations [9,10] imply that in real-world QCD the energy density must change very rapidly in a narrow temperature interval. This can be seen from the microscopic sound speed in the quark phase, $c_s \equiv (\partial p/\partial \varepsilon)_S^{1/2}$. Lattice QCD indicates that $3c_s^2(T_c) = \mathcal{O}(0.1)$ [10]. Thus, the cosmological time-temperature relation is strongly modified already before the nucleations, due to

$$\frac{dT}{dt} = -3c_s^2 \frac{T}{t_H}, \tag{3}$$

where $t_H \equiv 1/H = (3M_{\rm pl}^2/8\pi\varepsilon_q)^{1/2}$ with ε_q being the energy density in the quark phase. This behavior of the sound speed increases the nucleation distance because of the proportionality $\Delta t_{\rm nuc} \propto 1/[3c_s^2(T_f)]$ [11].

In the thin-wall approximation the nucleation action has the following explicit expression:

$$S(T) = \frac{C^2}{(1 - T/T_c)^2}, \qquad C \equiv 4\sqrt{\frac{\pi}{3}}\,\frac{\sigma^{3/2}}{l\sqrt{T_c}}, \tag{4}$$

for small supercooling. Assuming further that c_s does not change very much during supercooling, the following relation holds for the supercooling and nucleation scales:

$$\frac{\Delta t_{\rm sc}}{\Delta t_{\rm nuc}} = \frac{\Delta_{\rm sc}}{\Delta_{\rm nuc}} = \frac{2}{\pi^{1/3}}\,\bar{S}. \tag{5}$$

Here we denote by Δ a relative (dimensionless) temperature interval and by Δt a dimensionful time interval. $\bar{S} \equiv S(T_f)$ is the critical nucleation action, $\bar{S} = \mathcal{O}(100)$.

Surface tension and latent heat are provided by lattice simulations with quenched QCD only, giving the values $\sigma = 0.015T_c^3$, $l = 1.4T_c^4$ [8]. Scaling the latent heat for the physical QCD leads us to take $l = 3T_c^4$.

With these values for the latent heat and surface tension, the amount of supercooling is $\Delta_{\rm sc} = 2.3 \times 10^{-4}$. From Eq. (5) it follows that $\Delta_{\rm nuc} = 1.5 \times 10^{-6}$. Substituting $3c_s^2 = 0.1$ into Eq. (3), we find $\Delta t_{\rm nuc} = 1.5 \times 10^{-5} t_H$ for the duration of the nucleation period. The nucleation distance depends on the unknown velocity $v_{\rm heat}$ in Eq. (2). With the value 0.1 for $v_{\rm heat}$, the nucleation distance $d_{\rm nuc,hom}$ would have the value $2.9 \times 10^{-6} d_H$. One should take these values with caution, due to large uncertainties in l and σ. As our reference set of parameters, we take $\Delta_{\rm sc} = 10^{-4}$, $\Delta_{\rm nuc} = 10^{-6}$, and $\Delta t_{\rm nuc} = 10^{-5} t_H$.

In the real Universe the local temperature of the radiation fluid fluctuates. We decompose the local temperature $T(t, \mathbf{x})$ into the mean temperature $\bar{T}(t)$ and the perturbation $\delta T(t, \mathbf{x})$. The temperature contrast is denoted by $\Delta \equiv \delta T/\bar{T}$. On subhorizon scales in the radiation dominated epoch, each Fourier coefficient $\Delta(t, k)$ oscillates with constant amplitude, which we denote by $\Delta_T(k)$. Inflation predicts a Gaussian distribution,

$$p(\Delta)\, d\Delta = \frac{1}{\sqrt{2\pi}\,\Delta_T^{\rm rms}} \exp\left(-\frac{1}{2}\frac{\Delta^2}{(\Delta_T^{\rm rms})^2}\right) d\Delta. \tag{6}$$

We find [19] for the COBE normalized [12] rms temperature fluctuation of the radiation fluid (not of cold dark matter) $\Delta_T^{\rm rms} = 1.0 \times 10^{-4}$ for a primordial Harrison-Zel'dovich spectrum. The change of the equation of state prior to the QCD transition modifies the temperature-energy density relation, $\Delta = c_s^2 \delta\varepsilon/(\varepsilon + p)$. We may neglect the pressure p near the critical temperature since $p \ll \varepsilon_q$ at T_c. On the other hand, the drop of the sound speed enhances the amplitude of the density fluctuations proportional to $c_s^{-1/2}$ [14]. Putting all those effects together and allowing for a tilt in the power spectrum, the COBE normalized rms temperature fluctuation reads

$$\Delta_T^{\rm rms} \approx 10^{-4}(3c_s^2)^{3/4}\left(\frac{k}{k_0}\right)^{(n-1)/2}, \tag{7}$$

where $k_0 = (aH)_0$. For a Harrison-Zel'dovich spectrum ($n = 1$) and $3c_s^2 = 0.1$, we find $\Delta_T^{\rm rms} \approx 2 \times 10^{-5}$.

A small scale cutoff in the spectrum of primordial temperature fluctuations comes from collisional damping by neutrinos [14,20]. The interaction rate of neutrinos is $\sim G_F^2 T^5$. This has to be compared with the angular frequency $c_s k_{\rm ph}$ of the acoustic oscillations. At the QCD transition neutrinos travel freely on scales $l_\nu \approx 4 \times 10^{-6} d_H$. Fluctuations below the diffusion scale of neutrinos are washed out,

$$l_{\rm diff} = \left[\int_0^{t_c} l_\nu(\bar{t})\, d\bar{t}\right]^{1/2} \approx 7 \times 10^{-4} d_H. \tag{8}$$

In Ref. [14] the damping scale from collisional damping by neutrinos has been calculated to be $k_\nu^{\rm ph} = 10^4 H$ at $T = 150$ MeV. The estimate (8) is consistent with this damping scale. We assume $l_{\rm smooth} = 10^{-4} d_H$. The compression time scale for a homogeneous volume $\sim l_{\rm smooth}^3$ is $\delta t = \pi l_{\rm smooth}/c_s \sim 10^{-3} t_H$. Since $\delta t \gg \Delta t_{\rm nuc}$ the temperature fluctuations are frozen with respect to the time scale of nucleations. As long as $l_{\rm smooth}$ exceeds the Fermi scale homogeneous bubble nucleation applies within these small homogeneous volumes. This is a crucial difference to the scenario of heterogeneous nucleation [16], where bubbles nucleate at *ad hoc* impurities.

Let us now investigate bubble nucleation in a Universe with spatially inhomogeneous temperature distribution. Bubble nucleation effectively takes place while the temperature drops by the tiny amount $\Delta_{\rm nuc}$. To determine the mechanism of nucleation, we compare $\Delta_{\rm nuc}$ with the rms temperature fluctuation $\Delta_T^{\rm rms}$: (1) If $\Delta_T^{\rm rms} < \Delta_{\rm nuc}$, the probability to nucleate a bubble at a given time is *homogeneous* in space. This is the case of homogeneous nucleation. (2) If $\Delta_T^{\rm rms} > \Delta_{\rm nuc}$, the probability to nucleate a bubble at a given time is *inhomogeneous* in space. We call this inhomogeneous nucleation.

The quenched lattice QCD data and a COBE normalized flat spectrum lead to the values $\Delta_{\rm nuc} \sim 10^{-6}$ and $\Delta_T^{\rm rms} \sim 10^{-5}$. We conclude that the cosmological QCD transition may proceed via inhomogeneous nucleation. A sketch of inhomogeneous nucleation is shown in Fig. 1. The basic idea is that temperature inhomogeneities determine the location of bubble nucleation. Bubbles nucleate first in the cold regions.

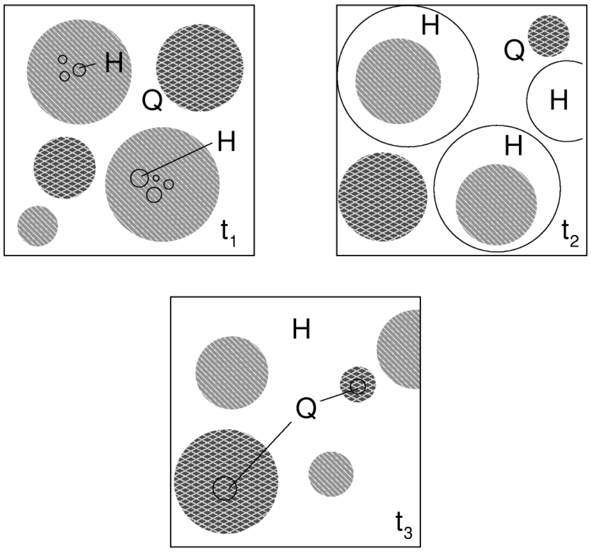

FIG. 1. Sketch of a first-order QCD transition in the inhomogeneous Universe. At t_1 the first hadronic bubbles (H) nucleate at the coldest spots (light gray), while most of the Universe remains in the quark phase (Q). At t_2 the bubbles inside the cold spots have merged and have grown to bubbles as large as the temperature fluctuation scale. At t_3 the transition is almost finished. The last quark droplets are found in the hottest spots (dark gray).

The temperature change at a given point is governed by the Hubble expansion and by the temperature fluctuations. For the fastest changing fluctuations, with angular frequency c_s/l_{smooth}, we find

$$\frac{dT(t,\mathbf{x})}{dt} = \frac{\bar{T}}{t_H}\left[-3c_s^2 + \mathcal{O}\left(\Delta_T \frac{t_H}{\delta t}\right)\right]. \tag{9}$$

The Hubble expansion is the dominant contribution, as typical values are $3c_s^2 = 0.1$ from quenched lattice QCD and $\Delta_T^{\mathrm{rms}} t_H/\delta t \approx 0.01$ from the discussion above. This means that the local temperature never does increase, except by the released latent heat during bubble growth.

To gain some insight in the physics of inhomogeneous nucleation, let us first inspect a simplified case. We have some randomly distributed cold spheres of diameter l_{smooth} with equal and uniform temperature, which is by the amount $\Delta_T^{\mathrm{rms}} T_c$ smaller than the again uniform temperature in the rest of the Universe. When the temperature in the cold spots has dropped to T_f, homogeneous nucleation takes place in them. Because of the Hubble expansion the rest of the Universe would need the time

$$\Delta t_{\mathrm{cool}} = \frac{\Delta_T^{\mathrm{rms}}}{3c_s^2}\, t_H \tag{10}$$

to cool down to T_f. Inside each cold spot there is a large number of tiny hadron bubbles, assumed to grow as deflagrations. They merge within Δt_{cool} if $\Delta_{\mathrm{nuc}} < (v_{\mathrm{def}}/v_{\mathrm{heat}})\Delta_T^{\mathrm{rms}}$. This condition should be clearly fulfilled for our reference set of parameters. Thus the cold spots have been fully transformed into the hadron phase while the rest of the Universe still is in the quark phase. The latent heat released in a cold spot propagates in all directions, which provides the length scale

$$l_{\mathrm{heat}} \equiv 2v_{\mathrm{heat}}\Delta t_{\mathrm{cool}}\,. \tag{11}$$

If the typical distance from the boundary of a cold spot to the boundary of a neighboring cold spot is less than l_{heat}, then no hadronic bubbles can nucleate in the intervening space. In this case the nucleation process is totally dominated by the cold spots, and the average distance between their centers gives the spatial scale for the resulting inhomogeneities. In the following analysis for a more realistic scenario we concentrate in this case, $l_{\mathrm{heat}} > l_{\mathrm{smooth}}$.

The real Universe consists of smooth patches of typical linear size l_{smooth}, their temperatures given by the distribution (6). As discussed above, the merging of tiny bubbles within a cold spot can here be treated as an instantaneous process. The fraction of space that is not reheated by the released latent heat (and not transformed to hadron phase) is given at time t by

$$f(t) \approx 1 - \int_0^t \Gamma_{\mathrm{ihn}}(t')V(t,t')\,dt', \tag{12}$$

where we neglect overlap and merging of heat fronts. At time t heat, coming from a cold spot which was transformed into hadron phase at time t', occupies the volume $V(t,t') = (4\pi/3)[l_{\mathrm{smooth}}/2 + v_{\mathrm{heat}}(t - t')]^3$. The other factor in Eq. (12), Γ_{ihn}, is the rate per volume at which smooth patches transform into the new phase, as a function of the mean temperature $T = \bar{T}(t)$. Γ_{ihn} is proportional to the fraction of space for which temperature is in the interval $[T_f, T_f(1 + d\Delta)]$. This fraction of space is given by Eq. (6) with $\Delta = T_f/T - 1$. Rewriting $d\Delta$ by means of Eq. (3) leads to the expression

$$\Gamma_{\mathrm{ihn}} = 3c_s^2 \frac{T_f}{T}\frac{1}{t_H \mathcal{V}_{\mathrm{smooth}}}\, p\left(\Delta = \frac{T_f}{T} - 1\right), \tag{13}$$

where the relevant physical volume is $\mathcal{V}_{\mathrm{smooth}} = (4\pi/3)\,(l_{\mathrm{smooth}}/2)^3$.

The end of the nucleation period, t_{ihn}, is defined through the condition $f(t_{\mathrm{ihn}}) = 0$. We introduce the variables $N \equiv (1 - T_f/T)/\Delta_T^{\mathrm{rms}}$ and $\mathcal{N} \equiv N(t_{\mathrm{ihn}})$. Since c_s may be assumed to be constant during the tiny temperature interval where nucleations actually take place, we find from Eq. (3): $1 - t/t_{\mathrm{ihn}} \approx 2/(3c_s^2)\Delta_T^{\mathrm{rms}}(N - \mathcal{N})$. Putting everything together we determine $\mathcal{N}$ from

$$\frac{l_{\mathrm{heat}}^3}{l_{\mathrm{smooth}}^3}\int_{\mathcal{N}}^{\infty} dN\, \frac{e^{-(1/2)N^2}}{\sqrt{2\pi}}\left(\frac{l_{\mathrm{smooth}}}{l_{\mathrm{heat}}} + N - \mathcal{N}\right)^3 = 1\,. \tag{14}$$

The COBE normalized spectrum gives $l_{\mathrm{heat}}/l_{\mathrm{smooth}} = 2v_{\mathrm{heat}}(3c_s^2)^{-1/4}(k/k_0)^{(n-1)/2}$. For $l_{\mathrm{heat}}/l_{\mathrm{smooth}} = 1, 2, 5, 10$ we find $\mathcal{N} \approx 0.8, 1.4, 2.1, 2.6$, respectively.

The effective nucleation distance in inhomogeneous nucleation is defined from the number density of those cold spots that acted as nucleation centers, $d_{\mathrm{nuc,ihn}} \equiv n^{-1/3}$.

We find

$$d_{\mathrm{nuc,ihn}} \approx \left[\int_0^{t_{\mathrm{ihn}}} \Gamma_{\mathrm{ihn}}(t)\, dt \right]^{-1/3} \qquad (15)$$

$$= \left\{ \frac{3}{\pi} [1 - \mathrm{erf}(\mathcal{N}/\sqrt{2})] \right\}^{-1/3} l_{\mathrm{smooth}}\,. \qquad (16)$$

With the above values $l_{\mathrm{heat}}/l_{\mathrm{smooth}} = 1, 2, 5, 10$ we get $d_{\mathrm{nuc,ihn}} = 1.4, 1.8, 3.0, 4.8 \times l_{\mathrm{smooth}}$, where $l_{\mathrm{smooth}} \approx 1$ m.

For a COBE normalized spectrum without any tilt and with a tilt of $n - 1 = 0.2$ [where $(k_{\mathrm{smooth}}/k_0)^{0.1} \approx 25$], together with $3c_s^2 = 0.1$ and $v_{\mathrm{heat}} = 0.1$, we find the estimate $l_{\mathrm{heat}}/l_{\mathrm{smooth}} \approx 0.4$ and 9, correspondingly. Notice that the values of v_{heat} and $3c_s^2$ are in principle unknown. Anyway, we can conclude that the case $l_{\mathrm{heat}} > l_{\mathrm{smooth}}$ is a realistic possibility.

With $2v_{\mathrm{heat}}(3c_s^2)^{-1/4}(10^{-4}d_H/l_{\mathrm{smooth}}) < 1$ and without positive tilt we are in the region $l_{\mathrm{heat}} < l_{\mathrm{smooth}}$, where the geometry is more complicated and the above quantitative analysis does not apply. In this situation nucleations take place in the most common cold spots ($\mathcal{N} \sim 1$), which are very close to each other. We expect a structure of interconnected baryon-depleted and baryon-enriched layers with typical surface l_{smooth}^2 and thickness $l_{\mathrm{def}} \equiv v_{\mathrm{def}}\Delta t_{\mathrm{cool}}$. In between $d_{\mathrm{nuc,hom}}$ would be the relevant length scale of inhomogeneities. An accurate analysis of this case requires computer simulations, which is beyond the scope of the present work. However, it is clear that the result will be different compared with homogeneous nucleation.

We emphasize that inhomogeneous and heterogeneous nucleation [16] are genuinely different mechanisms, although they give the same typical scale of a few meters by chance. If latent heat and surface tension of QCD would turn out to reduce Δ_{sc} to, e.g., 10^{-6}, instead of 10^{-4}, the maximal heterogeneous nucleation distance would fall to the centimeter scale, whereas on the distance in inhomogeneous nucleation this would have no effect.

We have shown that inhomogeneous nucleation during the QCD transition can give rise to an inhomogeneity scale exceeding the proton diffusion scale. The resulting baryon inhomogeneities could provide inhomogeneous initial conditions for nucleosynthesis. Observable deviations from the element abundances predicted by homogeneous nucleosynthesis seem to be possible in that case [4,5].

In conclusion, we found that inhomogeneous nucleation leads to nucleation distances that exceed by 2 orders of magnitude estimates based on homogeneous nucleation. We emphasize that this new effect appears for the (today) most probable range of cosmological and QCD parameters.

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